FUNCTIONS

SYNOPSIS

1. A and B are any two sets. If to every element of A exactly one element of B is associated, the association is said to form a function (mapping) from A to B, and written as f: A→ B.
2. If f : A →B, A is called the domain of f and B is called the codomain of f.
3. The set of all images in f: A→B i.e. the set of all values of f(x) is called the range of f and is denoted by f(A) and $f(A) \subseteq B$.
4. A function f: A→B is one-one or an injection if different elements of A have different images.
5. A function f: A→B is onto, if f(A)=B. i.e. if corresponding each $b \in B$, we can find an element $a \in A$ such that $f(a) = b$.
6. If a function is both one - one and onto, then the function is a bijection.
7. A function f : A→B is said to be invertible if f is one-one and onto.
8. Many-one mapping : If the mapping f:A→B is such that two distinct elements a_1, a_2 of A have the same f image in B, then f is called a many one mapping or many one function.
9. Into mapping: If f: A→B is such that there is at least one element of B which is not the f-image of any element of A, then f is an into function from A to B.
10. Two functions f and g are said to be equal if

    (i) They are defined on the same domain A

    (ii) f(x) = g(x) for every $x \in A$.
11. If f: A→B, g:B→C, h : C→D, then ho(gof) = (hog) of.
12. The function I: A → A is said to be the identity function on A, if $f(a) = a$ for every $a \in A$, and is denoted by I_A.
13. If f: A→B, g:B→A are such that gof = I_A and fog = I_B , then $g = f^{-1}$ and also $f = g^{-1}$.
14. If f: A→B, g:B→C are both one-one , then gof is also one-one (gof) is the composite function of f and g.
15. **i)** If f:A→B, g:B→C are both onto, then gof : A→ C is also onto.

ii) $f:A \rightarrow B$ $g: B \rightarrow C$ are one-one functions then $gof: A \rightarrow C$ is also one-one.

iii) If 0(A) = m , 0(B)=n and m>n, then the number of one-one functions from A to B is zero

16. A function $f(x)$ is a increasing function if $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

17. A function $f(x)$ is a decreasing function if $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$.

18. If O(A) = m, O(B) = n, then number of mappings from A to B is n^m .

19. If O(A) = m, O(B) = n and $m \le n$, then the number of one-one functions from A to B is ${}^{n}p_{m}$.

20. If O(A) = m, O(B) = n and $n \le m$, then the number of onto functions from A to B is $n^m - {}^{n}c_1 (n-1)^m + {}^{n}c_2(n-2)^m - {}^{n}c_3(n-3)^m + \ldots\ldots$

21. If O(A) = n, O(B) = n, the number of bijections from A to B is ${}^{n}p_n = n!$

22. If two sets A and B have exactly same number of elements then everyone-one function from A to B is also onto and every onto function from A to B is also one-one.

23. If O(A) = n, O(B) = 2, the number of surjections from A to B is 2^n -2.

24. **(i)** If O(A) = n, the number of binary operations defined on A is n^{n^2}.

(ii) If 0(A)=n, the number of binary operations defined on A which are commutative is $n^{\frac{n(n+1)}{2}}$

25. If f: R→R is such that $f(x)$ = |x| or x^2, then f is neither one-one nor onto.

26. If f: R→R is such that $f(x) = x|x|$, then f is a bijection.

27. If D_1 and D_2 are the domains of f_1 and f_2 , then domain of $f_1 + f_2$ is $D_1 \cap D_2$ and the domain of $f_1 f_2$ is also $D_1 \cap D_2$.

28. If $f(x) = \frac{ax+b}{cx-a}$ then (fof)(x) (or) f[f(x)] = x

29.

