INVERSE TRIGONOMETRIC FUNCTIONS

SYNOPSIS AND FORMULAE

1.

Function	Domain	Range
$\sin^{-1}(x)$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}(x)$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}(x)$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$
$\cot^{-1}(x)$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}$	$(-\infty,-1] \cup [1, \infty)$	$[0, \pi/2)\cup (\pi/2, \pi]$
$\text{cosec}^{-1}(x)$	$(-\infty,-1] \cup [1, \infty)$	$[-\pi/2, 0) \cup(0, \pi/2]$

2. a) $\sin(\sin^{-1}x) = x$ for $x \in [-1, 1]$

$\sin^{-1}(\sin x) = x$ for $x \in [-\pi/2, \pi/2]$

b) $\cos(\cos^{-1}x) = x$ for $x \in [-1, 1]$

$\cos^{-1} (\cos x) = x$ for $x \in [0, \pi]$

c) $\tan(\tan^{-1}x) = x$ for $x \in (-\infty, \infty)$

$\tan^{-1}(\tan x) = x$ for $x \in (-\pi/2, \pi/2)$

d) $\cot(\cot^{-1}x) = x$ for $x \in (-\infty, \infty)$

$\cot^{-1}(\cot x) = x$ for $x \in (0, \pi)$

e) $\sec(\sec^{-1}x) = x$ for $x \in(-\infty, -1] \cup [1, \infty)$

$\sec^{-1}(\sec x) = x$ for $x \in [0, \pi/2) \cup (\pi/2, \pi]$

f) $\text{cosec}(\text{cosec}^{-1}x) = x$ for $x \in (-\infty, -1] \cup [1, \infty)$

$\text{cosec}^{-1}(\text{cosec } x) = x$ for $x \in [-\pi/2, 0)\cup(0, \pi/2]$

3. a) $\sin^{-1}(-x) = -\sin^{-1}(x)$ for $x \in [-1, 1]$

   b) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ for $x \in [-1, 1]$

   c) $\tan^{-1}(-x) = -\tan^{-1}(x)$ for $x \in (-\infty, \infty)$

   d) $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$ for $x \in (-\infty, \infty)$

   e) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ for $x \in (-\infty,-1]\cup[1, \infty)$

   f) $\mathrm{cosec}^{-1}(x) = -\mathrm{cosec}^{-1}(x)$ for $x \in (-\infty,-1]\cup[1, \infty)$

4. a) $\sin^{-1}(x) = \mathrm{cosec}^{-1}(1/x)$; for $x \in [-1, 0) \cup (0, 1)$

   b) $\cos^{-1}(x) = \sec^{-1}(1/x)$; for $x \in [-1, 0) \cup (0, 1]$

   c) $\tan^{-1}(x) = \cot^{-1}(1/x)$ for $x \in (0, \infty)$ and

   $\tan^{-1}(x) = -\pi + \cos^{-1}(1/x)$ for $x \in (-\infty, 0)$

   d) $\cot^{-1} = \tan^{-1}(1/x)$ for $x \in (0, \infty)$

   $= + \tan^{-1}(1/x)$ for $x \in (-,\infty\ 0)$

5. a) $\sin^{-1}x + \cos^{-1}x = \pi/2$ for $x \in [-1, 1]$

   b) $\tan^{-1}x + \cot^{-1}x = \pi/2$for $x \in (-\infty, \infty)$

   c) $\sec^{-1}x + \mathrm{cosec}^{-1}(x) = \pi/2$for $x \in (-\infty, 1] \cup [1, \infty)$

6. a) $\sin^{-1}(x) = \cos^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ for $0 \le x \le 1$

   b) $\cos^{-1}(x) = \sin^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ for $0 < x \le 1$

   c) $\tan^{-1}(x) = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$ for $x \ge 0$

7. If $0 \le x \le 1$; $0 \le y \le 1$ then

   a) $\sin^{-1}x + \sin^{-1}y = s^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}$ for $x^2 + y^2 \le 1$

b) $\sin^{-1} + \sin^{-1}y = \pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}$ for $x^2 + y^2 > 1$

c) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}$

d) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-y^2} \cdot \sqrt{1-x^2})$for $x^2 + y^2 \le 1$.

e) $\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}(\sqrt{1-x^2} \cdot \sqrt{1-y^2} - xy)$ for $x^2 + y^2 > 1$

f) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}(xy + \sqrt{1-x^2} \cdot \sqrt{1-y^2})$

8. For $x > 0, y > 0$

a) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}(\frac{x+y}{1-xy})$ if $xy < 1$

$= \pi + \tan^{-1}(\frac{x+y}{1-xy}$ if $xy > 1$.

$= \frac{\pi}{2}$ if $xy = 1$

b) If x and y are of same signs then $\tan^{-1}x - \tan^{-1}y = \tan^{-1}(\frac{x-y}{1+xy})$.

9. a) $2\sin^{-1}(x) = \sin^{-1}(2x\sqrt{1-x^2})\cos^{-1}(1 - 2x^2)$

b) $2\cos^{-1}(x) = \cos^{-1}(2x^2 - 1) = \sin^{-1}(2x\sqrt{1-x^2})$.

c) $2\tan^{-1}(x) = \tan^{-1}(\frac{2x}{1-x^2}) = \sin^{-1}(\frac{2x}{1-x^2}) = \cos^{-1}(\frac{1-x^2}{1+x^2})$

10. a) $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$.

b) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

c) $3\tan^{-1}x = \tan^{-1}(\frac{3x-x^3}{1-3x^2})$

11. a) $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \tan^{-1}\left(\frac{x+y+z-xyz}{(1-xy-yz-zx)}\right)$

b) If $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \frac{\pi}{2}$ then $xy + yz + zx = 1$.

c) If $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \pi$ then $x + y + z = xyz$.

12. a) $\tan^{-1}\frac{n}{m} + \tan^{-1}\left(\frac{m-n}{m+n}\right) = \pi/4$ or $-3\pi/4$.

b) $\tan^{-1}\frac{m}{n} - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \pi/4$ or $-3\pi/4$.

c) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \pi/4$

d) $\tan^{-1} 2 + \tan^{-1} 3 = 3\pi/4$

e) $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$.

f) If $\tan^{-1} x + \tan^{-1} y = \pi/2$ then $xy = 1$.

g) If $\cot^{-1} x + \cot^{-1} y = \pi/2$ then $xy = 1$

h) If $\sin^{-1} a/x + \sin^{-1} b/x = \pi/2$ or

$\cos^{-1} a/x + \cos^{-1} b/x = \pi/2$ then $x = \sqrt{a^2+b^2}$.

i) If $\tan^{-1} a/x + \tan^{-1} b/y = \pi/2$ then $x = \sqrt{ab}$

j) If $\cos^{-1} x/a + \cos^{-1} y/b = 0$ then $\frac{x^2}{y^2} - \frac{2xy}{ab}\cos\theta + \frac{b}{y} = \sin^2\theta$

k) If $\sin^{-1} x/a + \sin^{-1} y/b = \theta$, then $\frac{x^2}{y^2} + \frac{2xy}{ab}\cos\theta + \frac{b}{y} = \sin^2\theta$.

l) $\tan^{-1}\frac{1}{1+x(x+1)} + \tan^{-1}\frac{1}{1+(x+1)(x+2)} + \ldots.\ \tan^{-1}\frac{1}{1+(x+n-1)(x+n)}$

$= \tan^{-1}(x+n) - \tan^{-1} x$.