DEFINITE INTEGRATION

SYNOPSIS

1.If $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$.

2. $\int_a^a f(x)dx = 0$.

3. $\int_a^b f(x)dx = \int_a^b f(y)dy = \int_a^b f(t)dt$.

4. $\int_a^b f(x)dx = \int_b^a f(x)dx$

5. $\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + ... + \int_{c_n}^b f(x)dx$ Where $a < c_1 < c_2 < \ldots\ldots\ldots < c_{n-1} < c_n < b$.

6. $\int_0^a f(x)dx = \int_a^a f(a-x)dx$

7. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

8. $\int_a^b f(x)dx = 0$ if $f(a + x) = -f(b - x)$.

9. $\int_a^b f(x)dx = 2\int_a^{\frac{a+b}{2}} f(x)dx$ if $f(a + x) = f(b - x)$.

10. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$; if $f(x)$ is an even function.

= 0; if $f(x)$ is an odd function.

11. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$; if $f(2a-x) = f(x)$

$= 0$; if $f(2a-x) = -f(x)$.

12. $\int_0^a xf(x)dx = \frac{a}{2}\int_0^a f(x)dx$; if $f(a-x) = f(x)$

$= a .. \int_0^{a/2} f(x)dx$

13. $\int_a^b xf(x)dx = \frac{a+b}{2}\int_a^b f(x)dx$; if $f(a+b-x) = f(x)$.

14. If f(x) is a periodic function with period "T "then $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx. n \in N$.

15. If $f : R \to R$ is a continuous periodic function with period 'T' and $a \in R$ and n is a positive integer then $\int_a^{a+nT} f(x)dx = n\int_a^{a+T} f(x)dx = n\int_0^T f(x)dx$, $n \in N$.

16. If f(x) is a periodic function with period T and $a \in R^+$ then $\int_{nT}^{a+nT} f(x)dx = \int_a^a f(x)dx$, $n \in N$.

17. If f(x) is a periodic function with period 'T' then

(i) $\int_{a+T}^{b+T} f(x)dx = \int_a^b f(x)dx$ Or $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$ where $n \in Z$;

(ii) $\int_{mT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$ Where $m, n \in Z$.

18. $\frac{d}{dx}\left(\int_{\varphi(x)}^{\phi(x)} f(t)dt\right) = f(\phi(x))\phi'(x) - f(\varphi(x))\varphi'(x)$.

19. If $f(x) \geq 0$ in $[a, b]$ then $\int_a^b f(x)dx \geq 0$.

20.If f(x) ≤ g(x) on [a, b] then $\int_a^b f(x)dx \le \int_a^b g(x)dx$.

21.If m, M are smallest and greatest value of a function $f(x)$ defined on [a, b] then $m\,(b - a) \le \int_a^b f(x)dx \le M\,(b - a)$.

22.If $I_n = \int_a^{\pi/2} \sin^n x dx = \int_a^{\pi/2} \cos^n x dx$, then $I_n = \frac{n-1}{n} . I_{n-2}$ where $n \in N$.

$$\int_a^{\pi/2} \sin^n x dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right).....\frac{1}{2}.\frac{\pi}{2}\text{; if n is even.}$$

$$= \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right).....\frac{2}{3}\text{; if n is odd.}$$

23.If $I_n = \int_0^{\pi/4} \tan^n x dx$ then $I_n = \frac{1}{n-1} - I_{n-2}$ and hence $I_n = \frac{1}{n-1} - \frac{1}{n-3} + \frac{1}{n+5} - \frac{1}{n-7} + ...I_0$ or I_1 according as n is even or odd where $I_0 = \frac{\pi}{4}$; $I_1 = \frac{1}{2}\log 2$.

24. $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)dx = \frac{1}{n-1}$.

25. $\int_{\pi/4}^{\pi/2} (\cot^n x + \cot^{n-2} x)dx = \frac{1}{n-1}$.

26. If $I_n = \int_0^{\pi/4} \sec^n x dx$ then $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$.

27. $\int_0^{\pi/2} \sin^m x.\cos^n x dx = \frac{[(m-1)(m-3)(m-5).....][(n-1)(n-3)(n-5).....]}{(m+n)(m+n-2)(m+n-4)(m+n-6).....}.K$. If m and n are both even then $K = \frac{\pi}{2}$, otherwise K = 1.

