DIFFERENTIAL EQUATIONS

SYNOPSIS

An equation involving one dependent variable, one or more independent variables and the differential coefficients of dependent variable with respect to independent variable is called a **Differential Equation**.

The order of a differential equation is the order of the highest order derivative appearing in the equation.

The degree of the differential equation is the power of the highest derivative involved in the differential equation, when the equation has been expressed in the form of a polynomial by eliminating radicals or fractional powers of the derivatives.

The degree of the differential equation $y = \cos\frac{dy}{dx}$ and $x = y + \log\frac{dy}{dx}$ cannot be determined and hence undefined because these equations cannot be expressed as a polynomial equation in $\frac{dy}{dx}$.

A relation between the variables without derivatives of a differential equation is said to be a solution.

The solution which containing as many as arbitrary constants as the order of the differential equation is called general solution.

Solution obtained by giving particular values of the arbitrary constants in the general solution of the differential equation is called a particular solution.

3. i) If $\frac{dy}{dx} = \frac{f(x)}{g(y)} \Rightarrow \int g(y)dy = \int f(x)dx + c$. ii) $\frac{dy}{dx} = f(ax+by)$ then put ax + by = z.

4. i) $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ is a homogeneous differential equation of first order where f(x, y) and g(x, y) are homogeneous functions of the same order.

ii) It can be reduced to variables separable form by substituting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

iii) On putting y = vx, the solution is obtained from $\int \frac{dv}{f(1,v)-v} = \log(cx)$.

5. $\frac{dy}{dx} = \frac{ax+by+c}{a^1x+b^1y+c^1}$, where a, b, c, a^1, b^1, c^1 are constants, is called the non-homogeneous differential equation of first order.

i) If $\frac{a}{a^1} = \frac{b}{b^1}$ then put ax + by = z.

ii) If $\frac{a}{a^1} \neq \frac{b}{b^1}$ then put x = X + h, y = Y + k$\Rightarrow$ where (h, k) is obtained by solving $ax+by+c= 0$, $a^1x + b^1y + c^1 = 0$.

iii) If $a_2 + b_1 = 0$ then the solution is obtained by proper grouping of the terms.

6. i) The general form of a linear differential equation of first order is $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x (or) constants.

ii) $e^{\int Pdx}$ is called the integrating factor (I.F.)

iii) The solution of the above equation is given by y(I.F.) = $\int Q.(I.F.)dx + C$.

iv) Similarly, the solution of the equation $\frac{dx}{dy} + Px = Q$ is x (I.F.)= $\int Q.(I.F.)dy + c$, where I.F.= $e^{\int pdy}$.

v) An equation of the form $\frac{dy}{dx}$ + P(x). y = Q(x). y^n where P(x) and Q(x) are functions of x only is called a Bernoulli's equation. Divide both sides with y^n we get

$y^{-n}\frac{dy}{dx} + P.\, y^{-n+1} = Q$ put $y^{-n+1} = v$.

7. **Equation of family of curves** — **Differential Equations**

(a) $y = A_1 e^{\alpha_1 x} + A_2 e^{\alpha_2 x}$ — $y_2 - (\alpha_1 + \alpha_2) \cdot y_1 + \alpha_1 \alpha_2 y = 0$

(b) $y = e^{\alpha x}(A_1 x + A_2)$ — $y_2 - 2\alpha y_1 + \alpha^2 y = 0$

(c) $y = e^{\alpha x}(A_1 \cos\beta x + A_2 \sin\beta x)$ — $y_2 - 2\alpha y_1 + (\alpha^2 + \beta^2) y = 0$

d) $y = A, x^m + A_2 x^n$ — $x^2 y_2 - (m+n-1) xy_1 + mny = 0$

e) $y = A_1 e^{\alpha_1 x} + A_2 e^{\alpha_2 x} + A_3 e^{\alpha_3 x}$ — $y_3 - (\alpha_1+\alpha_2+\alpha_3) y_2 + (\alpha_1 \alpha_2 + \alpha_2 \alpha_3 + \alpha_3 \alpha_1)$

$y_1 - \alpha_1 \ \alpha_2 \ \alpha_3 y = 0$

f) $y = e^{\alpha x}(Ax^2 + Bx + C)$ — $y_3 - 3\alpha y_2 + 3\alpha^2 y_1 - \alpha^3 y = 0$

g) $y = A_1 e^{\alpha x} + e^{\beta x}(A_2 \cos \gamma x + A_3 \sin\gamma x)$ — $y_3 - (\alpha+2\beta)y_2 + (2\alpha\beta+\beta^2+\gamma^2)y_1 - \alpha(\beta^2+\gamma^2)y = 0$

8. $xdx + ydy = \frac{1}{2} d(x^2 + y^2)$

9. $xdy + ydx = d(xy)$

10. $\frac{ydx - xdy}{y^2} = \frac{-(xdy - ydx)}{y^2} = d\left(\frac{x}{y}\right)$

11. $\frac{xdy - ydx}{x^2} = \frac{-(ydx - xdy)}{x^2} = d\left(\frac{y}{x}\right)$

12. $\frac{xdy - ydx}{xy} = -\left(\frac{ydx - xdy}{xy}\right) = d\left(\log \frac{y}{x}\right)$

13. $\frac{xdy - ydx}{x^2 + y^2} = \frac{-(ydx - xdy)}{x^2 + y^2} = d\left(\mathrm{Tan}^{-1} \frac{y}{x}\right)$