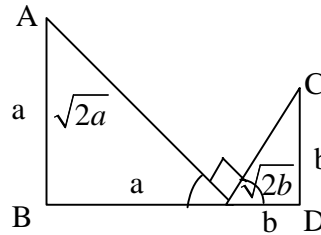# HEIGHTS AND DISTANCES

PREVIOUS EAMCET BITS

1. P is a point on the segment joining the feet of two vertical poles of heights a and b. The angles of elevation of the tops of the poles from P are 45° each. Then the square of the distance between the tops of the poles is **[EAMCET 2009]**

   1) $\dfrac{a^2 + b^2}{2}$
   2) $a^2 + b^2$
   3) $2(a^2 + b^2)$
   4) $4(a^2 + b^2)$

   Ans: 3

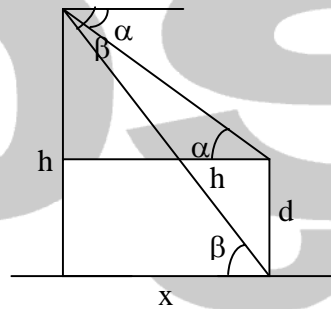   Sol. $AC^2 = (\sqrt{2}a)^2 + (\sqrt{2}b)^2$

   $2(a^2 + b^2)$

2. From the top of the hill h meters high the angles of depressions of the top and the bottom of a pillar are α and β respectively. The height (in metres) of the pillar is **[EAMCET 2008]**

   1) $\dfrac{h(\tan\beta - \tan\alpha)}{\tan\beta}$
   2) $\dfrac{h(\tan\alpha - \tan\beta)}{\tan\alpha}$
   3) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta}$
   4) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\alpha}$

   Ans: 1

   Sol. $x = h\cot\beta, x = (h-d)\cot\alpha$

   $\Rightarrow h\cot\beta = (h-d)\cot\alpha$

   $\Rightarrow h\tan\alpha = (h-d)\tan\beta$

   $\Rightarrow d\tan\beta = h(\tan\beta - \tan\alpha)$

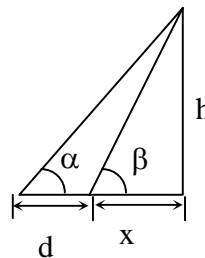 $\Rightarrow d = h\dfrac{(\tan\beta - \tan\alpha)}{\tan\beta}$

3. The angle of elevation of an object from a point P on the level ground is α. Moving d metres on the ground towards the object, the angle of elevation is found to be β. Then the height (in metres) of the object is **[EAMCET 2007]**

   1) $d\tan\alpha$
   2) $d\tan\beta$
   3) $\dfrac{d}{\cot\alpha + \cot\beta}$
   4) $\dfrac{d}{\cot\alpha - \cot\beta}$

   Ans: 4

   Sol. $h\cot\alpha = d + x$

   $h\cot\beta = x$

   $d = h(\cot\alpha + \cot\beta)$

   $\therefore h = \dfrac{d}{\cot\alpha - \cot\beta}$

4. The locus of the point $z = x + iy$ satisfying the equation $\left|\dfrac{z-1}{z+1}\right| = 1$ is given by **[EAMCET 2006]**

1

1) x = 0   2) y = 0   3) x = y   4) x + y = 0

Ans: 1

Sol. $|z-1|^2 = |z+1|^2$

$(x-1)^2 + y^2 = (x+1)^2 + y^2$

$\Rightarrow 4x = 0 \Rightarrow x = 0$

5. The product of the distinct $(2n)^{th}$ roots of $1+i\sqrt{3}$ is equal to   **[EAMCET 2006]**

1) 0   2) $-1-i\sqrt{3}$ 3) $1+i\sqrt{3}$ 4) $-1+i\sqrt{3}$

Ans: 2

Sol. by substitution method put n = 1

Then $\left(1+i\sqrt{3}\right)^{\frac{1}{2}} = \left(2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right)^{1/2} = 2^{\frac{1}{2}}\left(cis\frac{\pi}{3}\right)^{\frac{1}{2}}$

$= 2^{\frac{1}{2}}cis\left(2k\pi+\frac{\pi}{3}\right)^{\frac{1}{2}}$

If k = 0, $\alpha_1 = 2^{\frac{1}{2}cis\frac{\pi}{6}}$

k = 0, $\alpha_2 = 2^{1/2}cis\left(\pi+\frac{\pi}{6}\right) = 2^{1/2}cis\frac{7\pi}{6}$

Product of roots $\alpha_1\alpha_2 = 2^{1/2}2^{1/2}cis\frac{\pi}{6}.cis\left(\frac{7\pi}{6}\right)$

$= 2cis\left(\frac{\pi}{6}+\frac{7\pi}{6}\right)$

$= 2cis\frac{8\pi}{6} = 2cis\left(\frac{4\pi}{3}\right)$

$= -1-i\sqrt{3}$

6. A tower, of x meters high, has a Flagstaff at its top. The tower and the Flagstaff subtend equal angle at a point distant y metres from the foot of the tower. Then the length of the Flagstaff in metres is   **[EAMCET 2005]**
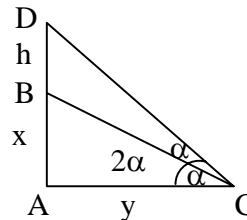
1) $\dfrac{y\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)}$ 2) $\dfrac{x\left(y^2 + x^2\right)}{\left(y^2 - x^2\right)}$ 3) $\dfrac{x\left(x^2 + y^2\right)}{\left(x^2 - y^2\right)}$ 4) $\dfrac{x\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)}$

