PARABOLA

SYNOPSIS

1.S is the focus and the line 'l' is the directrix. If a variable point P is such that $\frac{SP}{PM} = 1$ where PM is perpendicular to the directrix, then the locus of P is a parabola.

2.. $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents

(i) A circle if $\Delta \neq 0$ $a = b$, $h = 0$

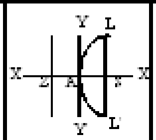
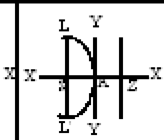
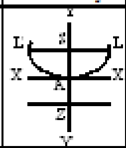
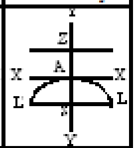
(ii) A parabola if $\Delta \neq 0$ $h^2 = ab$ and $af^2 \neq bg^2$

(iii) An ellipse if $\Delta \neq 0$ $h^2 < ab$

(iv) An hyperbola if $\Delta \neq 0$ $h^2 > ab$

3.

Four standard forms of the parabola

S. No.	Content	I	II	III	IV
	Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
	Figure				
1.	Vertex (A)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
2.	Focus (S)	(a, 0)	(–a, 0)	(0, a)	(0, –a)
3.	Point of intersection of axis and directrix (Z)	(–a, 0)	(a, 0)	(0, –a)	(0, a)
4.	End points of latus rectum (L, L')	(a, ±2a)	(–a, ±2a)	(±2a, a)	(±2a, –a)
5.	Equation of axis	y = 0	y = 0	x = 0	x = 0
6.	Equation of Directrix	x = –a	x = a	y = –a	y = a
7.	Equation of tangent at vertex	x = 0	x = 0	y = 0	y = 0
8.	Equation of latus rectum	x = a	x = –a	y = a	y = –a
9.	Length of latus rectum (LL')	4a	4a	4a	4a
10.	Distance from focus to directrix (SZ)	2a	2a	2a	2a
11.	SA = AZ	a	a	a	a

4. i) If the axis of a parabola is parallel to x-axis, equation of the parabola will be of the form $(y - \beta)^2 = 4a(x - \alpha)$ (or) $(y - \beta)^2 = -4a(x - \alpha)$ (or) $x = ay^2 + by + c$.

ii) If the axis of the parabola is parallel to y-axis, equation of the parabola will be of the form $(x - \alpha)^2 = 4a(y - \beta)$ (or) $(x - \alpha)^2 = -4a(y - \beta)$ (or) $y = ax^2 + bx + c$.

5. In the equation of the parabola $(y - \beta)^2 = 4a(x - \alpha)$.

i) Vertex = (α, β). ii) Focus = $(\alpha + a, \beta)$ iii) Ends of latustrectum = $(\alpha + a, \beta \pm \beta\, 2a)$

iv) Equation of axis is $y = \beta$

v) Equation of directrix is $x = \alpha - a$.

vi) Equation of latustrectum is $x = \alpha + a$

vii) Length of latustrectum = 4a.

6. In the equation of the parabola $(x - \alpha)^2 = 4a(y - \beta)$

i) Vertex = (α, β)

ii) Focus = $(\alpha, \beta + a)$

iii) Ends of latustrectum = $(\alpha \pm 2a, \beta + a)$

iv) Equation of axis is $x = \alpha$

v) Equation of directrix is $y = \beta - a$

vi) Equation of latustrectum is $y = \beta + a$

vii) Length of latustrectum = 4a.

7. The focal distance of the point $P(x_1, y_1)$ on the parabola

i) $y^2 = 4ax$ is $SP = |x_1 + a|$

ii) $x^2 = 4ay$ is $SP = |y_1 + a|$.

8. The equation of the tangent at (x_1,y_1) to the parabola $y^2=4ax$ is $yy_1=2a(x+x_1)$ i.e. $S_1= 0$.

Slope of this tangent is $\frac{2a}{y_1}$

9. The line $y = mx + c$ is a tangent to $y^2 = 4ax$ if $c = \frac{a}{m}$.

(i) The line $y = mx + c$ is a tangent to $x^2 = 4ay$ if $c = am^2$.

