

## TRIGONOMETRIC EQUATIONS

1.

Equation	Interval in which principal solution lies	General solution
$\sin \theta = k$ $(-1 \leq k \leq 1)$	$[-(\pi/2), \pi/2]$	$\theta = n\pi + (-1)^n \alpha$ $n \in \mathbb{Z}$
$\cos \theta = k$ $(-1 \leq k \leq 1)$	$[0, \pi]$	$\theta = 2n\pi \pm \alpha$ $n \in \mathbb{Z}$
$\tan \theta = k$ $(k \in \mathbb{R})$	$[-(\pi/2), \pi/2]$	$\theta = n\pi + \alpha$ ; $n \in \mathbb{Z}$

2. The solution of  $\sin \theta = k$  ( $|k| \leq 1$ ) lying between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$  is called the principle solution of the equation.
3. The solution of  $\cos \theta = k$  ( $|k| \leq 1$ ) lying between 0 and  $\pi$  is called the principal solution of the equation.
4. That solution of  $\tan \theta = k$ , lying between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$  is called the principal solution of the equation.
5. The general value of  $\theta$  satisfying  $\cos \theta = k$  ( $|k| \leq 1$ ) is given by  $\theta = 2n\pi \pm \alpha$  where  $n \in \mathbb{Z}$ .
6. The general value of  $\theta$  satisfying  $\sin \theta = k$  ( $|k| \leq 1$ ) is given by  $\theta = n\pi + (-1)^n \alpha$  where  $n \in \mathbb{Z}$ .
7. The general value of  $\theta$  satisfying  $\tan \theta = k$  is given by  $\theta = n\pi + \alpha$  where  $n \in \mathbb{Z}$ . (in each of the above cases,  $\alpha$  is the principal solution).
8. If  $\sin \theta = k$ ,  $\tan \theta = k$  are given equations, then the general value of  $\theta$  is given by  $\theta = 2n\pi + \alpha$  where  $\alpha$  is that solution lying between 0 and  $2\pi$ .

9. The equation  $a \cos\theta + b \sin\theta = c$  will have no solution or will be inconsistent if  $|c| > \sqrt{a^2 + b^2}$
10. If  $\sin A = \sin B$  and  $\cos A = \cos B$ , then  $\sin \frac{A-B}{2} = 0$  and  $A = 2n\pi + B$ .
11.  $\cos n\pi = (-1)^n$ ,  $\sin n\pi = 0$ .
12. If  $\sin^2\theta = \sin^2\alpha$  or  $\cos^2\theta = \cos^2\alpha$  or  $\tan^2\theta = \tan^2\alpha$ , then  $\theta = n\pi \pm \alpha$ ;  $n \in \mathbb{Z}$ .