FUNCTIONS

PREVIOUS EAMCET BITS

1. If f: [2, 3] $\to$ IR is defined by $f(x) = x^3 + 3x - 2$, then the range f(x) is contained in the interval : **[EAMCET 2009]**

   1) $[1,12]$ 2) $[12,34]$ 3) $[35,50]$ 4) $[-12,12]$

   Ans: 2

Sol. $f(2) = 12$ and $f(3) = 34$

$\therefore$ Range = [12, 34]

2. $\left\{x \in IR : \frac{2x-1}{x^3+4x^2+3x} \in IR\right\} =$ **[EAMCET 2009]**

   1) $IR - \{0\}$ 2) $IR - \{0,1,3\}$ 3) $IR - \{0,-1,-3\}$ 4) $IR - \left\{0,-1,-3,+\frac{1}{2}\right\}$

   Ans: 3

Sol. $\frac{2x-1}{x\left(x^2+4x+3\right)} = \frac{2x-1}{x(x+1)(x+3)}$

is not defined if $x(x+1)(x+3) = 0 \quad \Rightarrow x = -3, -1, 0$

3. Using mathematical induction, the numbers a_n 's are defined by $a_0 = 1, a_{n+1} = 3n^2 + n + a_n \ (n \geq 0)$, then a_n = **[EAMCET 2009]**

   1) $n^3 + n^2 + 1$ 2) $n^3 - n^2 + 1$ 3) $n^3 - n^2$ 4) $n^3 + n^2$

   Ans: 2

Sol. $a_0 = 1, a_1 = 1, a_2 = 3 + 1 + a_1 = 5$ and so on. Verify (2) is correct

4. The number of subsets of {1, 2, 3, …….9} containing at least one odd number is **[EAMCET 2009]**

   1) 324 2) 396 3) 496 4) 512

   Ans: 3

Sol. Number of subsets $= 2^9 - 2^4 = 512 - 16 = 46$

4. If $\mathbb{R} \to C$ is defined by $f(x) = e^{2ix}$ for $x \in R$, then f is (where C denotes the set of all complex numbers) **[EAMCET 2008]**

   1) one-one 2) onto 3) one-one and onto 4) neither one-one nor onto

   Ans: 4

Sol. $f(x) = e^{2ix} = \cos 2x + i \sin 2x$

$f(0) = f(\pi) = 1 \Rightarrow f$ is not one one

There exists not $x \in R \ni f(x) = 2 \Rightarrow f$ is not onto.

5. If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = |x|$ and $g(x) = [x-3]$ for $x \in R$, then $\left\{g(f(x)) : -\frac{8}{5} < x < \frac{8}{5}\right\} =$ **[EAMCET 2008]**

   1) [0, 1] 2) [1, 2] 3) {–3, –2} 4) {2, 3}

Ans: 3

Sol. $-\frac{8}{5} < x < \frac{8}{5} \Rightarrow 0 \le |x| < \frac{8}{5} \Rightarrow -3 \le |x| - 3 < \frac{8}{5} - 3$

$\Rightarrow -3 \le |x| - 3 < -\frac{7}{5} \Rightarrow [|x| - 3] = -3 \text{ or } -2$

$\Rightarrow \left\{ g(f(x)) : -\frac{8}{5} < x < \frac{8}{5} \right\} = \{-3, -2\}$

6. If $f : [-6, 6] \to R$ defined by $f(x) = x^2 - 3$ for $x \in R$ then $(fofof)(-1) + (fofof)(0) + (fofof)(1) =$ **[EAMCET 2008]**

1) $f(4\sqrt{2})$ 2) $f(3\sqrt{2})$ 3) $f(2\sqrt{2})$ 4) $f(\sqrt{2})$

Ans: 1

Sol. $(fofof)(-1) + (fofof)(0) + (fofof)(1) = -2 + 33 - 2 = 29$

$f(4\sqrt{2}) = 32 - 3 = 29$

7. If Q denotes the set of all rational numbers and $f\left(\frac{p}{q}\right) = \sqrt{p^2 - q^2}$ for any $\frac{p}{q} \in Q$, then observe the following statements **[EAMCET 2007]**

I) $f\left(\frac{p}{q}\right)$ is real for each $\frac{p}{q} \in Q$

II) $f\left(\frac{p}{q}\right)$ is complex number for each $\frac{p}{q} \in Q$

Which of the following is correct ?

