PROBABILITY

SYNOPSIS

1. If the results of an experiment is not certain and is any one of the several possible outcomes, then the experiment is called Random experiment.

2. The result of any single repetition of a random experiment is called an elementary event or simple event.

3. The list of all elementary events in a trial is called list of exhaustive events.

4. Elementary events are said to be equally likely if they have the same chance of happening.

5. A combination of one or more elementary events in a trial is called an event.

6. Every elementary event in a trial is an event.

7. The favourable cases to a particular event of an experiment are called successes and the remaining cases are called failures with respect to that event.

8. **Classical definition of Probability:** If a random experiment has n mutually exclusive, equally likely and exhaustive outcomes and m of them are favourable to the happening of an event 'A' then probability of A is given by

$$P(A) = m/n = \frac{\text{no. of favourable cases}}{\text{Total outcomes}}$$

9. The limits of Probability are 0, 1.

10. If P(A) = 0 then A is known as impossible event.

11. If P(A) = 1 then A is known as certain event.

12. The probability of non happening of E is denoted by $P(\overline{E})$ and $P(\overline{E}) = 1 - P(E)$.

13. Odds infavour of an event E = $P(E) : P(\overline{E})$

14. Odds against an even E=$P(\overline{E}) : P(E)$

15. If $P(E) : P(\overline{E}) = m : n$ then $P(E) = \frac{m}{m+n}$, $P(\overline{E}) = \frac{m}{m+n}$

16. Addition Theorem

If A and B are any two events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

If A and B are mutually exclusive $P(A \cup B) = P(A) + P(B)$.

17. (i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

(ii) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(iii) $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B)$

(iv) $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

(v) $P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$

(vi) If $A \subseteq B, P(A) \le P(B)$

(vii) If $A \subseteq B, P(B - A) = P(B) - P(A)$

18. Conditional Probabilities

The conditional probability of A, given B, is P(A/B) = $\dfrac{P(A \cap B)}{P(B)}, P(B) \ne 0$

The conditional probability of B, given A, is P(B/A) = $\dfrac{P(A \cap B)}{P(A)}, P(A) \ne 0$.

If A and B are independent then P(A/B) = P(A)

P(B/A) = P(B) and $P(A \cap B) = P(A).P(B)$

19. Multiplication Theorem

(i) If E_1 and E_2 are any two events in a random experiment not impossible, then

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$

(ii) If E_1, E_2 are any two independent events in a random experiment, then

$P(E_1 \cap E_2) = P(E_1). P(E_2)$

20. **i)** If A, B, C are three events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

ii) If A, B, C are three events, then

$$P[A \cap (B \cup C)] = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

21. If A, B, C .... n persons in that order, attempt an event and the probabilities of succeeding in one attempt is p and not succeeding is q, then their respective chances of succeeding if they attempt the event indefinitely are

$$\frac{p}{1-q^n}, \frac{pq}{1-q^n}, \frac{pq^2}{1-q^n}, \ldots\ldots \frac{pq^{n-1}}{1-q^n}.$$

22**.** Mean of (ax+b)= E(ax+b) = aE(x) + b

Variance of (ax+b) = V(ax+b) = a^2V(x)

23**.** Out of (2n + 1) tickets consecutively numbered, three are drawn at random. The chance that the numbers on them are in A.P is $\frac{3n}{4n^2-1}$.

24. Probability of obtaining different types of functions

A and B are two non-empty finite sets, n(A) = r and n(B) = s; s ≥r. A mapping is selected at random from the set of mappings from A into B. Then the probability that the mapping is

i) A one-one function is : ${}^sP_r / s^r$.

ii) A many to one function is : $\frac{s^r - {}^sP_r}{s^r}$

iii) A constant function is : s / s^r

iv) abijective function is : $\frac{r!}{s^r}$, iff r = s

25.Experiment with insertion of n letters in n addressed envelops

i) Probability of inserting all the n letters in right envelope = $\frac{1}{n!}$

ii) Probability of keeping atleast one letter in wrong envelope = 1 - $\frac{1}{n!}$

iii) Probability of keeping all the n letters in wrong envelopes

(p) = $\frac{1}{2!} - \frac{1}{3!} + \ldots\ldots + \frac{(-1)^n}{n!}$.

iv) Probability of keeping atleast one letter in right envelope = 1 – p.

26. There are n horizontal lines and n vertical lines forming a square containing n^2 squares. If 2 squares are chosen at random, thus the chance that they have common side is $\frac{2n(n-1)}{{}^{n^2}c_2}$.

27. n boys and n girls sit on a row at random. The probability that the boys and girls sit together is $\frac{2(n!)^2}{(2n)!}$

28. n boys and n girls sit in a row at random. The probability that the boys and girls sit alternately is $\frac{2(n!)^2}{(2n)!}$

29. **Description of normal pack of cards:**

1) King, Queen and Jacky cards are called face cards.
2) Ace, King, Queen, Jacky cards are called honour cards (or) court cards.
3) Hearts, Diamonds, Clubs, spades are called 4 suits of pack.

30. Independent events are not exclusive.

31. Exclusive events are not independent.

32. $P\left(\frac{E_1}{E_2}\right)+P\left(\overline{\frac{E_1}{E_2}}\right) = 1$

33. If $P(A\cup B) = P(A\cap B)$, then $P(A) = P(B)$.

34. If A and B tossed n coins each simultaneously then probability of getting equal number of heads is $\frac{{}^{2n}C_n}{2^{2n}}$.

35. If A and B tossed n and (n+1) coins respectively, then the probability of B getting more number of heads then A is 1/2.

36. If a coin is tossed (m+n) times (m > n), then the probability of at least m consecutive heads is $\frac{n+2}{2^{m+1}}$.

37. If A, B, C are pair wise independent events, then $P(A\cup B\cup C) = 1-P(\overline{A})\,P(\overline{B})P(\overline{C})$.

38.	**Description**	**Notation**
	1) One of A, B to occur	$A \cup B$
	2) Both of them to occur	$A \cap B$
	3) Exactly one of them to occur	$(A \cap \overline{B}) \cup (\overline{A} \cap B)$
	4) Neither A nor B occurs	$\overline{A} \cap \overline{B}$
	5) Non occurrence of one of them	$\overline{A} \cup \overline{B}$

39. 2n boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is $\frac{n}{2n-1}$

40. n persons, among whom are A and B, sit down at random in a row. The probability that there are m persons between A and B is $\frac{(n-m-1)2!(n-m)!}{n!}$.

41. If A_1, A_2, ... A_n are n mutually exclusive events in a sample space S then $P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = P(A_1) + P(A_2) + P(A_3) + \ldots\ldots + P(A_n)$