TRANSFORMATIONS

SYNOPSIS AND FORMULAE

1. $\sin C + \sin D = 2\sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$.
2. $\sin C - \sin D = 2\cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$.
3. $\cos C + \cos D = 2\cos \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$.
4. $\cos C - \cos D = 2\sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2}$.
5. $2\sin A \cos B = \sin(A + B) + \sin(A - B)$
6. $2\cos A \sin B = \sin(A + B) - \sin(A - B)$
7. $2\cos A \cos B = \cos(A + B) + \cos(A - B)$
8. $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

   (Or)

   $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$.
9. $\frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right)$.
10. If $\sin A + \sin B = x$, and $\cos A + \cos B = y$. Then

    i) $\tan\left(\frac{A+B}{2}\right) = \frac{x}{y}$

    ii) $\sin(A + B) = \frac{2xy}{y^2 + x^2}$

    iii) $\cos(A + B) = \frac{y^2 - x^2}{y^2 + x^2}$

    iv) $\tan(A + B) = \frac{2xy}{y^2 - x^2}$
11. If $A + B + C = 180^\circ$ then

    i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

    ii) $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \cos C$

    iii) $\cos 2A + \cos 2B + \cos 2C = 1 - 4\cos A \cos B \cos C$

    iv) $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A \sin B \cos C$

    v) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

    vi) $\sin A - \sin B + \sin C = 4 \sin \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$

    vii) $\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

    viii) $\cos A + \cos B - \cos C = 1 + 4 \cos\frac{A}{2} \cos\frac{B}{2}\sin\frac{C}{2}$

    ix) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$.

    x) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$.

xi) $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

xii) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

12. If in a triangle ABC, $\cos^2 A + \cos^2 B + \cos^2 C = 1$ (or) $\sin^2 A + \sin^2 B + \sin^2 C = 2$ then the triangle is right angled.

13. If in a triangle ABC, angles A, B, C are in A.P. then $B = \frac{A+C}{2}$ and $B = 60^o$.

14. If $\sin(y + z - x)$, $\sin(z + x - y)$, $\sin(x + y - z)$ are in A.P. then tan x, tan y, tan z are in A.P.