CIRCLES

SYNOPSIS

1. The equation of the circle with centre at the origin and radius 'a' is $x^2 + y^2 = a^2$.

   The equation of the circle with centre (h, k) and radius 'r' is $(x - h)^2 + (y - k)^2 = r^2$.

2. The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre (-g, -f) and radius $\sqrt{g^2 + f^2 - c}$.

3. The centre and radius of the circle $ax^2 + ay^2 + 2gx + 2fy + c = 0$ are $\left(-\frac{g}{a}, -\frac{f}{a}\right)$ and $\frac{\sqrt{g^2 + f^2 - ac}}{|a|}$

4. a. The equation of the circle on the line joining (x_1, y_1), (x_2, y_2) as diameter is

   $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

   b. The equation of the circle on the line joining (x_1, y_1), (x_2, y_2) as diameter is

   $x^2+y^2 - (x_1+x_2)x - (y_1+y_2)y + x_1x_2 + y_1y_2 = 0$.

5. The equation of the circle through the points (0, 0), (a, 0), (0, b) is $x^2 + y^2 - ax - by = 0$.

6. If the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ cut the co-ordinate axis in concyclic points, then $a_1a_2 = b_1b_2$ and the equation of the circle is $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$ leaving the xy term.

7. i) Through two points, infinite circles can be drawn.

   ii) Through three non collinear points, only one circle can be drawn and through three collinear points, no circle can be drawn.

   iii) If three lines are concurrent, no circle can be drawn which touches all the three lines.

   iv) If three lines are not concurrent and no two of them are parallel, four circles can be drawn which touch all the three lines.

   v) If three lines are not concurrent and two of them are parallel, only two circles can be drawn touching them.

8. The equation of the circle passing through the points A, B is of the form:

   equation of the circle on AB as diameter + k(equation of AB) = 0

9. The equations $x = x_1+r\cos\theta$, $y = y_1 + r\sin\theta$ where r is a constant and θ is the parameter represent the circle with centre (x_1, y_1) and radius 'r'.

10. The conditions for the general equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent a circle are i) $a = b$ ii) $h = 0$ iii) $g^2 + f^2 - ac \geq 0$

11. The circle passing through A and B and having least radius is the circle on AB as diameter.

12. Two circles having the same centre are said to be concentric.

13. The equation of the circle concentric with the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is of the form $x^2+y^2 + 2gx + 2fy + \lambda = 0$

14. The point $P(x_1, y_1)$ lies outside, on or inside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ according as $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c \gtreqless 0$.

15. Power of $P(x_1, y_1)$ w.r.t. $x^2 + y^2 + 2gx + 2fy + c = 0$ is $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$, (S_{11}).

16. If P is any point in the plane of a circle with centre 'O' and radius 'r', the power of P w.r.t. the circle is $OP^2 - r^2$.

17. If a secant through P cuts the above circle in A and B, power of P w.r.t. the circle is PA.PB

18. If P lies outside the circle, power of P w.r.t. the circle is PT^2 where PT is the length of the tangent from P to the circle.

19. The least and the greatest distances of a given point from any point on the circle are (d-r) and (d+r) where d is the distance of the given point from the centre of the circle and r is its radius.

20. The equation of the chord joining 'α', 'β' on the circle

$$S = 0 \text{ is } (x+g)\cos\frac{\alpha+\beta}{2} + (y+f)\sin\frac{\alpha+\beta}{2} = r\cos\frac{\alpha-\beta}{2}$$

21. Equation of chord with midpoint (x_1, y_1) w.r.t. the circle $S = 0$ is $S_1 = S_{11}$.

22. The equation of the tangent at (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $S_1 = 0$.

i.e., $xx_1+yy_1+g(x+x_1)+ f(y+y_1) + c = 0$. Slope of this tangent $= -(x_1 + g)/(y_1 +f)$

23. Any line will be tangent to a given circle if the length of the perpendicular from the centre of the circle to it is equal to the radius of the circle.

