

DIFFERENTIATION PREVIOUS EAMCET BITS

1. $x = \frac{1-\sqrt{y}}{1+\sqrt{y}} \Rightarrow \frac{dy}{dx} =$

[EAMCET 2009]

- 1) $\frac{4}{(x+1)^2}$ 2) $\frac{4(x-1)}{(1+x)^3}$ 3) $\frac{x-1}{(1+x)^3}$ 4) $\frac{4}{(1+x)^3}$

Ans: 2

Sol. Using Componendo and dividendo, then find y and $\frac{dy}{dx}$

2. $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right), y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right) \Rightarrow \frac{dy}{dx} =$

[EAMCET 2009]

- 1) 0 2) $\tan t$ 3) 1 4) $\sin t \cos t$

Ans: 3

Sol. $x = \tan^{-1} t, y = \tan^{-1} t$

$$\Rightarrow y = x \Rightarrow \frac{dy}{dx} = 1$$

3. $\frac{d}{dx} \left[a \tan^{-1} + b \log \left(\frac{x-1}{x+1} \right) \right] = \frac{1}{x^4-1} \Rightarrow a - 2b =$

[EAMCET 2009]

- 1) 1 2) -1 3) 0 4) 2

Ans: 2

Sol. $\frac{a}{1+x^2} + \frac{b}{x-1} - \frac{b}{x+1} = \frac{1}{x^4-1}$

$$\Rightarrow \frac{a}{x^2+1} + \frac{2b}{x^2-1} = \frac{1}{x^4-1}$$

Put $x = 0; a - 2b = 1$

4. If $x = \left\{ \cos \theta + \log \tan \left(\frac{\theta}{2} \right) \right\}$ and $y = a \sin \theta$ then $\frac{dy}{dx} =$

[EAMCET 2008]

- 1) $\cot \theta$ 2) $\tan \theta$ 3) $\sin \theta$ 4) $\cos \theta$

Ans: 2

Sol. $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \cos \theta}{a \left[-\sin \theta + \frac{1}{\tan(\theta/2)} \times \sec^2 \left(\frac{\theta}{2} \right) \cdot \frac{1}{2} \right]}$

$$= \frac{\cos \theta}{-\sin \theta + \frac{1}{2 \sin \left(\frac{\theta}{2} \right) \cos \left(\frac{\theta}{2} \right)}} = \frac{\cos \theta}{-\sin \theta + \frac{1}{\sin \theta}} = \frac{\cos \theta \sin \theta}{1 - \sin^2 \theta} = \frac{\cos \theta \cdot \sin \theta}{1 - \sin^2 \theta} = \tan \theta$$

5. If $y = \sin(\log_e x)$ then $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} =$ [EAMCET 2008]

- 1) $\sin(\log_e x)$ 2) $\cos(\log_e x)$ 3) y^2 4) $-y$

Ans: 4

Sol. $y = \sin(\log x) \Rightarrow \frac{dy}{dx} = \cos(\log x) \frac{1}{x} \Rightarrow x \frac{dy}{dx} = \cos(\log x)$

$$\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin(\log x) \frac{1}{x} \Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y$$

6. If $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$ then $\frac{dy}{dx}$ [EAMCET 2007]

- 1) $\frac{3y-4x-1}{2y-3x+2}$ 2) $\frac{3y+4x+1}{2y+3x+2}$ 3) $\frac{3y-4x+1}{2y-3x-2}$ 4) $\frac{3y-4x+1}{2y+3x+2}$

Ans: 1

Sol. $\frac{dy}{dx} = \frac{-\frac{\partial f}{\partial x}}{\left(\frac{\partial f}{\partial y}\right)} = \frac{3y-4x-1}{2y-3x+2}$

7. If $y = \log \left\{ \left(\frac{1+x}{1-x} \right)^{1/4} \right\} - \frac{1}{2} \tan^{-1}(x)$, then $\frac{dy}{dx} =$ [EAMCET 2007]

- 1) $\frac{x}{1-x^2}$ 2) $\frac{x^2}{1-x^4}$ 3) $\frac{x}{1+x^4}$ 4) $\frac{x}{1-x^4}$

Ans: 2

Sol. $y = \log \left(\frac{1+x}{1-x} \right)^{1/4} - \frac{1}{2} \tan^{-1}(x)$

$$= \frac{1}{4} (\log(1+x) - \log(1-x)) - \frac{1}{2} \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{4} \left(\frac{1}{1+x} - \frac{1}{1-x} \right) - \frac{1}{2(1+x^2)} = \frac{x^2}{1-x^4}$$

