PERMUTATIONS AND COMBINATIONS

SYNOPSIS

Fundamental Principle of Counting

Multiplication Principle: If an operation can be performed in ‘m’ different ways : following which a second operation can be performed in ‘n’ different ways, then the two operations in succession can be performed in m $\times$ n different ways.

Addition Principle: If an operation can be performed in ‘m’ different ways and another operation, which is independent of the first operation, can be performed in ‘n’ different ways, then either of the two operations can be performed in (m + n) ways.

Note: The above two principles can be extended for any finite number of operations.

Permutations

Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a Permutation.

Note: Permutation of things means arrangement of things. The word arrangement is used if order of this is taken into account. Thus, if order of different things changes, then their arrangement also changes.

Important Results on Permutations

- $^nP_r = \dfrac{n!}{(n-r)!}$

  = n(n – 1) (n – 2) ….. {n – (r – 1)}, $0 \le r \le n$
- $\dfrac{^nP_r}{^nP_{r-1}}$ = n – r + 1.
- $^nP_r + r.{}^nP_{r-1} = {}^{(n+1)}P_r$
- ^nP_r is always a positive integer.

- ${}^{n}P_{r} = n.{}^{n-1}P_{r-1}$
- Number of Permutations of n different things taken all at a time is ${}^{n}P_{n}(=n!)$
- The number of permutations of n things taken all at a time, out of which p are alike and are of one type. q are alike and are of second type and rest are all different is $\frac{n!}{p!d!}$.
- The number permutations of n different things taken r at a time when each things may be repeated any number of time is n′.

Permutations under Restrictions

- Number of Permutations of n different things taken r at a time, when a particular thing is to be always inclined in each arrangements, is $r.{}^{n-1}P_{r-1}$.
- Number of Permutations of n different things taken r at a time, when a particular things is never taken in each arrangement, is ${}^{n-1}P_{r}$.
- Number of Permutations of n different things taken all at a time, when m specified things never come together, is $n! - (m! \times (n-m+1)!)$.
- Number of permutations of n dissimilar things taken ‘r’ at time when at least one thing is repeated, is $n^{r} - {}^{n}P_{r}$.
- The number of permutations of ‘n’ different things taken not more than ‘r’ at a time when each thing may occur any number of time is $\frac{n(n'-1)}{n-1}$.
- The number of significant numbers consisting of ‘r’ digits and formed out of ‘n’ digits including zero, no digit being repeated in any number is ${}^{n}P_{r} - {}^{(n-1)}P_{(r-1)}$.

Note

- The number of permutations of ‘n’ different things where order of ‘r’ things is not to be considered is $\frac{n!}{r!}$.

Circular Permutations

- Number of Circular arrangements (permutations) of n different things is (n – 1)!
- Number of circular arrangements (Permutations) of n different things when clock wise and anticlockwise arrangements are not different, is $\frac{1}{2}$(n – 1)!.
- Number of Circular Permutations of n different things taken r at a time, when clockwise and anticlockwise orders are taken as different, is $= \frac{{}^nP_r}{r}$.
- Number of Circular Permutations of n different things taken r at a time, when clockwise and anticlockwise order are not different, is $= \frac{{}^nP_r}{2r}$.

Derangement

- If n things form an arrangement in a row, the number of ways in which they can be arranged so that no one of them occupies its original place is

$$= n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ... + (-1)^n \frac{1}{n!}\right]$$

Some Important Results

- The sum of digits in unit places of all numbers formed with the help of $a_1, a_2, ..., a_n$ all at a time is $(n-1)!(a_1 + a_2 + ... + a_n)$.
- The sum of all possible numbers formed out of all the 'n' digits without zero is $(n-1)!$ (sum of all the digits) (1111 ……….. n times)
- The sum of all possible number formed out of all the 'n' digits which includes zero is [$(n-1)!$ (sum of all digits) (111 ……… n times)] – [(n – 2)! (Sum of all digits) (111 ……. (n – 1) times]
- Sum of all the r – digit numbers formed by taking the given n digits (including 0) is (sum of all the n digits) $[{}^{n-1}P_{r-1} \times (111.....r \text{ times}) - {}^{n-2}P_{r-2} \times (111.....(r-2) \text{ times})]$

- Sum of all the r – digits numbers formed by taking the given n digits (excluding 0) is (sum of all the n digits) $[{}^{n-1}P_{r-1} \times (111.....r \text{ times})]$.

Important Points to Remember

- The number of mappings that can be defined from set A containing 'm' elements to set B containing 'n' elements is n^m.
- The number of one-one functions (injections) that can be defined from set A containing 'm' elements to set B containing 'n' elements is ${}^nP_m$.
- The number of one-one onto functions (bijections) that can be defined from set 'A' of 'n' elements to a set 'B' of 'n' elements is ${}^nP_n$ or n!.
- The number of ways in which n different things can be distributed into r different groups where each group must have at least one thing is equal to number of onto function with n(A) = m, n(B) = n, is given by $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \ldots\ldots$ Considered ${}^nC_r = 0$ if $n < r$

  Note:

  - The number of ways in which n different things can be arranged into r different group is ${}^{(n+r-1)}P_n$.