MULTIPLE AND SUBMULTIPLE ANGLES

SYNOPSIS

Multiple Angles: If A is an angle, then 2A, 3A, 4A, etc……. are called multiple angles of A and A/2, A/2, A/3, 3A/4, 2A/5 ….. etc are called submultiple angles of A.

I. i) $\text{Sin } 2A = 2 \sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$.

ii) $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2} = \dfrac{2\tan A/2}{1+\tan^2 A/2}$.

iii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

iv) $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2} = 2\cos^2\dfrac{A}{2} - 1 = 1 - 2\sin^2\dfrac{A}{2} = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

v) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

vi) $\tan A = \dfrac{2\tan A/2}{1-\tan^2 A/2} = \operatorname{cosec} 2A - \cot 2A$

vii) $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$

viii) $\cot A = \dfrac{\cot^2 (A/2) - 1}{2\cot A/2} = \operatorname{cosec} 2A + \cot 2A$

ix) $\tan\left(\dfrac{\pi}{4} + \dfrac{A}{2}\right) = \dfrac{1+\tan A/2}{1-\tan A/2}$

$= \dfrac{\cos A/2 + \sin A/2}{\cos A/2 - \sin A/2} = \dfrac{1+\sin A}{\cos A} = \dfrac{\cos A}{1-\sin A} = \sec A + \tan A = \cot\left(\dfrac{\pi}{4} - \dfrac{A}{2}\right)$.

x) $\tan\left(\dfrac{\pi}{4} - \dfrac{A}{2}\right) = \dfrac{1-\tan A/2}{1+\tan A/2} = \dfrac{\cos A/2 - \sin A/2}{\cos A/2 + \sin A/2} = \sqrt{\dfrac{1-\sin A}{1+\sin A}} = \dfrac{1-\sin A}{\cos A}$

$= \sec A - \tan A = \cot\left(\dfrac{\pi}{4} + \dfrac{A}{2}\right)$

II. i) $\sin 3A = 3\sin A - 4\sin^3 A$

ii) $\cos 3A = 4\cos^3 A - 3\cos A$

iii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

iv) $\sin^3 A = \dfrac{1}{4}[3\sin A - \sin 3A]$

v) $\cos^3 A = \dfrac{1}{4}[\cos 3A + 3\cos A]$

III. $\sin^2 A = \dfrac{1-\cos 2A}{2}$; $\sin A = \pm\sqrt{\dfrac{1-\cos 2A}{2}}$

$\cos^2 A = \dfrac{1+\cos 2A}{2}$; $\cos A = \pm\sqrt{\dfrac{1+\cos 2A}{2}}$

$\tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A}$; $\tan A = \pm\sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}}$

$\sin^2 \dfrac{A}{2} = \dfrac{1-\cos A}{2}$; $\sin \dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

$\cos^2 \dfrac{A}{2} = \dfrac{1+\cos A}{2}$; $\cos \dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

$\tan^2 \dfrac{A}{2} = \dfrac{1-\cos A}{1+\cos A}$; $\tan \dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

IV. i) $\cot A + \tan A = 2\,\mathrm{cosec}\, 2A$

ii) $\cot A - \tan A = 2\cot 2A$

iii) $\tan(\dfrac{\pi}{4} + A) + \tan(\dfrac{\pi}{4} - A) = 2\sec 2A$

iv) $\tan(\dfrac{\pi}{4} + A) - \tan(\dfrac{\pi}{4} - A)\ 2\tan 2A$

v) $\tan A + 2\tan 2A + 2^2 \tan 2^2 A + \ldots 2^{n-1} \tan 2^{n-1} A \quad + 2^n \cot 2^n A = \cot A.$

vi) $\sin\theta \sin(\alpha-\theta)\sin(\alpha+\theta) = \frac{1}{4}\sin 3\theta$ where $\alpha = 60^o$ or 120^o or 240^o or 300^o

vii) $\cos\theta \cos(\alpha-\theta)\cos(\alpha+\theta) = \frac{1}{4}\cos 3\theta$ where $\alpha = 60^o$ or 120^o or 240^o or 300^o

viii) $\tan\theta \tan(\alpha-\theta)\tan(\alpha+\theta) = \frac{1}{4}\tan 3\theta$where α = 60o or 120o or 240o or 300o