8. $x = \cos \theta, y = \sin 5\theta \Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} =$ [EAMCET 2007]

- 1) $-5y$ 2) $5y$ 3) $25y$ 4) $-25y$

Ans: 4

Sol. $\frac{dy}{dx} = \frac{-5 \cos 5\theta}{\sin \theta} = \frac{-5\sqrt{1-\sin^2 5\theta}}{\sqrt{1-\cos^2 \theta}}$

$$y_1 = -5 \sqrt{\frac{1-y^2}{1-x^2}}$$

$$(1-x^2)y_1^2 = 25(1-y^2) \Rightarrow (1-x^2)y_2 - xy_1 = -25y$$

9. $x^y = y^x \Rightarrow x(x-y \log x) \frac{dy}{dx} =$ [EAMCET 2006]

- 1) $y(y-x \log y)$ 2) $y(y+\log y)$ 3) $x(x+y \log x)$ 4) $x(y-x \log y)$

Ans: 1

Sol. $x^y = y^x$
 $\Rightarrow y \log x = x \log y$
 $\Rightarrow x(x - y \log) \frac{dy}{dx} = y(y - x \log y)$

10. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is an even function which is twice differentiable on \mathbb{R} and $f''(\pi) = 1$, then $f''(-\pi)$
 =
 1) -1 2) 0 3) 1 4) 2
[EAMCET 2005]

Ans: 3

Sol. Consider $f(x) = \frac{x^2}{2}$
 $f'(x) = \frac{2x}{2} = x, f''(x) = 1$
 $f''(\pi) = 1 = f''(-\pi)$

11. Observe the following statements : **[EAMCET 2005]**

I : $f(x) = ax^{41} + bx^{-40} \Rightarrow \frac{f''(x)}{f(x)} = 1640x^{-2}$

II : $\frac{d}{dx} \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \frac{1}{1+x^2}$

Which of the following is correct ?

- 1) I is true, but II is false 2) Both I and II are true
 3) Neither I nor II is true 4) I is false, but II is true

Ans: 1

Sol. I) $f''(x) = 1640ax^{39} + 1640x^{-42} \cdot b$
 $= \frac{1640}{x^2} (ax^{41} + bx^{-40}) = \frac{1640}{x^2} f(x)$
 $\frac{f''(x)}{f(x)} = 1640x^{-2}$ True

II) $\therefore \tan^{-1} \left(\frac{2x}{1-x^2} \right) = 2 \tan^{-1} x$

$\frac{d}{dx} (2 \tan^{-1} x) = \frac{2}{1+x^2}$ false

12. $f(x) = 10 \cos x + (13 + 2x) \sin x \Rightarrow f''(x) + f(x) =$ **[EAMCET 2005]**
 1) $\cos x$ 2) $4 \cos x$ 3) $\sin x$ 4) $4 \sin x$

Ans: 2

Sol. $f'(x) = -10 \sin x + (13 + 2x) \cos x + 2 \sin x$
 $f''(x) = -10 \cos x - (13 + 2x) \sin x + 2 \cos x + 2 \cos x$
 $= -f(x) + 4 \cos x$
 $f''(x) + f(x) = 4 \cos x$

13. $x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow \frac{dy}{dx} =$ **[EAMCET 2005]**

- 1) $\frac{1}{(1+x)^2}$ 2) $\frac{-1}{(1+x)^2}$ 3) $\frac{1}{1+x^2}$ 4) $\frac{1}{1+x^2}$

Ans: 2

Sol. $x\sqrt{1+y} = -y\sqrt{1+x}$

$$x^2 + x^2y = y^2 + y^2x$$

$$x^2 - y^2 = -xy(x - y)$$

$$x + y = -xy$$

$$y = \frac{-x}{1+x} \Rightarrow y' = \frac{-1}{(1+x)^2}$$

14. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is an even function having derivatives of all orders, then an odd function among the following is **[EAMCET 2004]**

- 1) f'' 2) f''' 3) $f' + f''$ 4) $f'' + f'''$

Ans: 2

Sol. f''' is odd, since 'f' is even

15. $x > 0, x^y = e^{x-y} \Rightarrow \frac{dy}{dx} =$ **[EAMCET 2004]**

- 1) $\frac{1}{(1+\log x)^2}$ 2) $\frac{\log x}{(1+\log x)^2}$ 3) $\left(\frac{\log x}{1+\log x}\right)^2$ 4) $\frac{(\log x)^2}{1+\log x}$

