CONTINUITY

PREVIOUS EAMCET BITS

1. If f = IR $\rightarrow$ IR is defined by $f(x)=\begin{cases}\dfrac{2\sin x-\sin 2x}{2x\cos x} & \text{if } (x\neq 0)\\ a & \text{if } (x=0)\end{cases}$

Then the value of a so that f is continuous at 0 is **[EAMCET 2009]**

1) 2 2) 1 3) – 1 4) 0

Ans: 4

2. If $f:R\rightarrow R$ is defined by $f(x)=\begin{cases}\dfrac{\cos 3x-\cos x}{x^2} & \text{for} \quad x\neq 0\\ \lambda & \text{for} \quad x=0\end{cases}$ and if f is continuous at x = 0,

then λ = **[EAMCET 2008]**

1) – 2 2)– 4 3) – 6 4) – 8

Ans: 2

Sol. f is continuous at

$x=0\Rightarrow \underset{x\to 0}{Lt}\, f(x)=f(0)$

$\Rightarrow \lambda=\underset{x\to 0}{Lt}\dfrac{\cos 3x-\cos x}{x^2}=\underset{x\to 0}{Lt}\dfrac{-3\sin 3x+\sin x}{2x}$

$=\underset{x\to 0}{Lt}\dfrac{-9\cos 3x+\cos x}{2}=\dfrac{-9+1}{2}=-4$

3. If $f(x)=\begin{cases}x-5 & \text{for} \quad x\leq 1\\ 4x^2-9 & \text{for} \quad 1<x<2\\ 3x+4 & \text{for} \quad x\geq 2\end{cases}$ then $f(2^+)-f(2^-)=$ **[EAMCET 2007]**

1) 0 2) 2 3) 3 4) 4

Ans: 3

Sol. $f(2^+)-f(2^-)$

$=(3(2)+4)-\left(4(2)^2-9\right)=3$

4. If $f(x)=\begin{cases}\dfrac{1-\sqrt{2}\sin x}{\pi-4x} & \text{if} \quad x\neq \dfrac{\pi}{4}\\ a & \text{if} \quad x=\dfrac{\pi}{4}\end{cases}$ is continuous at, $\dfrac{\pi}{4}$ then a = **[EAMCET 2006]**

1) 4 2) 2 3) 1 4) 1/4

Ans: 4

Sol. f(x) is continuous at x = $\dfrac{\pi}{4}$

$\lim\limits_{x\to\frac{\pi}{4}} f(x)=f\left(\dfrac{\pi}{4}\right)$

$$a = \lim_{x\to\frac{\pi}{4}} \frac{1-\sqrt{2}\sin x}{\pi - 4x} = \lim_{x\to\frac{\pi}{4}} \frac{f'(x)}{g'(x)}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\cos x}{-4} = \frac{\sqrt{2}}{4}.\frac{1}{\sqrt{2}} = \frac{1}{4}$$

5. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x) = \begin{cases} \frac{x-2}{x^2-3x+2} & \text{if } x \in R-(1,2) \\ 2 & \text{if } x=1 \\ 1 & \text{if } x=2 \end{cases}$ then $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} =$

[EAMCET 2005]

1) 0 2) –1 3)1 4) – 1/2

Ans: 2

Sol. $\underset{x\to 2}{\text{Lt}} \frac{f(x)-f(2)}{x-2} = \underset{x\to 2}{\text{Lt}} \frac{\frac{x-2}{x^2-3x+2}-1}{x-2}$

$= \underset{x\to 2}{\text{Lt}} \frac{\frac{1}{x-1}-1}{x-2} = \underset{x\to 2}{\text{Lt}} \frac{-(x-2)}{x-2} = -1$

6. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x) = \begin{cases} \frac{x+2}{x^2+3x+2} & \text{if } x \in \mathbb{R}\{-1,-2\} \\ -1 & \text{if } x=-2 \\ 0 & \text{if } x=-1 \end{cases}$ then f is continuous on the set

[EAMCET 2005]

1) $\mathbb{R}$ 2) $\mathbb{R}-\{-2\}$ 3) $\mathbb{R}-\{-1\}$ 4) $\mathbb{R}-\{-1,-2\}$

Ans: 3

Sol. $\underset{x\to -1-}{\text{Lt}} f(x) = \underset{x\to -1-}{\text{Lt}} \frac{x+2}{x^2+3x+2} =$

$\underset{x\to -1-}{\text{Lt}} \frac{1}{x+1} = -\infty, \underset{x\to -1+}{\text{Lt}} f(x) = \infty$

$\therefore \underset{x\to -1-}{\text{Lt}} \neq \underset{x\to -1+}{\text{Lt}}$

$\therefore$ f(x) is not continuous at x = – 1

7. If f(x) =- $\begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & \text{for } -1\le x<0 \\ 2x^2+3x-2 & \text{for } 0\le x\le 1 \end{cases}$ is continuous at x = 0, then k = … **[EAMCET 2003]**

1) –4 2) –3 3) – 2 4) –1

Ans: 3

Sol. $\lim_{x\to 0} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}$

$= \lim_{x\to 0}(2x^2+3x-2)$

$= \lim_{x\to 0} \frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}} = -2$

k = – 2

8. If $f : |R \to R|$ defined by $f(x) = \begin{cases} a^2\cos^2 x + b^2 \sin^2 x & (x \le 0) \\ e^{ax+b} & (x > 0) \end{cases}$ is a continuous function, then

[EAMCET 2002]

1) $b = 2\log|a|$ 2) $2b = \log|a|$ 3) $b = \log|2a|$ 4) $b^2 = \log|a|$

Ans: 1

Sol. $\underset{x\to 0}{\text{Lt}}\, f(x) = f(0)$

$= \underset{x\to 0^+}{\text{Lt}}\, e^{ax+b} = a^2(1) + b^2(0)$

$e^b = a^2 \Rightarrow b = 2\log|a|$

9. If $f(x) = \begin{cases} 2x+b & (x < \alpha) \\ x+d & (x \ge \alpha) \end{cases}$ is such that $\lim_{x\to\alpha} f(x) = \ell$, then $\ell =$ **[EAMCET 2001]**

1) 2d – b 2) 2b – d 3) 2d + b 4) b – 2d

Ans: 1

Sol. $\lim_{x\to\alpha^-} f(x) = \lim_{x\to\alpha^+} f(x)$

$\lim_{x\to\alpha^-}(2x+b) = \lim_{x\to\alpha^+}(x+d)$

$2\alpha + b = \alpha + d \Rightarrow \alpha = d - b$

$\therefore \lim_{x\to\alpha}(2x+b) = \ell \Rightarrow 2\alpha + b = \ell$

$\Rightarrow 2(d-b) + b = \ell$

$\therefore \ell = 2d - b$

10. If the function $f(x) = \begin{cases} \frac{\sin 3x}{x} & (x \ne 0) \\ \frac{k}{2} & (x = 0) \end{cases}$ is continuous at x = 0 then k = **[EAMCET 2000]**

1) 3 2) 6 3) 9 4) 12

Ans: 2

Sol. $\lim_{x\to 0}\left(\frac{\sin 3x}{x}\right) = \frac{k}{2} \Rightarrow 3 = \frac{k}{2}$

$\Rightarrow \therefore k = 6$