1) Both I and II are true 2) I is true, II is false
3) I is false, II is true 4) Both I and II are false

Ans: 3

Sol. $f\left(\frac{1}{2}\right) = \sqrt{1-4} = \sqrt{-3}$ is an imaginary $\Rightarrow$ I is false

$f\left(\frac{p}{q}\right) = \sqrt{p^2 - q^2}$ it is a complex number $\Rightarrow$ II is true

8. If $f : R \to \mathbb{R}$ is defined by $f(x) = \frac{1}{2 - \cos 3x}$ for each $x \in R$, then the range of f is **[EAMCET 2007]**

1) $\left(\frac{1}{3}, 1\right)$ 2) $\left[\frac{1}{3}, 1\right]$ 3) (1, 2) 4) [1, 2]

Ans: 2

Sol. Max. and Min. values of 2 – cos3x are 3 and 1

$\therefore$ Range = $\left[\frac{1}{3}, 1\right]$

9. If $f : R \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by f(x) = x – [x] and g(x) = [x] for $x \in \mathbb{R}$, where [x] is The greatest integer not exceeding x, then for every $x \in \mathbb{R}$, $f(g(x)) =$ **[EAMCET 2007]**

1) x 2) 0 3) f(x) 4) g(x)

Ans: 2

Sol. $f\left(g\left(x\right)\right)$

$= g\left(x\right) - \left[g\left(x\right)\right]$

$= \left[x\right] - \left[x\right] = 0$

10. If $f = \mathbb{R} \to \mathbb{R}$ is defined by $f\left(x\right) = x - \left[x\right] - \frac{1}{2}$ for $x \in \mathbb{R}$, where [x] is the greatest integer not exceeding x, then $\left\{x \in \mathbb{R} : f\left(x\right) = \frac{1}{2}\right\} = \ldots\ldots$ **[EAMCET 2006]**

1) Z, the set of all integers 2) IN, the set of all natural number

3) ϕ, the empty set 4) $\mathbb{R}$

Ans: 3

Sol. $f\left(x\right) = x - \left[x\right] - \frac{1}{2}, x \in \mathbb{R}$

$f\left(x\right) = \frac{1}{2}$

$\Rightarrow x - \left[x\right] - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow x - \left[x\right] = 1$

$\Rightarrow \left\{x\right\} = 1$ which is not possible, where {x} denotes the fractional part

11. If $f = \mathbb{R} \to \mathbb{R}$ is defined by $f\left(x\right) = \left[2x\right] - 2\left[x\right]$ for $x \in \mathbb{R}$. where [x] is the greatest integer not exceeding x, then the range of f is **[EAMCET 2006]**

1) $\left\{x \in \mathbb{R} : 0 \le x \le 1\right\}$ 2) {0, 1}

3) $\left\{x \in \mathbb{R} : x > 0\right\}$ 4) $\left\{x \in \mathbb{R} : x \le 0\right\}$

Ans: 2

Sol. $f\left(x\right) = \left[2x\right] - 2\left[x\right], x \in \mathbb{R} = 0$

$= \forall x \in \mathbb{R}$ where $x = a + f$

$\ni 0 < f < 0.5$

$= 1, \forall x \in \mathbb{R}$

$x = a + f$ where $0.5 \le a < 1$

$\therefore$ Range = {0, 1}

12. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f\left(x\right) = \begin{cases} x + 4 & \text{for} \quad x < -4 \\ 3x + 2 & \text{for} \quad -4 \le x < 4 \\ x - 4 & \text{for} \quad x \ge 4 \end{cases}$ then the correct matching of List I from List II is **[EAMCET 2006]**

