TRIGONOMETRIC RATIOS

SYNOPSIS AND FORMULAE

1. One right angle = $\frac{\pi}{2}$ radians = 90^o.

   π radians = 2 right angles = 180^o

   $1^o = 60^1$, $1^1 = 60^{11}$

   1^o = 0.01745 radians.

   $1^c = 57^o 17^1 45^{11}$ (approx)

2. **Relations:**

   i) $\sin\theta \operatorname{cosec}\theta = 1$

   ii) $\cos\theta \sec\theta = 1$

   iii) $\tan\theta \cot\theta = 1$

   iv) $\sin^2\theta + \cos^2\theta = 1$

   v) $1 + \tan^2\theta = \sec^2\theta$

   $\rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1.$

   $\rightarrow \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta} = 1$

   vi) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

   $\rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

   $\rightarrow \operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

   vii) $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \,.\, \operatorname{cosec}^2\theta$

   viii) $\tan^2\theta - \sin^2\theta = \tan^2\theta \,.\, \sin^2\theta$;

$\cot^2\theta - \cos^2\theta = \cot^2\theta \,.\, \cos^2\theta$

ix) $\sin^2\theta + \cos^4\theta = 1 - \sin^2\theta\cos^2\theta$

$= \sin^4\theta + \cos^2\theta$

x) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

xi) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

xii) $\sin^2 x + \text{cosec}^2 x \geq 2$

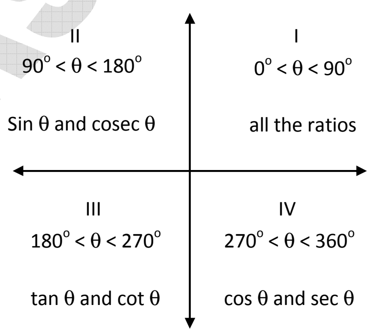
xiii) $\cos^2 x + \sec^2 x \geq 2$

xiv) $\tan^2 x + \cot^2 x \geq 2$.

3. Values of trigonometric ratios of certain angles

angle ↓	0^o	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined
cot	undefined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
cosec	undefined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	undefined

4. **Signs of Trigonometric Ratios:** If θ lies in I, II, III, IV quadrants then the signs of trigonometric ratios are as follows.



Note: i) $0^o, 90^o, 180^o, 270^o$. $360^o, 450^o$, ….. etc. are called quadrant angles.

ii) With "ALL SILVER TEA CUPS" symbol we can remember the signs of trigonometric ratios.

5. **Increasing and Decreasing Behavior of Trigonometrical Ratios:**

In Q_1: sin θ, tan θ, sec θ are increasing functions and cos θ, cot θ, cosec θ are decreasing functions.

In Q_2: sin θ, cos θ, cot θ are decreasing functions and tan θ , sec θ, cosec θ are increasing.

In Q_3: sin θ, cot θ, sec θ are decreasing functions and tan θ, cos θ, cosec θ are increasing functions.

In Q_4: sin θ, cos θ, tan θ are increasing functions and cosec θ, sec θ, cot θ are decreasing functions.

6. **Coterminal Angles:** If two angles differ by an integral multiples of 360^o then two angles are called coterminal angles.

Thus 30^o, 390^o, 750^o, 330^o etc., are coterminal angles.

Fn	$90 \mp \theta$	$180 \mp \theta$	$270 \mp \theta$	$360 \mp \theta$
$\sin\theta$	$\cos\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\sin\theta$
$\cos\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\sin\theta$	$\cos\theta$
$\tan\theta$	$\pm\cot\theta$	$\mp\tan\theta$	$\pm\cot\theta$	$\mp\tan\theta$
$\text{cosec}\,\theta$	$\sec\theta$	$\pm\text{cosec}\,\theta$	$-\sec\theta$	$\mp\text{cosec}\,\theta$
$\sec\theta$	$\pm\text{cosec}\,\theta$	$-\sec\theta$	$\mp\text{cosec}\,\theta$	$\sec\theta$
$\cot\theta$	$\pm\tan\theta$	$\mp\cot\theta$	$\pm\tan\theta$	$\mp\cot\theta$

7. Sin (n. 360^o + θ) = sin θ

Cos (n. 360^o + θ) = cos θ

Tan (n. 360^o + θ) = tan θ

Sin (n. 360^o – θ) = sin (–θ) = –sin θ

Cos (n. 360^o – θ) = cos (–θ) = cos θ

Tan (n. 360^o - θ) = tan (–θ) = –tan θ

8. **Complementary Angles:** Two Angles A, B are said to complementary $\Rightarrow$ A + B = 90^o

1) sin A = cos B and cos A = sin B.

2) $\sin^2 A + \sin^2 B = 1$, and $\cos^2 A = \sin^2 B$.

3) tan A . tan B = 1 and cot A cot B = 1.

9. **Supplementary angles:** Two angles A, B are said to be supplementary $\Rightarrow$ A + B = 180^o.

1) sin A – sin B = 0

2) cos A + cos B = 0

3) tan A + tan B = 0

Note: 1) If A – B = 180^o then i) cos A + cos B = 0

ii) sin A + sin B = 0

iii) tan A – tan B = 0

2) If A + B = 360^o then i) sin A + sin B = 0

ii) cos A – cos B = 0

iii) tan A + tan B = 0

10.

Functons	Domain	Range
Sin	R	$[-1,1]$
Cos	R	$[-1,1]$
Tan	$R-\{(2n+1)\frac{\pi}{2}, n\in z\}$	R
Cot	$R-\{n\pi, n\in z\}$	R
Sec	$R-\{(2n+1)\frac{\pi}{2}, n\in z\}$	$(-\infty-1]\cup[1,\infty)$
Cosec	$R-\{n\pi, n\in z\}$	$(-\infty-1]\cup[1,\infty)$

Note: 1) If a cos θ + b sin θ = c then

$$a\sin\theta - b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$$

2) If $a\cos\theta - b\sin\theta = c$ then

$$a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$$

11. $\sin\theta + \sin(\pi+\theta) + \sin(2\pi+\theta) + \ldots\ldots + \sin(n\pi+\theta) = 0$, if n is odd

$= \sin\theta$, if n is even.

12. $\cos\theta + \cos(\pi+\theta) + \cos(2\pi+\theta) + \ldots\ldots + \cos(n\pi+\theta) = 0$, if n is odd

$= \cos\theta$, if n is even.