# MATHEMATICAL INDUCTION

## SYNOPSIS

**Principle of finite Mathematical Induction**

Let $\{P(n) / n \in N\}$ be a set of statements. If

(i) p(1) is true

(ii) p (m) is true $\Rightarrow$ p (m+1) is true ; then p (n) is true for every n $\in$ N.

Principle of complete induction

Let {P (n) / n N } be a set of statements. If p (1) is true and p(2), p(3) .... p(m-1) are true => p(m) is true, then p (n) is true for every n $\in$ N.

**Note**

(i) The principle of mathematical induction is a method of proof of a statement.

(ii) We often use the finite mathematical induction, hence or otherwise specified the mathematical induction is the finite mathematical induction.

Some important formulae:

1. $\sum n = \frac{n(n+1)}{2}$

2. $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$;

3. $\sum n^3 = \frac{n^2(n+1)^2}{4}$

4. $a, (a+d), (a+2d), ..............$ are in a.p n th term $t_n = a+(n-1)d$, sum of n terms $S_n = \frac{n}{2}\left[2a+(n-1)d\right] = \frac{n}{2}[a+l]$ , a a = first term, l= last term.

5. $a, ar, ar^2, ........$ is a g.p.

Nth term $t_n = a.r^{n-1}$. a = 1st term, r = common ratio.

Sum of n terms $s_n = a\frac{(r^n-1)}{r-1}; r>1, \quad = a\left(\frac{1-r^n}{1-r}\right); r<1$

6. In Infinite G.P, Sum of Infinite terms IS $S_\infty = \frac{a}{1-r}$

7. Sum of the first 'n' odd +ve integers = $n^2$

8. Sum of the first 'n' even +ve integers = n(n+1)

9. The sum of cubes of three consecutive natural numbers is always divisible by 9

10. $x^n - y^n$ is divisible by $x+y$ when 'n' is even.