(ii). The condition for $x = my + c$ is a normal to $x^2 = 4ay$ is $c = -2am - am^3$

10. If m is the slope of any tangent to $y^2 = 4ax$, then its equation is $y = mx + \frac{a}{m}$ and its point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

11. The line $lx + my + n = 0$ is a tangent to the parabola $y^2 = 4ax$ if $am^2 = ln$ and

point of contact $= \left(\frac{n}{\ell}, \frac{-2am}{\ell}\right)$

12. The line $lx + my + n = 0$ is a tangent to the parabola $x^2 = 4ay$, if $al^2 = mn$. and

point of contact $= \left(\frac{-2a\ell}{m}, \frac{n}{m}\right)$

13. The line $y = mx + c$, touches the parabola $y^2 = 4a(x + a)$, if $c = am + a/m$

14. i) The line $y = mx + c$ doesnot intersect the parabola $y^2 = 4ax$ $(a > 0)$ then $mc > a$

ii) The line $y = mx+c$ intersects the parabola $y^2 = 4ax$ $(a > 0)$ the $mc < a$.

15. The slopes of the tangents to the parabola $y^2=4ax$ passing through (x_1, y_1) are given by the equation $m^2x_1 - my_1 + a = 0$

16. Sum of the slopes $= y_1/x_1$

Product of the slopes $= a/x_1$

17. Angle between the tangents drawn to the parabola $y^2 = 4ax$ from (x_1, y_1) is $\alpha = \tan^{-1}\dfrac{\sqrt{S_{11}}}{x_1 + a}$

18. The equation of the locus of the point of intersection of tangents to the parabola $y^2=4ax$ which include an angle 'α' is $(y^2 - 4ax)\cot^2\alpha=(x+a)^2$.

19. The locus of the points of intersection of perpendicular tangents to a parabola is the directrix of the parabola.

20. The tangents at the ends of a focal chord of the parabola meet on the directrix at right angles.

21. If the tangents are drawn from any point on the latusrectum to the parabola, they make complementary angles with the axis of the parabola.

22. **a.** The equation of the tangent at $(at^2, 2at)$ is $yt = x + at^2$

b. Slope of the tangent at $(at^2, 2at) = \dfrac{1}{t}$

23. Point of intersection of the tangents at 't_1', 't_2' is $(at_1t_2, a\overline{t_1+t_2})$

24. If the tangents at P and Q on a parabola meet in T and if S is the focus then $ST^2 = SP.SQ$

25. If m is the slope of any normal to $y^2 = 4ax$ then its equation is $y = mx - 2am - am^3$ and foot of the normal is $(am^2, -2am)$

26. The equation of the normal at (x_1, y_1) is $y - y_1 = -\dfrac{y_1}{2a}(x - x_1)$

27. **a.** The equation of the normal at $(at^2, 2at)$ to $y^2 = 4ax$ is $xt + y = 2at + at^3$.

b. Slope of the normal at $(at^2, 2at) = -t$

28. The line $lx + my + n = 0$ is a normal to the parabola $y^2 = 4ax$ if $al^3 + 2alm^2 + m^2n = 0$

29. If the normals at $(x_1, y_1), (x_2, y_2)$ on the parabola $y^2 = 4ax$ meet again on the parabola, then $x_1x_2 + y_1y_2 = 12a^2$

30. From any point, three normals can be drawn to a parabola and the sum of the ordinates of the normals is zero.

31. The point of intersection of normals drawn at 't_1', 't_2' to the parabola $y^2 = 4ax$ is $[a(2+t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2)]$

32. If the normal at $(at_1^2, 2at_1)$ cuts the curve again at $(at_2^2, 2at_2)$, then $t_2 = -t_1 - \frac{2}{t_1}$

33. If the normals at the points 't_1', 't_2' meet on the curve again at 't_3', then

(i) $t_1t_2=2$ (ii) $t_1 + t_2 = -t_3$

34. If the normals at P, Q on the parabola meet on the curve at R, the centroid of the triangle PQR lies on the axis of the parabola.