Function	**Domain**	**Range**
1. sinx	R	[-1, 1]
2. cosx	R	[-1, 1]
3. tanx	$R-\left\{(2n+1)\frac{\pi}{2} / n \in Z\right\}$	R
4. cosecx	$R - \{n\pi / n \in Z\}$	R - (-1, 1)

5.	secx	R - $\left\{(2n+1)\frac{\pi}{2} / n \in Z\right\}$	R - (-1, 1)
6.	cotx	R - $\{n\pi / n \in Z\}$	R
7.	$\sin^{-1} x$	[-1, 1]	[-π/2, π/2]
8.	$\cos^{-1} x$	[-1, 1]	[0, π]
9.	$\tan^{-1} x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
10.	$\text{cosec}^{-1} x$	R-(-1, 1)	$\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$
11.	$\sec^{-1} x$	R - (-1, 1)	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
12.	$\cot^{-1} x$	R	(0, π)
13.	logx	R^+	R
14.	[x]	R	Z
15.	e^x	R	R^+
16.	a^x (a > 0)	R	(0, ∞)
17.	sin hx	R	R
18.	coshx	R	[1, ∞)
19.	tanhx	R	(-1, 1)
20	cothx	R–{0}	R – [-1,1]
21.	sechx	R	(0, 1]
22.	cosechx	R - {0}	R - {0}
23.	$\sinh^{-1} x$	R	R
24.	$\cosh^{-1} x$	[1, ∞)	[0, ∞)
25.	$\tanh^{-1} x$	(-1, 1)	R
26.	$\coth^{-1} x$	R – [-1,1]	R – {0}
27.	$\text{sech}^{-1} x$	(0,1]	[0, ∞)
28.	$\text{cosech}^{-1} x$	R – {0}	R –{0}

29. $\lvert x \rvert$	R	$[0, \infty)$
30. $\sqrt{x}$	$[0,\infty)$	$[0, \infty)$

30. Let $x \in R$, f(x) = [x] where f(x) = n (an integer) such that $n \le x < n+1$.[x +k] = [x] +k where 'k' is an integer.

31. If 0(A) = m, 0(B)=n, then the number of constant functions from A to B is n

32. If f(x+y) = f(x) + f(y), then f(x) is an odd function.

33. If f: A $\to$ B and g: R $\to$ C are functions such that gof: A $\to$ C is one – one, then 'f' is necessarily one –one

34. If f: A $\to$ B and g : B $\to$ C are functions such that gof: A $\to$ C is onto, then 'g' is necessarily onto.

35. If 0(A) =m, 0(B)=n and m<n, then the number of onto functions form A to B is zero

36. If f(x+y) = f(x), then f(x) is a constant function

37. If $f(x) + f\left(\frac{1}{x}\right) = f(x).\, f\left(\frac{1}{x}\right)$, then $f(x) = 1 \pm x^n$.

38. If f(xy) = f(x). f(y) then $f(x) = x^n$

39. If f(x+y) = f(x). f(y) then $f(x) = a^x$.

40. If f(x+y) = f(x) + f(y) then f(n) = nf(1) where $n \in W$

41. If f(xy) = f(x) + f(y) then f(x) = k logx

42. If f(x+y) f(x-y) = 2f(x)f(y) then $f(x) = \frac{a^x + a^{-x}}{2}$

43. If a function is strictly increasing or decreasing then it is an injection.

44. The graph of the function y = f(x) is symmetrical about the line x=a, then f(a+x) = f(a-x).

45. If f(x,y) =0, then the image of the curve with respect to the x-axis is f(-x,y)=0

46. If f(x,y) =0 then

i) The image of the curve with respect to the X-axis is f(-x,y) =0

ii) The image of the curve with respect to the Y-axis is f(x,-y) =0

iii) The image of the curve with respect to the to Origin is f(-x,-y)=0

47. $f(x) = \frac{1}{a\cos x + b\sin x + c}$

Case I: If $0 \notin \left(c-\sqrt{a^2+b^2}, c+\sqrt{a^2+b^2}\right)$ then Range of f(x) is $\left[\frac{1}{c+\sqrt{a^2+b^2}}, \frac{1}{c-\sqrt{a^2+b^2}}\right]$

Case II: If $0 \in \left(c-\sqrt{a^2+b^2}, c+\sqrt{a^2+b^2}\right)$ then Range of f(x) is

$$\left(-\infty, \frac{1}{c-\sqrt{a^2+b^2}}\right] \cup \left[\frac{1}{c+\sqrt{a^2+b^2}}, \alpha\right)$$

Case III: If $c-\sqrt{a^2+b^2}=0$ then Range of f(x) is $\left[\frac{1}{c+\sqrt{a^s+b^2}}, \infty\right)$

Case IV: If $c+\sqrt{a^2+b^2}=0$ then Range of f(x) is $\left(-\infty, \frac{1}{c-\sqrt{a^2+b^2}}\right]$

* * *