List – I	List – II
A) $f\left(-5\right) + f\left(-4\right)$	i) 14
B) $f\left(\left\|f\left(-8\right)\right\|\right)$	ii) 4
C) $f\left(f\left(-7\right) + f\left(3\right)\right)$	iii) – 11

D) $f\left(f\left(f\left(f(0)\right)\right)\right)+1$ iv) – 1

v) 1

vi) 0

	A	B	C	D		A	B	C	D
1)	iii	vi	ii	v	2)	iii	iv	ii	v
3)	iv	iii	ii	i	4)	iii	vi	v	ii

Ans: 1

Sol. $(A)f(-5)+f(-4)=(-5+4)+3(-4)+2=-11$

$(B)f(-8+4)=f(-4)=3\Rightarrow f(4)=0$

$(C)f\left[(-3)+11\right]=f(8)=4$

$(D)f\left(f\left(f(2)\right)\right)=f\left(f(8)\right)+1=f(4)+1=0+1=1$

13. $\left\{x\in\mathbb{R}:\left[x-|x|\right]=5\right\}=$ **[EAMCET 2005]**

1) $\mathbb{R}$, the set of all real numbers 2) ϕ, the empty set

3) $\left\{x\in\mathbb{R}:x<0\right\}$ 4) $\left\{x\in\mathbb{R}:x>0\right\}$

Ans: 2

Sol. $x-|x|=2x,\forall x<0$

$=0,\forall x\geq 0$

$\therefore$ x - |x| $\neq 5$

14. The function $f:c\to c$ defined by $f(x)=\dfrac{ax+b}{cx+d}$ for $x\in c$ where $bd\neq 0$ reduces to a constant function if **[EAMCET 2005]**

1) a = c 2) b = d 3) ad = bc 4) ab = cd

Ans: 3

Sol. $f(x)=\dfrac{ax+b}{cx+d}$

$cx+d)\,ax+b\,(a/c$

$\underline{ax+ad/c}$

$\underline{bc-ad}$

c

$f(x)=\dfrac{a}{c}+\dfrac{bc-ad}{c(cx+d)}$ = constant bc = ad

2004

15. For any integer $n\geq 1$, the number of positive divisors of n is denoted by d(n). Then for a prime P, $d\left(d\left(d\left(P^7\right)\right)\right)=$ **[EAMCET 2004]**

1) 1 2) 2 3) 3 4) P

Ans: 3

Sol. $d\left(d\left(d\left(p^7\right)\right)\right)=d\left(d(8)\right)=d\left(d\left(2^3\right)\right)=d(4)$

$=d\left(2^2\right)=2+1=3$

16. If $f : N \to Z$ is defined by $f(n) = \begin{cases} 2 & \text{if} \quad n = 3k, k \in Z \\ 10 & \text{if} \quad n = 3k+1, k \in Z \\ 0 & \text{if} \quad n = 3k+2, k \in Z \end{cases}$ then $\{n \in N : f(n) > 2\} =$ **[EAMCET 2004]**

1) {3, 6, 4} 2) {1, 4, 7} 3) {4, 7} 4) {7}

Ans: 2

Sol. $f(n) > 2 \Rightarrow n = 3k+1$

$\Rightarrow n = 1; n = 4; n = 7$

17. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3^{-x}$. Observe the following statements of it :

I. f is one-one II) f is onto III) f is a decreasing function **[EAMCET 2004]**

Out of these, true statements are

1) only I, II 2) only II, III 3) only I, III 4) I, II, III

Ans:

Sol. $f : R \to R; f(x) = 3^{-x}$

$\therefore$ f(x) is one-one and it is decreasing function

18. If $f(x) = \begin{cases} [x] & \text{if} \quad -3 < x \le -1 \\ |x| & \text{if} \quad 1 < x < 1 \\ [[x]] & \text{if} \quad 1 \le x < 3 \end{cases}$, then $(x : f(x) \ge 0) =$ **[EAMCET 2004]**

