DIRECTION RATIOS AND DIRECTION COSINES

PREVIOUS EAMCET BITS

1. The angle between the lines whose direction cosines satisfy the equation $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$ is **[EAMCET 2009]**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Ans: 3

Sol. $n = -(1 + m)$

$\therefore \ell^2 + m^2 - n^2 = 0 \Rightarrow \ell = 0 (\text{or}) m = 0$

$\ell = 0 \Rightarrow n = -m \Rightarrow \frac{\ell}{0} = \frac{m}{-1} = \frac{n}{1}$

$m = 0 \Rightarrow n = -1 \Rightarrow \frac{\ell}{-1} = \frac{m}{0} = \frac{n}{1}$

$\therefore \cos\theta = \frac{0+0+1}{\sqrt{2}\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = 60°$

60. If a line in the space makes angles α, β and γ with the coordinates axes, then $\cos 2\alpha + \cos^2\beta + \cos 2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma =$ **[EAMCET 2009]**

1) –1 2) 0 3) 1 4) 2

Ans: 3

Sol. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\therefore 1 - 2\sin^2\alpha + 1 - 2\sin^2\beta + 1 - 2\sin^2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= 3 - 2 = 1$

52. The angle between the lines whose direction consines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$ is **[EAMCET 2008]**

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

Ans: 3

Sol. $\cos\theta = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{\sqrt{3}}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = \frac{3}{16} + \frac{1}{16} - \frac{3}{4} = -\frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

52. The cosine of the angle A of the triangle with vertices A(1, –1, 2), B(6, 11, 2), C(1, 2, 6) is **[EAMCET 2007]**

1) $\frac{63}{65}$ 2) $\frac{36}{65}$ 3) $\frac{16}{65}$ 4) $\frac{13}{64}$

Ans: 2

Sol. $\overrightarrow{AB} = 5i + 12J; \overrightarrow{AC} = 3J + 4k$

$$\cos A = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|} = \frac{36}{65}$$

51. If the direction cosines of two lines are such that $\ell + m + n = 0, \ell^2 + m^2 - n^2 = 0$, then the angle between them is **[EAMCET 2006]**

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Ans: 2

Sol. $\ell + m + n = 0 \Rightarrow \ell = -(m+n)$

Substituting

m = 0 or m = – n

$\ell_1 : m_1 : n_1 = -1; 0 : 1$ and $\ell_2 : m_2 : n_2 = 0 : -1 : 1$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

51. The direction cosines of the line passing through P(2, 3, –1) and the origin are **[EAMCET 2005]**

1) $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ 2) $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ 3) $\frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ 4) $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}$

Ans: 3

Sol. Direction cosines are $\pm\frac{x_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}}$,

$\pm\frac{y_1}{\sqrt{x_1^2 + y_1^2 + z_2^2}}, \pm\frac{z_1}{\sqrt{x_1^2 + y_1^2 + z_2^2}}$

52. If the direction ratio of two lines are given by $\ell + m + n = 0$, $mn - 2\ell n + \ell m = 0$, then the angle between the lines is **[EAMCET 2004]**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) 0

Ans: 3

Sol. $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0 \Rightarrow \theta = 90°$

22. If the direction ratios of two lines are given by $3\ell m - 4\ell n + mn = 0$ and $\ell + 2m + 3n = 0$, then the angle between the lines is **[EAMCET 2003]**

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Ans: 1

Sol. 1 = –(2m + 3n)

$-3(2m+3n)m + 4(2m+3n)n + mn = 0$

$\Rightarrow \frac{m}{n} = \pm\sqrt{2}$

$\frac{\ell_1}{n_1} = -2\sqrt{2} - 3 \Rightarrow \frac{\ell_1}{-3-2\sqrt{2}} = \frac{m_1}{\sqrt{2}} = \frac{n_1}{1}$

$\frac{\ell_2}{-3+2\sqrt{2}} = \frac{m_2}{-\sqrt{2}} = \frac{n_2}{1}$

$\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0 \Rightarrow \theta = 90°$

21. The acute angle between the two lines whose direction ratios are given by $\ell + m - n = 0$ and $\ell^2 + m^2 - n^2 = 0$ is **[EAMCET 2002]**

1) 2) 3) 4)

Ans: 4

Sol. n = *l* + m substituting in

$\ell^2 + m^2 = n^2 \Rightarrow \ell m = 0$

If $\ell = 0 \Rightarrow m = n \Rightarrow \frac{\ell}{0} = \frac{m}{1} = \frac{n}{1}$

$m = 0 \Rightarrow \ell = n \Rightarrow \frac{\ell}{1} = \frac{m}{0} = \frac{n}{1}$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} = \frac{0+0+1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$\theta = \frac{\pi}{3}$

22. The direction ratios of a normal to the plane passing through (0, 0, 1) (0, 1, 2) and (1, 2, 3) are **[EAMCET 2002]**

1) $[0,1,-1]$ 2) $[1,0,-1]$ 3) $[0,0,-1]$ 4) $[1,0,0]$

Ans: 1

Sol. $A(0,0,1); B(0,1,2); C(1,2,3)$

$\overline{AB} = (0,1,1)$

$\overline{AC} = (1,2,2)$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = (0,1,-1)$$

21. The direction ratios of two lines are given by a + b + c = 0. 2ab + 2ac – bc = 0. Then the angle between the lines is **[EAMCET 2001]**

1) π 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$

Ans: 2

Sol. Given a + b + c = 0 and 2ab + 2ac – bc = 0

2a(b + c) = bc $\Rightarrow -2(b+c)^2 = bc \quad (\because a = -(b+c))$

$\Rightarrow 2b^2 + 5bc + 2c^2 = 0$

$\Rightarrow (2b+c)(b+2c) = 0$

$\therefore$ C = - 2b (or) – b/2

If C = 2b $\Rightarrow$ a = b

$\Rightarrow$ a : b : c = 1: 1: –2

If C = - b/2 $\Rightarrow$ a = - b /2

$\Rightarrow$ a : b: c = 1 : –2 : 1

If θ is the angle between the lines then $\cos\theta = \frac{(1)(1)+(1)(-2)+(-2)(1)}{\sqrt{1+1+4}\sqrt{1+4+1}} = -\frac{1}{2}$

$\therefore\ \theta = \frac{2\pi}{3}$

22. If a line makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the x–axis and y –axis respectively, then the angle made by the line with z-axis is

[EAMCET 2001]

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{5\pi}{12}$

Ans: 2

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1 \Rightarrow \cos^2\gamma = \frac{1}{4} \quad \therefore \gamma = \frac{\pi}{3}$

24. If P = (0, 1, 2). Q = (4, –2, 1), O = (0, 0, 0), then $\angle POQ =$ **[EAMCET 2001]**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Ans: 4

Sol. $x_1x_2 + y_1y_2 + z_1z_2 = 0 - 2 + 2 = 0$

$\therefore\ \theta = 90°$

24. If $\ell^2 + m^2 = 1$, then the maximum value of $\ell + m$ is **[EAMCET 2000]**

1) 1 2) $\sqrt{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) 2

Ans: 2

Sol. Let $\ell = \cos\theta$ and $m = \sin\theta$

$\ell^2 + m^2 = 1$

The max.value of $\cos\theta + \sin\theta$ is $\sqrt{2}$

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