INDEFINITE INTEGRATION

SYNOPSIS

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c.$ $(n \neq -1)$

2. $\int \frac{dx}{x} = \log|x| + c$

3. $\int \sin x dx = -\cos x + c$

4. $\int \cos x dx = \sin x + c$

5. $\int \sec^2 x dx = \tan x + c$

6. $\int \operatorname{cosec}^2 x dx = -\cot x + c$

7. $\int \sec x \tan x dx = \sec x + c$

8. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$

9. $\int e^x dx = e^x + c, \int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$

10. $\int a^x dx = \frac{a^x}{\log a} + c$

11. i) $\int \frac{f^1(x)}{f(x)} dx = \log|f(x)| + c$

ii) $\int \frac{f^1(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

iii) $\int [f(x)]^n f^1(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$

12. i) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$ $(n \neq -1)$

ii) $\int \frac{dx}{ax+b} = \frac{\log|(ax+b)|}{a} + c$

iii) $\int \sin(ax+b) dx = \frac{-\cos(ax+b)}{a} + c$

13. $\int \tan x dx = \log|\sec x| + c$

14. $\int \cot x dx = \log|\sin x| + c$

15. $\int \sec x dx = \log|(\sec x + \tan x)| + c$

16. $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| + c$

17. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$ or $-\cos^{-1} x + c$

18. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$ or $-\cot^{-1} x + c$

19. $\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + c$ or $-\operatorname{cosec}^{-1} x + c$

20. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

21. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

22. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c.$

23. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log(x + \sqrt{x^2+a^2}) + c$ or $\sinh^{-1} \frac{x}{a} + c$

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25. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$

26. $\int e^x \{f(x) + f^1(x)\} dx = e^x f(x) + c$

27. $\int e^x f(x) dx = e^x \{f(x) - f^1(x) + f^{11}(x) - f^{111}(x) .....\}$ Where f(x) is a polynomial in x.

28. $\int e^x \left[f(x) - f^{11}(x)\right] dx = e^x \left[f(x) - f^1(x)\right] + c$

29. a. $\int udv = uv - \int vdu$ (Integration by parts formula)

b. If u, v are functions of x and u^1, u^{11}, ... are derivatives of u and v_1, v_2 ,.... are successive integration of v then $\int uv\,dx = uv_1 - u^1 v_2 + u^{11} v_3 - u^{111} v_4 + \ldots$ where 'u' should be polynomial and v should have continuous integration.

30. $\int \frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx = \frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\tan x\right) + c$

31. $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}dx = \left(\frac{ac+bd}{c^2+d^2}\right)x + \frac{ad-bc}{c^2+d^2}\log|c\cos x + d\sin x| + k.$

32. $\int \log x dx = x\log x - x + c$

33. $\int \sin^{-1} x dx = x\sin^{-1} x + \sqrt{1-x^2} + c$

34. $\int \tan^{-1} x dx = x\tan^{-1} x - \frac{1}{2}\log(1+x^{2)} + c$

35. $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$ (or) $\frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

36. $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$ (or) $\frac{e^{ax}}{\sqrt{a^2+b^2}}\cos\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

37 $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + c$ or $\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x+\sqrt{x^2+a^2}) + c$

38. $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a} + c$ or $\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x+\sqrt{x^2-a^2}) + c$

39. $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

40. $\int (xf^1 + f)dx = xf + c$

41. $\int |x|dx = \frac{x|x|}{2} + c$

42. $\int \sin^n x\,dx = \frac{-\sin^{n-1}x\cdot\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx\ (n\in N)$

43. $\int \cos^n x\,dx = \frac{\cos^{n-1}x\cdot\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx\ (n\in N)$

44. $\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx\ (n\geq 2, n\in N)$

(Or)

If $I_n = \int \tan^n x\,dx$ then $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$

45. $\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+c$

46. $\int x\tan^{-1}x\,dx = \frac{1}{2}\left[(1+x^2)\tan^{-1}x - x\right]+c$

47. $\int x\sin^{-1}x\,dx = \frac{1}{4}\left[(2x^2-1)\sin^{-1}x + x\sqrt{1-x^2}\right]+c$

48. $\int\sqrt{\frac{1+x}{1-x}}\,dx = \sin^{-1}x - \sqrt{1-x^2}+c$

49. $\int \frac{1}{a\cos x + b\sin x}dx = \frac{1}{\sqrt{a^2+b^2}}\log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}-\frac{1}{2}\tan^{-1}\frac{b}{a}\right)\right]+c$

50. $\int \frac{\sec^2\theta}{(\sec\theta+\tan\theta)^n}d\theta = -\frac{1}{2}\left[\frac{1}{(n-1)t^{n-1}}+\frac{1}{(n+1)t^{n+1}}\right]$ when $t = \sec\theta + \tan\theta$.

51. $\int \frac{\sec x\,dx}{(\sec x+\tan x)^n} = -\frac{1}{n(\sec x+\tan c)^n}+C$

52. $\int x\sin x\,dx = -x\cos x + \sin x + C$

53. $\int x\cos x\,dx = -x\sin x + \cos x + C$

54. $\int x\,e^x\,dx = e^x(x-1)+C$

55. $\int \frac{dx}{a\cos x+b\sin x} = \frac{1}{\sqrt{a^2+b^2}}\left[\log\tan\left(\frac{\pi}{4}+\frac{x}{2}-\frac{1}{2}\tan^{-1}\frac{b}{a}\right)\right]$