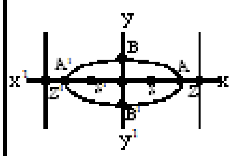
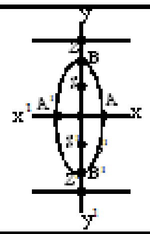
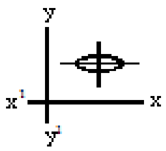
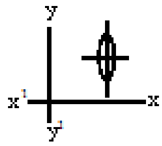
ELLIPSE

SYNOPSIS

1. S is a fixed point (focus) and 'l' is a fixed line (directrix). If P is a variable point such that $\frac{SP}{PM} = e$, where e < 1 and PM is perpendicular to 'l', the locus of P is an ellipse.

2.

Four Standard forms of an ellipse

S. No. Content	I	II	III	IV
Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ where $b^2=a^2(1-e^2)$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a<b)$ where $a^2=b^2(1-e^2)$	$\frac{(x-\alpha)^2}{a^2}+\frac{(x-\beta)^2}{b^2}=1(a>b)$ where $b^2=a^2(1-e^2)$	$\frac{(x-\alpha)^2}{a^2}+\frac{(x-\beta)^2}{b^2}=1(a<b)$ where $a^2=b^2(1-e^2)$
Figure				
1. Centre (C)	(0, 0)	(0, 0)	(α, β)	(α, β)
2. Vertices	$A, A^1 = (\pm a, 0)$	$B, B^1 = (0, \pm b)$	$(\alpha, \pm a, \beta)$	$(\alpha, \beta \pm b)$
3. Foci (S, S^1)	$(\pm ae, 0)$	$(0, \pm be)$	$(\alpha \pm ae, \beta)$	$(\alpha, \beta \pm be)$
4. Z, Z^1	$(\pm a/e, 0)$	$(0, \pm b/e)$	$(\alpha \pm a/e, \beta)$	$(\alpha, \beta \pm b/e)$
5. End points of latararecta	$(\pm ae, \pm b^2/a)$	$(\pm a^2/b, \pm be)$	$(\alpha \pm ae, \beta \pm b^2/a)$	$(\alpha \pm a^2/b, \beta \pm be)$
6. Eqn. of major axis	$y = 0$	$x = 0$	$y = \beta$	$x = \alpha$
7. Eqn. of minor axis	$x = 0$	$y = 0$	$x = \alpha$	$y = \beta$
8. Eqn's of latusrectum	$x = \pm ae$	$y = \pm be$	$x = \alpha \pm ae$	$y = \beta \pm be$
9. Eqn's of directrices	$x = \pm a/e$	$y = \pm b/e$	$x = \alpha \pm ae$	$y = \beta \pm be$
10. Length of major axis	2a	2b	2a	2b
11. Length of minor axis	2b	2a	2b	2a
12. Length of latusrectum	$2b^2/a$	$2a^2/b$	$2b^2/a$	$2a^2/b$
13. Eccentricity (e)	$\sqrt{\frac{a^2-b^2}{a^2}}$	$\sqrt{\frac{b^2-a^2}{b^2}}$	$\sqrt{\frac{a^2-b^2}{a^2}}$	$\sqrt{\frac{b^2-a^2}{b^2}}$
14. Sum of focal distance (focal radii) of a point p on the ellipse	$SP + S^1P = 2a$	$SP + S^1P = 2b$	$SP + S^1P = 2a$	$SP + S^1P = 2b$
15. Distance between the foci	$SS^1 = 2ae$	$SS^1 = 2be$	$SS^1 = 2ae$	$SS^1 = 2be$
16. Distance between vertices	$AA^1 = 2a$	$BB^1 = 2b$	$AA^1 = 2a$	$BB^1 = 2b$
17. Distance between	$ZZ^1 = 2a/e$	$ZZ^1 = 2b/e$	$ZZ^1 = 2a/e$	$ZZ^1 = 2b/e$

3. **Notation:**

i) $S = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$ ii) $S_1 = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$ iii) $S_{11} = \frac{{x_1}^2}{a^2} + \frac{{y_1}^2}{b^2} - 1$ iv) $S_{12} = \frac{x_1x_2}{a^2} + \frac{y_1y_2}{b^2} - 1$

4. Let $P(x_1, y_1)$ be a point $S = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ be an ellipse. Then

i) P lies on the ellipse if $S_{11} = 0$

ii) P lies inside the ellipse if $S_{11} < 0$

iii) P lies outside the ellipse if $S_{11} > 0$

5. Two tangents can be drawn to an ellipse from an external point.

6. The equation of tangent at $P(x_1, y_1)$ to the ellipse $S = 0$ is $S_1 = 0$.

7. The equation of the auxilary circle is $x^2 + y^2 = a^2$ for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$.

8. The locus of the point of intersection of perpendicular tangents to an ellipse is the Director circle of the ellipse and its equation is $x^2 + y^2 = a^2 + b^2$.

