# PARTIAL FRACTIONS

## SYNOPSIS

1. Method of resolving proper fraction $\frac{f(x)}{g(x)}$ into partial fractions.

**Type 1:** When the denominator g(x) contains non-repeated linear factors i.e. g(x) = (x – a) (x – b) (x – c).

$$\frac{f(x)}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} \text{ where}$$

$$A = \frac{f(a)}{(a-b)(a-c)},\ B = \frac{f(b)}{(b-a)(b-c)},\ C = \frac{f(c)}{(c-a)(c-b)}.$$

**Type 2:** When the denominator g(x) contains repeated and non repeated linear factors.

i.e. $g(x) = (x-a)^2 (x-b)$,

$$\frac{f(x)}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)} \text{ where}$$

$$A = \frac{f'(a)}{a-b} - \frac{f(a)}{(a-b)^2},\ B = \frac{f(a)}{a-b},\ C = \frac{f(b)}{(a-b)^2}.$$

**Type 3:** When the denominator g(x) contains non repeated irreducible quadratic factors.

i.e. $g(x) = (ax^2 + bx + c)(x-d)$.

$$\frac{f(x)}{(ax^2+bx+c)(x-d)} = \frac{Ax+B}{(ax^2+bx+c)} + \frac{C}{x-d}.$$

where $C = \frac{f(d)}{ad^2+bd+c}$

we write $f(x) = (x-d)(Ax+B) + C(ax^2+bx+c)$ and equating the coefficients, we get A and B.

**Type 4:** When the denominator g(x) contains repeated irreducible quadratic factors

i.e. $g(x) = (ax^2+bx+c)^2 (x-d)$

$$\frac{f(x)}{(ax^2+bx+c)^2(x-d)} = \frac{Ax+B}{(ax^2+bx+c)} + \frac{Cx+D}{(ax^2+bx+c)^2} + \frac{E}{x-d}$$

where E = $\frac{f(d)}{(ad^2 + bd + c)^2}$

We write

$f(x) = (Ax + B)(ax^2 + bx + c)(x - d) + (Cx + D)(x - d) + E(ax^2 + bx + c)^2$

and by equation the coefficients, we get A, B, C and D..