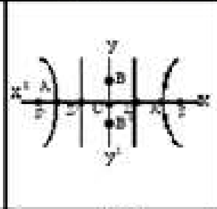
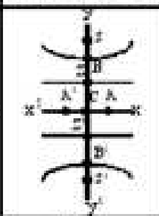
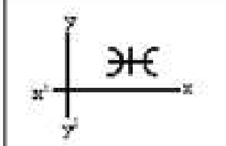
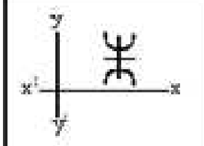
HYPERBOLA

SYNOPSIS

1. S is focus and the 'L' is the directrix. The locus of a point P is a hyperbola if $\frac{SP}{PM} = e\ (>1)$, e being a constant, PM being perpendicular to the fixed line 'L' from P.

2.

Four Standard forms of a hyperbola.

S. No.	Content	I	II	III	IV
	Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ where $b^2=a^2(e^2-1)$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ where $a^2=b^2(e^2-1)$	$\frac{(x-\alpha)^2}{a^2}-\frac{(y-\beta)^2}{b^2}=1$ where $b^2=a^2(e^2-1)$	$\frac{(x-\alpha)^2}{a^2}-\frac{(y-\beta)^2}{b^2}=-1$ where $a^2=b^2(e^2-1)$
	Figure				
1.	Centre (C)	(0, 0)	(0, 0)	(α, β)	(α, β)
2.	Vertices	$A, A^1 = (\pm a, 0)$	$B, B^1 = (0, \pm b)$	$(\alpha \pm a, \beta)$	$(\alpha, \beta \pm b)$
3.	Foci (S, S^1)	$(\pm ae, 0)$	$(0, \pm be)$	$(\alpha \pm ae, \beta)$	$(\alpha, \beta \pm be)$
4.	Z, Z^1	$(\pm a/e, 0)$	$(0, \pm b/e)$	$(\alpha \pm a/e, \beta)$	$(\alpha, \beta \pm b/e)$
5.	End of latus recta	$(\pm ae, \pm b^2/a)$	$(\pm a^2/b, \pm be)$	$(\alpha \pm ae, \beta \pm b^2/a)$	$(\alpha \pm a^2/b, \beta \pm be)$
6.	Eqn. of transverse axis	$y = 0$	$x = 0$	$y = \beta$	$x = \alpha$
7.	Eqn. of conjugate axis	$x = 0$	$y = 0$	$x = \alpha$	$y = \beta$
8.	Eqn's of latus rectum	$x = \pm ae$	$y = \pm be$	$x = \alpha \pm ae$	$y = \beta \pm be$
9.	Eqn's of directrices	$x = \pm a/e$	$y = \pm b/e$	$x = \alpha \pm ae$	$y = \beta \pm be$
10.	Length of transverse axis	2a	2b	2a	2b
11.	Length of conjugate axis	2b	2a	2b	2a
12.	Length of latus rectum	$2b^2/a$	$2a^2/b$	$2b^2/a$	$2a^2/b$
13.	Eccentricity (e)	$\sqrt{\frac{a^2+b^2}{a^2}}$	$\sqrt{\frac{b^2+a^2}{b^2}}$	$\sqrt{\frac{a^2+b^2}{a^2}}$	$\sqrt{\frac{b^2-a^2}{b^2}}$
14.	Diff of focal distance (focal radii) of a point p on the ellipse	$\lvert S^1P - SP \rvert = 2a$	$\lvert S^1P - SP \rvert = 2b$	$\lvert S^1P - SP \rvert = 2a$	$\lvert S^1P - SP \rvert = 2b$
15.	Distance between the foci	$SS^1 = 2ae$	$SS^1 = 2be$	$SS^1 = 2ae$	$SS^1 = 2be$
16.	Distance between vertices	$AA^1 = 2a$	$BB^1 = 2b$	$AA^1 = 2a$	$BB^1 = 2b$
17.	Distance between	$ZZ^1 = 2a/e$	$ZZ^1 = 2b/e$	$ZZ^1 = 2a/e$	$ZZ^1 = 2b/e$

3. The equation of the auxiliary circle is $x^2 + y^2 = a^2$

4. The equation of the director circle of the hyperbola is $x^2 + y^2 = a^2 - b^2$

5. The equation of the tangent at (x_1, y_1) to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1=0$ $(S_1 = 0)$

6. The line y = mx + c will be a tangent to the hyperbola if $c^2 = a^2m^2 - b^2$

7. If 'm' is the slope of any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then its equation is $y=mx\pm\sqrt{a^2m^2-b^2}$

8. The line lx + my + n = 0 will be a tangent to the hyperbola if $a^2 l^2 - b^2m^2 = n^2$.

9. The slopes of tangents drawn from (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$ are given by $m^2(x_1^2 - a^2) - 2mx_1y_1 + y_1^2 + b^2 = 0$.