9. The equation of the tangent at (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$ $(S_1 = 0)$

10. The line $y = mx + c$ is a tangent to the ellipse if $c^2 = a^2m^2 + b^2$

11. The line $lx + my + n = 0$ is a tangent to the ellipse if $a^2l^2 + b^2m^2 = n^2$

12. If m is the slope of any tangent to the ellipse, then its equation is $y = mx \pm \sqrt{a^2m^2 + b^2}$

13. If tangents are drawn from (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then their slopes are obtained from $m^2(x_1^2 - a^2) - 2mx_1y_1 + (y_1^2 - b^2) = 0$

14. If 'α' is the angle between the tangents drawn from (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

$$\tan^2\alpha = \frac{4s_{11}}{\left(x_1^2 + y_1^2 - a^2 - b^2\right)^2}$$

15. The tangent at any point p on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the tangents at A, A' in L and M respectively. Then $AL.A'M = b^2$

16. The minimum area of the triangle formed by any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the coordinate axes is ab

17. The equation of the normal at (x_1, y_1) is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}$ = a^2 - b^2

18. Equation of normal drawn at (ae, b^2/a) is x - ey = ae^3.

19. The line lx + my + n = 0 is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$.

20. The normal at one end of latusrectum of the ellipse passes through one end of minor axis. Then $e^4 + e^2 = 1$

21. The maximum distance of any normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from the centre is a-b

22. The tangents at the ends of the focal chord meet on the directrix.

23. The foot of the perpendicular from any focus to any tangent to the ellipse lies on the auxilary circle of the ellipse.

24. The product of the perpendiculars from the foci on any tangent to the ellipse is the square of the semi minor axis.

25. From any point, four normals can be drawn to an ellipse.

26. The equation of the chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 joining the points (a cosθ, b sinθ), (acosϕ, bsinϕ) is $\left(\frac{x}{a}\cos\frac{\theta+\phi}{2}+\frac{y}{b}\sin\frac{\theta+\phi}{2}\right)=\cos\frac{\theta-\phi}{2}$

27. If θ and ϕ are the ends of a focal chord, then e $\cos\left(\frac{\theta+\phi}{2}\right)$ = $\cos\left(\frac{\theta-\phi}{2}\right)$.

28. If the chord joining the variable points θ and ϕ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ subtends a right angle at the point (a, 0), then $\tan\frac{\theta}{2}.\tan\frac{\phi}{2}=\frac{-b^2}{a^2}$

29. The equation of the tangent at (a cosθ, bsinθ) to $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$

30. The equation of the normal at (a cosθ, b sinθ) is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}$ = a^2 - b^2

31. The equation of the chord of contact of tangents from (x_1, y_1) to $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is $S_1 = 0$

32. The equation of the polar of (x_1, y_1) w.r.t. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $S_1 = 0$

33. The pole of the line $lx + my + n = 0$ w.r.t. $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is $\left(\frac{-a^2l}{n}, \frac{-b^2m}{n}\right)$

34. Two lines $l_1x + m_1y + n_1 = 0$, $l_2x + m_2y + n_2 = 0$ are conjugate lines w.r.t. the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ if $a^2l_1l_2 + b^2m_1m_2 = n_1n_2$.

35. The midpoint of the chord of $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ on the line $lx + my + n = 0$ is $\left[\frac{-a^2ln}{a^2l^2+b^2m^2}, \frac{-b^2mn}{a^2l^2+b^2m^2}\right]$

36. Length of the chord intercepted by $y = mx + c$ on the ellipse is $\frac{2ab\sqrt{b^2+a^2m^2-c^2}}{b^2+a^2m^2}\sqrt{1+m^2}$

37. The centre of the ellipse or hyperbola is obtained by solving the equations $ax+hy+g=0$, $hx+by+f=0$.

38. From any point, 4 normals can be drawn to an ellipse and the sum of eccentric angles of the feet of the normals is an odd multiple of π.

39. A circle cuts an ellipse in 4 points real or imaginary and the sum of the eccentric angles of the points of intersection is an even multiple of π.

40. If the normals at (x_1, y_1) (x_2, y_2) (x_3, y_3) (x_4, y_4) to an ellipse are concurrent, then $(x_1+x_2+x_3+x_4)\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}\right) = 4$.

41. Area of the ellipse with axes 2a, 2b is πab.

42. The maximum area of a rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with sides parallel to the axes is 2ab.

43. The foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are S and S', $P(\theta)$ is a point on the ellipse then the in centre of $\Delta SPS' = \left(ae\cos\theta, \frac{be\sin\theta}{1+e}\right)$

44. If P is a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci S and S', then maximum area of Δ SPS' = abe

45. A, A' are the vertices and S, S' are the foci of an ellipse. The normal at any point P on the ellipse is the angular bisector of $\angle S'PS$

46. P is a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and Q is the corresponding point on the auxiliary circle. If the normals at P and Q meet in R, then the equation of locus of R is $x^2+y^2=(a+b)^2$

47. The tangents at the ends of the focal chord meet on the directrix.

48. The maximum area of a rectangle that can be inscribed in the ellipse S = 0 is 2ab sq. units and the sides are $a\sqrt{2}$, $b\sqrt{2}$.

49. Area of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 is πab sq. units.