QUADRATIC EQUATIONS

SYNOPSIS

1. The standard form of a quadratic equation is $ax^2 + bx + c = 0$ where a, b, $c \in R$ and $a \neq 0$

2. The roots of $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

3. For the equation $ax^2+bx+c = 0$, sum of the roots = $-\frac{b}{a}$, product of the roots = $\frac{c}{a}$.

4. If the roots of a quadratic are known, the equation is

x^2 - (sum of the roots)x +(product of the roots)= 0

5. Nature of the roots of $ax^2 + bx + c = 0$

Nature of the Roots	Condition
Imaginary	$b^2 - 4ac < 0$
Equal	$b^2 - 4ac = 0$
Real	$b^2 - 4ac \geq 0$
Real and different	$b^2 - 4ac > 0$
Rational	$b^2 - 4ac$ is a perfect square a, b, c being rational
Equal in magnitude and opposite in sign	$b = 0$
Reciprocal to each other	$c = a$
both positive	b has a sign opposite to that of a and c
both negative	a, b, c all have same sign
opposite sign	a, c are of opposite sign

6. "Irrational roots" of a quadratic equation with "rational coefficients" occur in conjugate pairs. If $p + \sqrt{q}$ is a root of $ax^2 + bx + c = 0$, then $p - \sqrt{q}$ is also a root of the equation.

7. "Imaginary" or "Complex Roots" of a quadratic equation with "real coefficients" occur in conjugate pairs. If p + iq is a root of $ax^2 + bx + c = 0$. Then p - iq is also a root of the equation

8. If exactly one root of $ax^2+bx + c = 0$ lies in the interval (k_1, k_2), then $f(k_1)f(k_2) < 0$.

9. The roots of $ax^2 + bx + c = 0$ are in the ratio m : n, if $mnb^2 = ac(m + n)^2$.

10. One root of $ax^2 + bx + c = 0$ is the square of the other if $ac(a + c) + b^3 = 3abc$.

11. One root of $ax^2 + bx + c = 0$ is n^{th} power of the other if $(a^n c)^{\frac{1}{n+1}} + (a.c^n)^{\frac{1}{n+1}} = -b$.

12. Two equations $a_1x^2 + b_1x + c_1 = 0$, $a_2x^2 + b_2x + c_2 = 0$ have exactly the same roots if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

13. The equations $a_1x^2 + b_1x + c_1 = 0$, $a_2x^2 + b_2x + c_2 = 0$ have a common root,

If $(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$ and the common root is $\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$ if $a_1b_2 \neq a_2b_1$

14. (i) If unity is a root of $ax^2 + bx + c = 0$, then the other root is $\frac{c}{a}$

(ii) If –1 is one root of $ax^2 + bx + c = 0$, then the other root is $-\frac{c}{a}$.

15. The difference between the roots of $ax^2 + bx + c = 0$ is $\frac{\sqrt{b^2 - 4ac}}{a}$.

16. If f(x) = 0 is a quadratic equation, then the equation whose roots are

(i) The reciprocals of the roots of f(x) = 0 is $f\left(\frac{1}{x}\right) = 0$

(ii) The roots of f(x) = 0, each 'increased' by k is f(x - k) = 0

(iii) The roots of f(x) = 0, each 'diminished' by k is f(x + k) = 0

(iv) The roots of f(x) = 0 with sign changed is f(-x) = 0

(v) The roots of f(x) = 0 each multiplied by k(≠0) is $f\left(\frac{x}{k}\right) = 0$

17. If the coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers, then the roots are not rational.

18. The number of quadratic equations which are unchanged by squaring their roots is four.

19. The standard form of a quadratic expression is $ax^2 + bx + c$ where $a, b, c \in R$ and $a \neq 0$.

20. The product (x - a)(x - b) (where a < b) is negative if a < x < b i.e if x lies between a and b.

21. The product (x - a)(x - b) (where a < b) is positive if x < a or x > b i.e. x does not lie between a and b.

22. The sign of the expression $ax^2 + bx + c$ is same as that of 'a' for all values of x if $b^2 - 4ac \le 0$ i.e. if the roots of $ax^2 + bx + c = 0$ are imaginary or equal.

23. If the roots of the equation $ax^2 + bx + c = 0$ are real and different i.e. $b^2-4ac > 0$, the sign of the expression is same as that of 'a' if x does not lie between the two roots of the equation and opposite to that of 'a' if x lies between the roots of the equation.

24. The expression $ax^2 + bx + c$ is positive for all real values of x if $b^2 - 4ac < 0$ and $a > 0$.

25. The expression $ax^2 + bx + c$ has a maximum value when 'a' is negative and $x = -\frac{b}{2a}$.

Maximum value of the expression = $\frac{4ac - b^2}{4a}$.

26. The expression $ax^2 + bx + c$, has a minimum when 'a' is positive and $x = -\frac{b}{2a}$. Minimum value of the expression = $\frac{4ac - b^2}{4a}$.

27. The minimum value of $k + (x + a)^2$ is k. and The maximum value of $k - (x + a)^2$ is k.

28. If $a^2 + b^2 + c^2 = 1$ then ab + bc + ca lies in $\left[-\frac{1}{2}, 1\right]$.

29. Range of $x + \frac{1}{x}$ is $[2, \infty)$ when $x > 0$, $(-\infty, -2]$ when $x < 0$.

30. If $f(x) = \frac{x^2 - ax + b}{x^2 + ax + b}$ where x is real then the range of f(x) is $\left[f\left(-\sqrt{b}\right), f\left(\sqrt{b}\right)\right]$.