10. 'α' is the angle between the tangents drawn from (x_1, y_1) to the hyperbola, then $\tan^2\alpha=\frac{4\left(-b^2x_1^2+a^2y_1^2+a^2b^2\right)}{\left(x_1^2+y_1^2-a^2+b^2\right)^2}$

11. The equation of the normal at (x_1, y_1) to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1} = a^2 + b^2$

12. The line lx + my + n = 0 is a normal to the hyperbola if $\frac{a^2}{\ell^2}-\frac{b^2}{m^2}=\frac{\left(a^2+b^2\right)^2}{n^2}$.

13. The equation of the chord joining the points (x_1,y_1) and (x_2,y_2) as the rectangular hyperbola $xy = c^2$ is $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$

14. The equation of the chord joining the points (a secθ, b tanθ) is

$$\frac{x}{a}\cos\frac{\theta-\phi}{2}-\frac{y}{b}\sin\frac{\theta+\phi}{2}=\cos\frac{\theta+\phi}{2}$$

15. The equation of the tangent at (a secθ, b tanθ) is $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$

16. The equation of the normal at (a secθ, b tanθ) is $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta} = a^2 + b^2$

17. From any point four normals can be drawn to a hyperbola.

18. The equation of the chord of contact of tangents from (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$is $S_1 = 0$.

19. The equation of the polar of (x_1, y_1) w.r.t. hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$is $S_1 = 0$

20. The pole of the line lx + my + n = 0 w.r.t. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\left(\frac{-a^2\ell}{n},\frac{b^2m}{n}\right)$

21. The lines $l_1x + m_1y + n_1 = 0$, $l_2x + m_2y + n_2 = 0$ are conjugate lines w.r.t. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $a^2l_1l_2-b^2m_1m_2 = n_1n_2$

22. The equation of the chord of the hyperbola S = 0 having its middle point at (x_1, y_1) is $S_1=S_{11}$

23. The midpoint of the chord of the hyperbola $\left[\frac{-a^2\ell n}{a^2\ell^2-b^2m^2},\frac{b^2mn}{a^2\ell^2-b^2m^2}\right]$

24. PN is the ordinate of any point P on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and AA' is the transverse axis. If Q divides AP in the ratio $a^2 : b^2$, then NQ is perpendicular to AP.

25. If e_1 and e_2 are the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$.

26. The equations of the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are $\frac{x}{a}-\frac{y}{b}=0$, $\frac{x}{a}+\frac{y}{b}=0$

27. The combined equation of the asymptotes of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

28. The angle between the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $2\sec^{-1}e$ **or** $2\tan^{-1}\left(\frac{b}{a}\right)$

29. If the asymptotes of a hyperbola are at right angles, then its eccentricity is $\sqrt{2}$

30. The hyperbola whose eccentricity is $\sqrt{2}$ is called a rectangular hyperbola.

31. The equation of a hyperbola and that of its asymptotes differ only in the constant term.

32. The polar of any point on one asymptote is parallel to that asymptote.

33. The points where the asymptotes meet the directricies lie on the auxilary circle of the hyperbola.

34. The foot of the perpendicular from the focus on any asymptote lies on the auxilary circle as well as on the corresponding directrix.

35. The equation of rectangular hyperbola w.r.t. the asymptotes as co-ordinate axes is $xy = c^2$.

36. The product of perpendiculars from any point on hyperbola to its asymptotes is $\frac{a^2b^2}{a^2+b^2}$

37. a) If the product of perpendiculars from a variable point to two given lines is a constant, then the locus of the point is a hyperbola.

b) If the two lines are perpendicular, then the locus of the point is a Rectangular Hyperbola

38. $x^2-y^2 = a^2$ takes the form $xy = \frac{a^2}{2}$ when the asymptotes are taken as its axes.

39. The tangent at a point P on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cuts one of its directrices in Q. Then PQ subtends a right angle at the corresponding focus.

40. The area of triangle formed by any tangent to the hyperbola and its asymptotes is ab.

41. The portion of any tangent to a hyperbola intercepted between the asymptotes is bisected at its point of contact.

42. The hyperbola and its conjugate hyperbola are having same asymptotes.

43. The tangent and normal at any point bisect the angle between the focal distances internally and externally.