1) (–1, 3) 2) [–1, 3) 3) (–1, 3] 4) [–1, 3]

Ans: 1

Sol. Verification

19. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are definite by f(x) = 2x + 3 and g(x) = x^2 + 7 then the values of x such that g(f(x)) = 8 are **[EAMCET 2003]**

1) 1, 2 2) –1, 2 3) –1, –2 4) 1, –2

Ans: 3

Sol. $g(f(x)) = 4x^2 + 12x + 16$

$\Rightarrow 4x^2 + 12x + 16 = 8$

$\Rightarrow (x+1)(x+2) = 0 \Rightarrow x = -1, -2$

20. Suppose $f : [-2, 2] \to \mathbb{R}$ is defined $f(x) = \begin{cases} -1 & \text{for} -2 \le x \le 0 \\ x-1 & \text{for}\, 0 \le x \le 2 \end{cases}$,

then $\{x \in [-2, 2] : x \le 0 \text{ and } f(|x|) = x\} = ...$ **[EAMCET 2003]**

1) {–1} 2) {0} 3) $\left\{-\frac{1}{2}\right\}$ 4) ϕ

Ans: 3

Sol. Now take $x = -\frac{1}{2}$

$\therefore f\left(\left|-\frac{1}{2}\right|\right) = f\left(\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

Hence f(|x|) = x

$\therefore$ Domain of $f(x) = \left\{-\frac{1}{2}\right\}$

21. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are given f(x) = |x| and g(x) = [x] for each $\{x \in \mathbb{R} : g(f(x)) \le f(g(x))\}$ **[EAMCET 2003]**

1) $z \cup (-\infty, 0)$ 2) $(-\infty, 0)$ 3) z 4) $\mathbb{R}$

Ans: 4

Sol. $f(x) = |x|; g(x) = [x]$

$g(f(x)) \le f(g(x))$

$g(f(x)) = g(|x|) = [|x|] = [x]$

$f(g(x)) = f\,|[x]| = |[x]|$

$[x] \le |[x]|$

$\therefore x \in \mathbb{R}$

22. If $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$, then $f(a) =$ **[EAMCET 2002]**

1) a 2) 0 3) 1 4) – 1

Ans: 2

Sol. $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$

$f'(x) = \frac{1}{2\sqrt{ax}}.a + a^2\left[-\frac{1}{2}(ax)^{-3/2} a\right]$

$f'(a) = \frac{a}{2a} - \frac{a^3.a^{-3}}{2} = 0$

23. If $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$ for $x \in R$, then f(2002)= **[EAMCET 2002]**

1) 1 2) 2 3) 3 4) 4

Ans: 1

Sol. $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$

$= \frac{1 - \frac{1}{4}\sin^2 2x}{1 - \frac{1}{4}\sin^{2x}} = 1$

$\Rightarrow f(2002) = 1$

24. The function $f : R \to R$ is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$ .Then f(R) =

[EAMCET 2002]

1) $\left(\frac{3}{4}, 1\right]$ 2) $\left[\frac{3}{4}, 1\right)$ 3) $\left[\frac{3}{4}, 1\right]$ 4) $\left(\frac{3}{4}, 1\right)$

Ans: 3

Sol. $f(x)=\cos^2 x+\sin^4 x$

$=\cos^2 x+\sin^2 x\left(1-\cos^2 x\right)$

$=1-\frac{1}{4}\sin^2 2x$

$\sin^2 2x\in[0,1]$

$\therefore$ Maximum of f(x) = $1-\frac{1}{4}(0)=1$

Minimum of f(x) = $1-\frac{1}{4}(1)=\frac{3}{4}$

$\therefore$ Range of f(x) = $\left[\frac{3}{4},1\right]$

25. If the functions f and g are defined by $f(x)=3x-4, g(x)=2+3x$ for $x\in\mathbb{R}$ respectively, then $g^{-1}\left(f^{-1}(5)\right)=$ **[EAMCET 2002]**

