SYSTEM OF CIRCLES

SYNOPSIS

1. If two circles S = 0 and $S^1 = 0$ intersect at P, then the angle between the tangents at P is called the angel between the circles.

2. Angle between two intersecting circles

   i. with centres C_1, C_2 and radii r_1, r_2 is $\text{Cos}^{-1}\left[\frac{C_1C_2^2 - r_1^2 - r_2^2}{2r_1r_2}\right]$.

   ii. With equations $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$. $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is

   $$\text{Cos}^{-1}\left[\frac{c_1 + c_2 - 2(g_1g_2 + f_1f_2)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}\right]$$

3. If the angle between two circles is a right angle, then the circles are said to cut orthogonally.

4. Two circles cut orthogonally

   (i) When C_1, C_2 are centres and r_1, r_2 are radii if $C_1C_2^2 = r_1^2 + r_2^2$.

   (ii) When equations are $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$, $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

5. **Radical Axis**:

   The locus of the point whose powers with respect to the two given circles are equal is called radical axis (R.A) of these two circles.

6. If S = 0, $S^1 = 0$ are two circles in standard form, then their radical axis is

   $S - S^1 = 0$. If $S = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$, $S^1 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ then it is $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.

7. Radical axis is a straight line.

8. If two circles intersect in A and B, then their common chord is radical axis.

   (i.e. $\overleftrightarrow{AB}$ is R.A.).

9. If two circles touch each other, then the common tangent at the point of contact is radical axis.

10. Radical axis bisects all the common tangents of two circles.

11. Radical axis of two circles is perpendicular to the line joining their centres.

12. If one circle lie in the other, then radical axis lies outside of both the circles.

13. If one circle lies outside the other, then radical axis lies in between both the circles.

14. The lengths of the tangents from any point on the radical axis to the two circles are equal.

15. The circle with centre on radical axis and length of tangent from it to the circles as radius cut the two circles orthogonally.

16. The centres of the circles which cuts two circles orthogonally lies on their radical axis.

17. The number of radical axes of n circles, no three of their centres are collinear is ${}^{n}c_2$.

12) If 'n' circles are given, then the maximum number of radical axes taken two circles at a time is ${}^{n}c_2$ and minimum number is zero. (i.e. when all the circles are concentric).

13) For three circles whose centres are non-collinear, there will be three radical axes and they are concurrent.

14) For concentric circles there is no radical axis.

15) If centres of three circles are non-collinear, then they have three radical axes and they are concurrent.

18 **Radical centre:** If centres of three circles are non-collinear, then the point of concurrence of the three radical axes is called as radical centre.

19. **Radical Centre (R.C)**:

1) The power of radical centre with respect to the three circles is equal.

2) The circle with radical centre as centre and length of tangent from radical centre to any of the circles as radius cuts the three circles orthogonally.

Note: Only when radical centre lies outside the three circles, we can have a circle cutting the three given circles orthogonally.

20. If $S = x^2 + y^2 + 2gx + 2fy + c = 0$, $S^1 = x^2 + y^2 + 2g^1x + 2f^1y + c^1 = 0$ are two circles, such that $S = 0$ bisects the circumference of $S^1 = 0$ then the centre of $S^1 = 0$ lies on common chord of $S = 0$, $S^1 = 0$ (i.e. $S - S^1 = 0$) i.e. $2(g - g^1)g^1 + 2(f - f^1)f^1 = (c - c^1)$.