Ans: 2

Sol. $\tan 2\alpha = \dfrac{x+h}{y}, \tan\alpha = \dfrac{x}{y}$

Use tan(2α) formula

Then $h = \dfrac{x\left(y^2 + x^2\right)}{\left(y^2 - x^2\right)}$

7. An aeroplane flying with uniform speed horizontally one kilometer above the ground is observed at an elevation of 60°. After 10 seconds if the elevation is observed to be 30°, then the speed of the plane (in km/hr) is   **[EAMCET 2004]**

1) $\dfrac{240}{\sqrt{3}}$ 2) $200\sqrt{3}$ 3) $240\sqrt{3}$ 4) $\dfrac{120}{\sqrt{3}}$
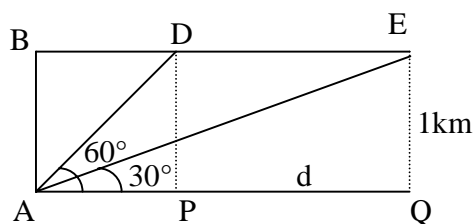
Ans: 3

Sol. In $\triangle APD \Rightarrow \tan 60° = \dfrac{1}{AP} \Rightarrow AP = \dfrac{1}{\sqrt{3}}$

$\Rightarrow AP + PQ = \sqrt{3}$

$PQ = \sqrt{3} - \dfrac{1}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$ km

$10\sec - \dfrac{2}{\sqrt{3}}$ km

$1hr - \dfrac{2}{\sqrt{3}} \times \dfrac{3600}{10} = 240\sqrt{3}$ km / hr

8. A tower subtends angles, α, 2α and 3α respectively at points A, B and C, all lying on a horizontal line through the foot of the tower. Then $\dfrac{AB}{BC} = \ldots\ldots$ **[EAMCET 2003]**

1) $\dfrac{\sin 3\alpha}{\sin 2\alpha}$ 2) $1 + 2\cos 2\alpha$ 3) $2\cos 2\alpha$ 4) $\dfrac{\sin 2\alpha}{\sin \alpha}$

Ans: 2

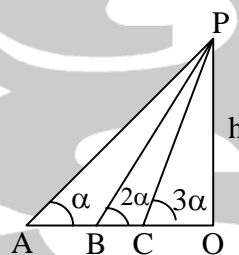Sol. Let height of the tower OP = h

$AB = OA - OB = h(\cot\alpha - \cot 2\alpha)$

$BC = OB - OC = h(\cot 2\alpha - \cot 3\alpha)$

$\dfrac{AB}{BC} = \dfrac{\cot\alpha - \cot 2\alpha}{\cot 2\alpha - \cot 3\alpha}$

$= \dfrac{\cos\alpha \sin 2\alpha - \cos 2\alpha \sin\alpha}{\sin\alpha \sin 2\alpha} \times \dfrac{\sin 2\alpha . \sin 3\alpha}{\cos 2\alpha \sin 3\alpha - \cos 3\alpha \sin 2\alpha}$

$= \dfrac{\sin 3\alpha}{\sin\alpha} = 1 + 2\cos 2\alpha$

9. From a point on the level ground, the angle of elevation of the top of a pole is 30°. On moving 20 mts nearer, the angle of elevation is 45°. Then the height of the pole in mts is

[EAMCET 2002]

1) $10\left(\sqrt{3} - 1\right)$ 2) $10\left(\sqrt{3} + 1\right)$ 3) 15   4) 20

Ans: 2

Sol. $\tan 30° = \dfrac{h}{h + 20}$

$\Rightarrow h = 10\left(\sqrt{3} + 1\right)$

10. The shadow of the two standing on a level ground is found to be 60 metres longer when the sun's altitude is 30° then when it is 45°. The height of the tower is **[EAMCET 2001]**
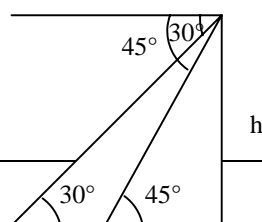
1) 60 m   2) 30 m   3) 60 $\sqrt{3}$ m   4) $30\left(\sqrt{3} + 1\right)$ m

Ans: 4

Sol. $h = \dfrac{60}{\cot 30° - \cot 45°}$

$h = \dfrac{60}{\sqrt{3} - 1} = 30\left(\sqrt{3} + 1\right)$

11. If two towers of height h_1 and h_2 subtend angles 60° and 30° respectively at the midpoint of the line joining their feet, then $h_1 : h_2 =$ **[EAMCET 2000]**

    1) 1 : 2   2) 1 : 3   3) 2 : 1   4) 3 : 1

    Ans: 4

Sol. $h_1 : h_2 = \tan\alpha : \tan\beta$

    $= \tan 60° : \tan 30°$

    $= 3 : 1$

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