24. The line $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(m^2 + 1)$.

25. If m is the slope of any tangent to the circle $x^2+y^2=a^2$, then its equation is $y=mx\pm a\sqrt{m^2+1}$.

26. The circle $x^2+y^2+2gx+2fy+c=0$ touches the x-axis if $g^2=c$ and the y-axis if $f^2=c$.

27. The equation of any circle touching the X-axis is of the form

$x^2+y^2+2gx+2fy+g^2=0$ and that touching y - axis is of the form

$x^2+y^2+2gx+2fy+f^2=0$

28. The equation of any circle touching both the axes is of the form $x^2+y^2\pm 2gx\pm 2gy+g^2=0$

29. The equation of the circle having radius 'a' and centre in the first quadrant and touching both the axes is $x^2+y^2-2ax-2ay+a^2=0$

30. If two circles which touch both the axes in the first quadrant meet in (x_1, y_1), then the length of the common chord is $\sqrt{2}|x_1-y_1|$

31. The length of the tangent from (x_1, y_1) to $S = x^2+y^2+2gx+2fy+c=0$ is $\sqrt{S_{11}}$ i.e. $\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$.

32. The length of the intercept made by a circle on a line =

$2\sqrt{(\text{radius})^2-(\text{length of the perpendicular from the centre to the line})^2}$

33. Length of the intercept made by the circle $x^2+y^2+2gx+2fy+c=0$ on the x-axis is $2\sqrt{g^2-c}$ and on the y-axis is $2\sqrt{f^2-c}$.

34. The equation of the pair of tangents from (x_1,y_1) to the circle $S=0$ is $S_1^2=SS_{11}$.

35. Angle between the tangents drawn from a point (x_1, y_1) to a circle $S=0$ is $2\tan^{-1}\left(\frac{r}{\sqrt{S_{11}}}\right)$

36. The pair of tangents from (0, 0) to the circle $x^2+y^2+2gx+2fy+c=0$ are at right angles if $g^2+f^2=2c$

37. Area of the quadrilateral formed by pair of tangents from (x_1, y_1) and the corresponding radii is $r.\sqrt{s_{11}}$

38. Two tangents from a point to a circle are at right angles if the length of the tangent from the point to the circle is equal to the radius of the circle.

    a. The locus of the point of intersection of two perpendicular tangents drwan to the circle $x^2+y^2=a^2$ is $x^2+y^2=2a^2$.

b. The locus of the point of intersection of the two tangents drawn to the circle $x^2+y^2=a^2$ which include an angle α is $x^2 + y^2 = a^2 \operatorname{cosec}^2 \alpha/2$.

c. The locus of the point of intersection of the two tangents drawn to the circle $x^2+y^2+2gx+2fy+c=0$ which include an angle α is $(x+g)^2 + (y+f)^2 = r^2 \operatorname{cosec}^2 \alpha/2$ where $r^2 = g^2 + f^2 - c$.

39. The equation of a tangent to the circle $x^2+y^2+2gx+2fy+c=0$ and having slope 'm' is $y+f = m(x+g) \pm \sqrt{g^2+f^2-c}\sqrt{1+m^2}$.

40. The equation of tangent at 'θ' on S = 0 is $(x+g)\cos\theta + (y+f)\sin\theta = r$.

41. The equation of tangent at 'θ' on $x^2 + y^2 = a^2$ is $x\cos\theta + y\sin\theta = a$.

42. The slopes of tangents drawn from an external point $P(x_1, y_1)$ to the circle S = 0 are given by $m^2(x_1^2 - a^2) - 2mx_1y_1 + y_1^2 - a^2 = 0$.

43. The equation of the normal at $P(x_1,y_1)$ to the circle $x^2 + y^2 = a^2$ is $xy_1 - yx_1 = 0$

44. The locus of poles of chords of the circle $x^2+ y^2=a^2$ subtending an angle α at the centre is $x^2 + y^2= a^2 \sec^2 \alpha/2$.

45. The normal through a point P to a circle is the line through P and the centre of the circle.

46. The square of the lengths of the tangetns from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ form an A.P. Then a^2, b^2, c^2 form A.P.

47. A tangent at a point on the circle $x^2 + y^2 = a^2$ intersects a concentric circle S at P and Q. The tangents of this circle at P, Q meet on the circle $x^2 + y^2 = b^2$. The equation of the concentric circle S is $x^2 + y^2 = ab$.