ix) $\cot\theta \cot(\alpha-\theta)\cot(\alpha+\theta) = \frac{1}{4}\cot 3\theta$ where $\alpha = 60^o$ or 120^o or 240^o or 300^o

x) $\cos\theta \,.\, \cos 2\theta \,.\, \cos 4\theta \ldots\ldots \cos((2^{n-1}\theta) = \frac{\sin(2^n\theta)}{2^n \sin\theta}$

xi) $\cos\theta \cos 2\theta \cos(2^2)\theta \ldots\ldots \cos(2^n\theta) = \frac{\sin(2^{n+1}\theta)}{2^{n+1}\sin\theta}$.

xii) If $\theta = \frac{\pi}{2^n+1}$ then $\cos\theta \,.\, \cos 2\theta \,.\, \cos 2^2\theta \ldots\ldots \cos 2^{n-1}\theta = \frac{1}{2^n}$

xiii) If $\theta = \frac{\pi}{2^n-1}$ then $\cos\theta \,.\, \cos 2\theta \,.\, \cos 2^2\theta \ldots\ldots \text{Cos}\, 2^{n-1}\theta = -\frac{1}{2^n}$

xiv) $\cos\theta + \cos 2\theta + \cos 3\theta + \ldots\ldots + \cos n\theta =$

$$\frac{\sin(\frac{n\theta}{2})\cos(\frac{n+1}{2})\theta}{\sin(\theta/2)}$$

V. i) $\sin^2\theta + \sin^2(\alpha-\theta) + \sin^2(\alpha+\theta) = \frac{3}{2}$

ii) $\cos^2\theta + \cos^2(\alpha-\theta) + \cos^2(\alpha+\theta) = \frac{3}{2}$

Where $\alpha = 60^o$ or 120^o or 240^o or 300^o

iii) $\cos^3\theta + \cos^3(120^o+\theta) + \cos^3(120^o-\theta)$

$$= \frac{3}{4}\cos 3\theta$$

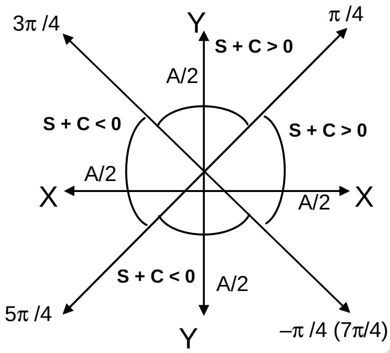
iv) $(1 + \sec 2\theta)(1 + \sec 4\theta)(1 + \sec 8\theta) \ldots\ldots$

$(1 + \sec 2^n \theta) = \tan 2^n \theta \cot \theta$

v) $(2\cos \theta - 1)(2\cos 2\theta - 1(2\cos 4\theta - 1) \ldots.$

$(2\cos 2^{n-1} \theta - 1) = \dfrac{2\cos 2^n \theta + 1}{2\cos\theta + 1}.$

VI. If $S = \sin\dfrac{A}{2}$ and $C = \cos\dfrac{A}{2}$ then



i) $S + C = \pm\sqrt{1+\sin A}$

ii) $S - C = \pm\sqrt{1-\sin A}$

iii) $2\sin\dfrac{A}{2} = \pm\sqrt{1+\sin A} \pm\sqrt{1-\sin A}$

iv) $2\cos\dfrac{A}{2} = \pm\sqrt{1+\sin A} \pm\sqrt{1-\sin A}$

v) a) $S + C > 0,\ S - C > 0$ if

$\dfrac{\pi}{4} < \dfrac{A}{2} < \dfrac{3\pi}{4}$

b) $S + C < 0,\ S - C > 0$ if

$\dfrac{3\pi}{4} < \dfrac{A}{2} < \dfrac{5\pi}{4}$

c) S + C < 0, S – C < 0 if

$$\frac{5\pi}{4} < \frac{A}{2} < \frac{7\pi}{4}$$

d) S + C > 0, S – C < 0 if

$$\frac{-\pi}{4} < \frac{A}{2} < \frac{\pi}{4}$$

VII.

	18^o	36^o	54^o	72^o
Sin	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$
Cos	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$