Ans: 2

Sol. $x - y = y \log x \Rightarrow y = \frac{x}{1+\log x}$

$$\Rightarrow \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

16. If $f(x) = \begin{cases} \frac{x-1}{3x^2-7x+5} & \text{for } x \neq 0 \\ 1/3 & \text{for } x = 1 \end{cases}$, then $f'(1) =$ **[EAMCET 2003]**

- 1) $-\frac{1}{9}$ 2) $-\frac{2}{9}$ 3) $-\frac{1}{3}$ 4) $\frac{1}{3}$

Ans: 2

Sol. $f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = -2/9$

17. If $f(x) = \frac{x}{1+|x|}$ for $x \in \mathbb{R}$ then $f'(0) = \dots$ **[EAMCET 2003]**

- 1) 0 2) 1 3) 2 4)

Ans: 2

Sol. $f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = 1$

18. Let $f(x) = e^x, g(x) = \sin^{-1} x$ and $h(x) = f(g(x))$, then $\frac{h'(x)}{h(x)} =$ **[EAMCET 2002]**

- 1) $\sin^{-1} x$ 2) $\frac{1}{\sqrt{1-x^2}}$ 3) $\frac{1}{1-x^2}$ 4) $e^{\sin^{-1} x}$

Ans: 2

Sol. $h(x) = f[g(x)] = f(\sin^{-1} x) = e^{\sin^{-1} x}$

$$h(x) = e^{\sin^{-1} x} \Rightarrow h'(x) = \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$$

$$\therefore \frac{h'(x)}{h(x)} = \frac{1}{\sqrt{1-x^2}}$$

19. If $h(x) = e^{e^x}$ then $\frac{h'(x)}{h(x)} =$ **[EAMCET 2001]**

- 1) $h(x)$ 2) $\frac{1}{h(x)}$ 3) $\log h(x)$ 4) $-\log h(x)$

Ans: 3

Sol. Given $h(x) = e^{e^x} \Rightarrow \log(h(x)) = e^x$

$$\Rightarrow \frac{h'(x)}{h(x)} = e^x = \log h(x)$$

20. If $f(x) = \frac{x^2}{x+a}$ then $f''(a) =$ **[EAMCET 2001]**

- 1) $4a$ 2) $\frac{1}{8a}$ 3) $\frac{1}{4a}$ 4) $8a$

Ans: 3

Sol. $f(x) = \frac{x^2}{x+a} = x - a + \frac{a^2}{x+a}$

$$f'(x) = 1 - \frac{a^2}{(x+a)^2}$$

$$f''(x) = \frac{2a^2}{(x+a)^3}$$

$$\therefore f''(a) = \frac{2a^2}{(a+a)^3} = \frac{1}{4a}$$

21. If $y = 2^{2^x}$, then $\frac{dy}{dx} =$ **[EAMCET 2000]**

- 1) $y(\log_{10} 2)^2$ 2) $y(\log_e 2)^2$ 3) $y2^x (\log_e^2)^2$ 4) $y \log_e 2$

Ans: 3

Sol. $y = 2^{2^x}$
 $\Rightarrow \log y = 2^x \log_e^2$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = 2^x \cdot (\log_e^2)^2$$

$$\therefore \frac{dy}{dx} = y \cdot 2^x \cdot (\log_e^2)^2$$

20. $\frac{d}{dx} \left\{ \cos^{-1} \left(\frac{4x^3}{27} - x \right) \right\} =$

[EAMCET 2000]

1) $\frac{3}{\sqrt{9-x^2}}$

2) $\frac{1}{\sqrt{9-x^2}}$

3) $\frac{-3}{\sqrt{9-x^2}}$

4) $\frac{-1}{\sqrt{9-x^2}}$

Ans: 3

Sol. $y = \cos^{-1} \left(\frac{4x^3}{27} - x \right)$

$$= \cos^{-1} \left[4 \left(\frac{x}{3} \right)^3 - 3 \left(\frac{x}{3} \right) \right] = 3 \cos^{-1} \left(\frac{x}{3} \right)$$

$$\therefore \frac{dy}{dx} = \frac{-3}{\sqrt{9-x^2}}$$

ନୀଳମଣି