48. The area of the triangle formed by the tangent at (x_1, y_1) to the circle $x^2 + y^2 = a^2$ and the coordinate axes is $\frac{a^4}{2|x_1y_1|}$.

49. OA, OB are tangents from origin to the circle $x^2 + y^2 + 2gx + 2fy + c=0$ and C is the centre of the circle. Then

i) O, A, B, C are concyclic

ii) Centre of the circumcircle of $\Delta OAB = \left(-\frac{g}{2}, -\frac{f}{2}\right)$.

iii) Radius of circumcircle of $\Delta OAB = \frac{\sqrt{g^2+f^2}}{2}$

iv) Area of the quadrilateral OACB = $\sqrt{c(g^2+f^2-c)}$.

v) Equation of the circumcircle of ΔOAB is $x^2 + y^2 + gx + fy = 0$.

50. A line meets the circle $x^2 + y^2 = a^2$ in two points P and Q equidistant d from (x_1, y_1) on the circle. The equation to PQ is $x_1x + y_1y = a^2 - \frac{d^2}{2}$

51. a. The equation of the chord of contact of tangents from (x_1, y_1) to the circle S = 0 is $S_1 = 0$.

b. The chord of contact of P to a circle is perpendicular to OP, O being the centre of the circle.

c. The area of the triangle formed by the tangents from (x_1, y_1) to the circle $x^2 + y^2 = a^2$ and its chord of contact is $\frac{a(x_1^2+y_1^2-a^2)^{3/2}}{x_1^2+y_1^2}$ i.e., $\frac{a(s_{11})^{3/2}}{x_1^2+y_1^2}$.

d. The area of the triangle formed by the tangents from (x_1, y_1) to the circle S = 0 and its chord of contact is $\left(\frac{r(S_{11})^{3/2}}{S_{11}+r^2}\right)$ where r is radius of the circle..

52. If (x_1, y_1) is the midpoint of the chord AB of the circle S = 0, then the length of the chord AB = $2\sqrt{|S_{11}|}$; where $S_{11} < 0$.

53. The middle point of the chord lx+my+n = 0 of the circle $x^2 + y^2 = r^2$ is $\left(-\frac{ln}{l^2+m^2}, \frac{-mn}{l^2+m^2}\right)$.

54. The equation to the polar of the point (x_1, y_1) w.r.t. the circle S = 0 is $S_1 = 0$.

55. The polar of a point P w.r.t. a circle is perpendicular to OP, O being the centre of the circle.

56. The pole of the line lx + my + n = 0 w.r.t. the circle $x^2 + y^2 = a^2$ is $\left(\frac{-a^2l}{n}, \frac{-a^2m}{n}\right)$.

57. The pole of the line lx + my + n = 0 w.r.t. the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is obtained from $\left(-g + \frac{lr^2}{lg+mf-n}, -f + \frac{mr^2}{lg+mf-n}\right)$.

58. If the polars of A and B meet at C w.r.t. a circle, then the polar of C w.r.t. the circle is AB.

59. The polars of collinear points w.r.t. a circle are concurrent.

60. The poles of concurrent lines w.r.t. a circle are collinear.

61. Two points $(x_1, y_1), (x_2, y_2)$ are conjugate points w.r.t. the circle $x^2 + y^2 = a^2$ if $x_1x_2 + y_1y_2 = a^2$.

62. Two points $(x_1, y_1), (x_2, y_2)$ are conjugate points w.r.t. the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ if $x_1x_2 + y_1y_2 + g(x_1 + x_2) + f(y_1 + y_2) + c = 0$ i.e., $S_{12} = 0$

63. Let P and Q be the conjugate points w.r.to the circle S = 0. Then the circle on PQ as diameter is orthogonal to S = 0

64. a. If two lines $l_1x + m_1y + n_1 = 0$, $l_2x + m_2y + n_2 = 0$ are conjugate w.r.t. $x^2 + y^2 = a^2$, then $a^2(l_1l_2 + m_1m_2) = n_1n_2$.

b. If two lines $l_1x + m_1y + n_1 = 0$, $l_2x + m_2y + n_2 = 0$ are conjugate w.r.t. $x^2+y^2+2gx+2fy+c = 0$, then $r^2 (l_1l_2 + m_1m_2) = (l_1g + m_1f - n_1)(l_2g + m_2f - n_2)$.