28. $\int_0^{\pi} (\sin^n x)dx = 2\int_0^{\pi/2} (\sin^n x)dx$ for all positive integral values of n.

29. If n is an even natural number, then $\int_0^{\pi} \cos^n x dx = 2\int_0^{\pi/2} \cos^n x dx$ and $\int_0^{2\pi} \cos^n x dx = 4\int_0^{\pi/2} \cos^n x dx$, and $\int_0^{2\pi} \sin^n x dx = 4\int_0^{\pi/2} \sin^n x dx$.

30. If n is an odd natural number then $\int_0^{2\pi} \sin^n x dx = 0$; $\int_0^{2\pi} \cos^n x dx = 0$.

Standard Results

31. $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx = \frac{a}{2}$.

32. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx = \frac{b-a}{2}$.

33. $\int_0^a \sqrt{a^2-x^2} dx = \frac{\pi a^2}{4}$.

34. $\int_0^a \frac{1}{\sqrt{a^2-x^2}} dx = \frac{\pi}{2}$.

35. $\int_0^{\infty} \frac{1}{x^2+a^2} dx = \frac{\pi}{2a}$.

36. $\int_a^b \sqrt{\frac{x-a}{b-x}} dx = \frac{\pi}{2}(b-a)$.

37. $\int_a^b \sqrt{(x-a)(b-x} dx = \frac{\pi}{8}(b-a)^2$.

38. $\int_a^b \frac{1}{\sqrt{(x-a)(b-x}} dx = \pi$

39. $\int_0^a x(a-x)^n dx = \frac{a^{n+2}}{(n+1)(n+2)}$.

40. $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx = \frac{\pi}{4}$.

41. $\int_0^{\pi/2} \frac{f(\tan x)}{f(\tan x)+f(\cot x)} dx = \frac{\pi}{4}$.

42. $\int_0^{\pi/2} \frac{f(\sec x)}{f(\sec x)+f(\operatorname{cosec} x)}dx = \frac{\pi}{4}$.

43. $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}dx = (a+b)\frac{\pi}{4}$.

44. $\int_0^{\pi/2} \frac{a\tan x + b\cot x}{\tan x + \cot x}dx = (a+b)\frac{\pi}{4}$.

45. $\int_0^{\pi/2} \frac{a\sec x + b\operatorname{cosec} x}{\sec x + \operatorname{cosec} x}dx = (a+b)\frac{\pi}{4}$.

46. $\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\sin 2x)dx = \frac{\pi}{2}\log 2$.

47. $\int_0^{\pi/2} \log\cos x dx = -\frac{\pi}{2}\log 2$.

48. $\int_0^{\pi/2} \log\sec x dx = \frac{\pi}{2}\log 2$.

49. $\int_0^{\pi/2} \log\operatorname{cosec} x dx = \frac{\pi}{2}\log 2$.

50. $\int_0^{\pi/2} \log\tan x dx = 0$.

51. $\int_0^{\pi/2} \log\cot x dx = 0$.

52. $\int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx = \sqrt{2}\log(\sqrt{2}+1)$.

53. $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{2ab}$.

54. $\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{ab}$.

55. $\int_0^{\infty} e^{-ax}\cos bx dx = \frac{a}{a^2+b^2}$.

56. $\int_0^\infty e^{-ax}\sin bx dx = \dfrac{b}{a^2+b^2}$.

57. $\int\limits_0^\infty \dfrac{1}{(x+\sqrt{x^2+1})^n}dx = \dfrac{n}{n^2-1}$.

58. $\int\limits_0^n [x]dx = \dfrac{n(n-1)}{2}$, n ∈ N.

59. $\int\limits_0^n [x^2]dx = n(n^2-1) - \left(\sqrt{1}+\sqrt{2}+.....+\sqrt{n^2-1}\right)$.

60. $\int\limits_0^{n^2} [\sqrt{x}]dx = n^2(n-1) - \left(1^2+2^2+.....+(n-1)^2\right)$.