1) 1 2) 1/2 3) 1/3 4) 1/4

Ans: 3

Sol. $f^{-1}(x)=\frac{x+4}{3}, g^{-1}(x)=\frac{x-2}{3}$

$f^{-1}(5)=3$ $g^{-1}\left(f^{-1}(5)\right)=g^{-1}(3)=\frac{1}{3}$

26. If $f(x)=\left(25-x^4\right)^{1/4}$ for $0<x<\sqrt{5}$ then $f\left[f\left(\frac{1}{2}\right)\right]=$ **[EAMCET 2001]**

1) 2^{-4} 2) 2^{-3} 3) 2^{-2} 4) 2^{-1}

Ans: 4

Sol. $f(x)=\left(25-x^4\right)^{1/4}$

$\Rightarrow f\left(f(x)\right)=\left[25-\left(25-x^4\right)\right]^{1/4}=x$

$\therefore f\left(f(1/2)\right)=\frac{1}{2}=2^{-1}$

27. Let z denote the set of all integers Define f : z $\to$ z by $f(x)=\begin{cases} x/2 & (x \text{ is even}) \\ 0 & (x \text{ is odd}) \end{cases}$. Then f is = **[EAMCET 2001]**

1) On to but not one-one 2) One –one but not onto

3) One-one and onto 4) Neither one-one nor onto

Ans: 1

Sol. ---

28. Let $f:R\to R$ be defined by $f(x)=\begin{cases} x+2 & (x\le -1) \\ x^2 & (-1\le x\le 1) \\ 2-x & (x\ge 1) \end{cases}$. Then the value of f(–1.75)+f(0.5) + f(1.5) is **[EAMCET 2001]**

1) 0 2) 2 3) 1 4) – 1

Ans: 3

Sol. $f(-1.75)+f(0.5)+f(1.5)$

$=(-1.75+2)+(0.5)^2+2-1.5=1$

29. The functions $f:\mathbb{R}\to\mathbb{R}, g:\mathbb{R}\to\mathbb{R}$ are defined as follows: **[EAMCET 2001]**

$f(x)=\begin{cases}0 & (x \text{ rational})\\ 1 & (x \text{ irrational})\end{cases}$; $g(x)=\begin{cases}-1 & (x \text{ rational})\\ 0 & (x \text{ irrational})\end{cases}$. The (f0g) (π) + (gof) (e) =

1) –1 2) 0 3) 1 4) 2

Ans: 1

Sol. O

$=f(0)+g(1)$ ($\because$ π and e are irrationals)

= 0 – 1 = - 1

30. If f : R →R is defined by f(x) = 2x + |x|, then f(2x) + f(–x) – f(x) = **[EAMCET 2000]**

1) 2x 2) 2|x| 3) –2x 4) –2|x|

Ans: 2

Sol. $f(x)=2x+|x|$

$\therefore\ f(2x)+f(-x)-f(x)$

$=2(2x)+|2x|+2(-x)+|-x|-(2x+|x|)=2|x|$

31. If f : R →R and g : R →R are defined by f(x) = 2x + 3 and g(x) = x^2+ 7, then the value of x for which f(g(x)) = 25 are **[EAMCET 2000]**

1) ±1 2) ±2 3) ±3 4) ±4

Ans: 2

Sol. $f(g(x))=25\Rightarrow f(x^2+7)=25$

$\Rightarrow 2(x^2+7)+3=25$

$\therefore\ x=\pm 2$

32. $\{x\in R:|x-2|=x^2\}=$ **[EAMCET 2000]**

1) {–1, 2} 2) {1, 2} 3) {–1, –2} 4) {1, –2}

Ans: 4

Sol. $\{1,-2\}$ satisfies

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