RANDOM VARIABLES

SYNOPSIS

1. Let S be a simple space of a random experiment. A real valued function X: S -- > R is called a random variable.

2. Let S be a sample space and X: S $\rightarrow$ R be a random variable. The function F: R $\rightarrow$R denoted by $F(x) = P(X \leq x)$, is called probability distribution function of the random variable X.

3. A set E is said to be countable, if there exists a one - one correspondence between E and the set of Natural numbers N

4. If a sample space is countable then it is called a discrete sample space. A real valued function defined on a discrete sample space is called a discrete random variable.

5. If X: S $\rightarrow$ R is a discrete random variable with range $\{x_1, x_2, x_3, \ldots.\}$ then $\sum_{r=1}^{\infty} P(X = x_r) = 1$

$$E(x) = \sum_{i=1}^{n} x_i P(x = x_i)$$

$$\text{var}(x) = \sum x_i^2 \, P(x = x_i) - (\text{mean})^2$$

i.e., $\sigma^2 = E(x^2) - [E(x)]^2$

6. In a poisson distribution the variance is m. The sum of the terms in the odd places of the distribution is $e^{-m}\cos hm$.

7. In the above case the sum of the terms in the even places of the distribution is $e^{-m}\sin hm$.

8. **Bernoulli experiment**

A random experiment in which the probability of occurrence of any event is a constant is called a Bernoulli experiment.

9. Binomial Distribution

If a Bernoulli experiment is conducted n times, then probability of obtaining x successes is $P(x) = {}^{n}c_{x} . P^{x} q^{n-x}$, x = 0, 1, 2, ... , n

10. Mean of Binomial distribution = np.

11. Variance of Binomial distribution = npq.

12. Poisson Distribution

Binomial distribution tends to Poisson distribution if

(i) The number of trials n is very large.

(ii) Probability of success p is very small so that

np = a constant = λ

The probability function is given by

$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$, x = 0, 1, 2, ...

13. Mean of Poisson distribution = λ

14. Variance of Poisson distribution = λ

15. Standard deviation = $\sqrt{\lambda}$

16. If the mean of a binomial distribution is λ, then standard deviation lies in the interval $[0, \sqrt{\lambda})$.

17. Maximum variance of binomial distribution is n/4.

18. In a binomial distribution mean > variance.

19. If (n+1)p is not an integer, then mode of binomial distribution is [(n+1)p].