PERIODICITY AND EXTREME VALUES

SYNOPSIS

Periodic Function and Period of Function

A real function $f : A \to B$ is such that $f(x + k) = f(x) \ \forall \ k \in R$; then f is called periodic function and least positive real number 'k' is called period of function.

(i.e.) 'k' is period of f(x) then (i) f(x + K) = f(x) and (ii) f(x + nk) = f(x).

* If the period of f(x) is a, then the period of –f(x) is also 'a'.

* The period of sin x , cos x, cosec x and sec x is 2π.

* The period of tan x, cot is π.

* The period of sin kx , cos kx, sec kx, cosec kx is $\frac{2\pi}{|k|}$.

* The period of tan kx and cot kx is $\frac{\pi}{|k|}$.

* The period of $\sin^n x$, $\cos^n x$, $\text{cosec}^n x$, $\sec^n x$.

  If n is even it is π.

  If n is odd it is 2π.

* The period of $\tan^n x$, $\cot^n x$ when 'n' is either even or odd is π.

* The period of |sin x|, |cos x|, |tan x|, |cosec x|, |sec x| and

  |cot x| is π.

* If a, b, $\in$ R and n $\in$ R. The period of

(i) $a \sin^n x + b \cos^n x$.

(ii) $a \tan^n x + b \cot^n x$.

(iii) $a \text{cosec}^n x + b \sec^n x$

	a = b	a ≠ b
n even	$\frac{\pi}{2}$	π
n odd	2π	2π

* The period of

(i) $a|\sin x| + b|\cos x|$

(ii) $a|\sin x| + b|\cot x|$

(iii) $a|\text{cosec } x| + b|\sec x|$ is $\frac{\pi}{2}$ if a = b AND is π if $a \neq b$.

* The period of $x - [x]$ is 1 [$\because$ Here [.] denotes greatest integer function $\leq x$]

* $f_1(x)$, $f_2(x)$, $f_3(x)$ and $f_4(x)$ are periodic functions with periods P_1, P_2, P_3 and P_4 respectively then the period of

(a) $a.f_1(x) \pm b\, f_2(x)$ is LCM of periods of $f_1(x)$ and $f_2(x)$ $(a \neq b)$

(b) $\frac{a.f_1(x) \pm b.f_2(x)}{c.f_3(x) \pm d.f_4(x)}$ is LCM of periods of $f_1(x)$, $f_2(x)$, $f_3(x)$ and $f_4(x)$.

* The LCM of functions $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$.

$\frac{\text{LCM of Nr}}{\text{HCF of Dr}}$ (i.e.) $\frac{\text{LCM of (a.c.e)}}{\text{HCF of (b.d.f)}}$

Extreme values.

* The range of sin x and cos x is [–1, 1]

* The range of tan x and cot x is $(-\infty, \infty)$

* The range of sec x and cosec x is $(-\infty, -1] \cup [1, \infty)$.

* The extreme values of a cos x + b sin x + c.

Min = $c - \sqrt{a^2 + b^2}$.

Max = $c + \sqrt{a^2 + b^2}$

Range = $[c - \sqrt{a^2+b^2}\ ,\ c + \sqrt{a^2+b^2}\ ]$

* The minimum value of

(i) $a^2\sin^2x + b^2\text{cosec}^2x$

(ii) $a^2\tan^2x + b^2\cot^2x$

(iii) $a^2\cos^2x + b^2\sec^2x$ is 2ab.

Range: $[2ab, \infty)$

* The extreme values of

$a\sin^2x + b\sin x\cos x + c\cos^2x$ Min = $\frac{a+c}{2} - \frac{\sqrt{b^2+(a-c)^2}}{2}$

Max = $\frac{a+c}{2} + \frac{\sqrt{b^2+(a-c)^2}}{2}$.

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