35. From any point three normals can be drawn to a parabola and

(i) At least one of the 3 normals is real

(ii) The sum of their slopes is '0'

(iii) The sum of the ordinates of their feet is '0'

36. The foot of the perpendicular from the focus on any tangent to a parabola lies on the tangent at the vertex.

37. The tangent at one end of a focal chord is parallel to the normal at the other end.

38. The orthocentre of the triangle formed by any three tangents to the parabola lies on the directrix of the parabola.

39. The orthocentre of the triangle formed by tangents at 't_1', 't_2', 't_3' to the parabola $y^2=4ax$ is $(-a, a(t_1 + t_2 + t_3 + t_1t_2t_3))$

40. The circum circle of the triangle formed by any three tangents to a parabola passes through its focus.

41. The length of the chord of contact of tangents drawn from (x_1, y_1) to the parabola $y^2 = 4ax$ is $\frac{\sqrt{y_1^2 - 4ax_1} \cdot \sqrt{y_1^2 + 4a^2}}{a}$ and area of the triangle formed by the tangents from (x_1, y_1) to the parabola $y^2 = 4ax$ and its chord of contact is $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$.

42. (a) Length of the chord cut-off by the parabola $y^2 = 4ax$ on the line $y = mx + c$ is

$\frac{4}{m^2}\sqrt{a^2 - amc}\sqrt{1+m^2}$,

(b) Length of the chord cut –off by any conic on the line y = mx+c is $|x_1 - x_2|\sqrt{1+m^2}$ where x_1 and x_2 are the roots of quadratic obtained by eliminating y.

43. The area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ where y_1, y_2, y_3 are the ordinates of the vertices.

44. The ends of focal chord of the parabola $y^2 = 4ax$ are $(x_1, y_1), (x_2, y_2)$. Then $x_1x_2 + y_1y_2 = -3a^2$

45. The equation of the chord of the parabola $y^2=4ax$ joining the points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ is $y(t_1 + t_2) = 2x + 2at_1t_2$.

46. If the points 't_1', 't_2' are ends of a focal chord, then $t_1t_2 = -1$and ends of focal chord are $(at^2, 2at)$, $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$.

47. The length of the chord joining the points 't_1', 't_2' is $a|t_1 - t_2|\sqrt{(t_1+t_2)^2+4}$

48. **a)** Least length of focal chord is 4a **b)** Length of focal chord drawn at 't' is $|a(t + 1/t)^2|$

49. If a focal chord of the parabola $y^2 = 4ax$ makes an angle 'θ' with the x-axis; then the length of the focal chord is $4a\operatorname{cosec}^2\theta$.

50. **a)** The length of the normal chord drawn at 't' is given by $4a(1 + t^2)^{3/2}/t^2$.

b) The least length of normal chord is $6\sqrt{3}a$, when drawn at $(2a, 2\sqrt{2}a)$

51. The length of the chord of the parabola $y^2 = 4ax$ passing through the vertex and making an angle 'θ' with the axes is $4a\cos\theta\operatorname{cosec}^2\theta$.

52. If the normal at 't' on the parabola $y^2 = 4ax$ subtends a right angle

(a) at its focus then $t = \pm 2$

(b) at its vertex then $t = \pm\sqrt{2}$.

53. The equation of the chord of contact of tangents from (x_1, y_1) to the parabola is $S_1 = 0$.

54. The equation of the chord of the parabola $y^2 = 4ax$ having its midpoint at (x_1, y_1) is $S_1=S_{11}$.

55. The semi lotus rectum of any conic is H.M. between the segments of a focal chord.

56. The circle described on any focal radius of a parabola as diameter touches tangent at the vertex.

57. The circle described on any focal chord of a parabola as diameter touches the directrix.

58. The equation of the common tangent to two parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $a^{1/3}x + b^{1/3}y + a^{2/3}b^{2/3} = 0$