65. If A, B, C are pair wise conjugate points w.r..t a circle having the centre at 'O', then O is the ortho centre of ΔABC and ΔABC is obtuse angled.

66. If A and B are conjugate points w.r.to the circle $x^2 + y^2 = r^2$, then $OA^2 + OB^2 - AB^2 = 2r^2$.

67. Polar of the centre of a circle w.r.to the same circle doesnot exist.

68. Pole of the diameter of a circle w.r.to the same circle doesnot exist.

69. If t_1 & t_2 are the lengths of tangents drawn from 2 conjugate points A, B; then $t_1^2 + t_2^2 = AB^2$.

70. Two points P and Q are said to be inverse points w.r.to circle of centre "O" and radius "r" if

(i) O, P, Q lie on a line (ii) $OP.OQ = r^2$.

71. If P lies on the circle then the inverse of P is itself.

72. The inverse point of (x_1, y_1) w.r.to $x^2 + y^2 = a^2$ is $\left(\frac{a^2x_1}{x_1^2 + y_1^2}, \frac{a^2y_1}{x_1^2 + y_1^2}\right)$

73.i) The inverse of a point P w.r.to a circle is the foot of the perpendicular from P to the polar of P.

ii) The inverse of (x_1, y_1) w.r.t $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$\left(\frac{r^2(x_1 + g)}{(x_1 + g)^2 + (y_1 + f)^2}, \frac{r^2(y_1 + f)}{(x_1 + g)^2 + (y_1 + f)^2}\right),$$ where $r^2 = g^2 + f^2 - c$.

74. Two circles

i)do not intersect when $d > r_1 + r_2$

ii) touch each other externally when $d = r_1 + r_2$

iii) intersect each other when $|r_1 - r_2| < d < r_1 + r_2$

iv) touch each other internally when $d = |r_1 - r_2|$

v) are such that one lies inside the other when $d < |r_1 - r_2|$

75. The point dividing the line of centres in the ratio of the radii internally is the internal centre of similitude of the circles.

76. The point dividing the line of centres in the ratio of the radii externally is the external centre of similitude of the circles.

77. Length of the direct common tangent is $\sqrt{d^2 - (r_1 - r_2)^2}$

78. Length of the indirect common tangent= $\sqrt{d^2 - (r_1 + r_2)^2}$, where d is distance between the centres

79. a. The locus of the middle points of the chords of the circle $x^2 + y^2 = a^2$ subtending a right angle at the centre is $x^2 + y^2 = a^2/2$.

b. The locus of the middle points of the chords of the circle $x^2 + y^2 = a^2$ subtending an angle α at the centre is $x^2 + y^2 = a^2 \cos^2\alpha/2$.

c. The lcous of the midpoints of the chords of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ subtending an angle 'α' at the centre is $(x+g)^2 + (y+f)^2 = r^2 \cos^2 \alpha/2$ where $r^2 = g^2 + f^2 - c$.

80. One circle bisects the circumference of another if their common chord is a diameter of the circle which is to be bisected

81. The circle $x^2+y^2+2g_1x + 2f_1y + c_1 = 0$ bisects the circumference of $x^2+y^2+2g_2x+2f_2y+c_2= 0$ if $2g_2(g_1 - g_2) + 2f_2 (f_1 - f_2) = c_1 - c_2$.

82. a. The angle between two intersecting circles is defined as the angle between their tangents at a point of intersection.

b. The angle between the radii joining a point of intersection is $\pi - \theta$ where θ is angle between the circles.

c. If θ is the angle between the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$,

$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$, then $\cos\theta = \dfrac{(c_1 + c_2) - 2(g_1g_2 + f_1f_2)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}$.

83. Two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$, $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cut each other orthogonally if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ or $d^2 = r_1^2 + r_2^2$.

84. Two circles whose radii are r_1 and r_2 cut each other orthogonally. Then the length of their common chord is $\dfrac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$

CIRCLES

OBJECTIVES

1. **If a circle passes through the point (0, 0), (*a*, 0), (0, *b*), then its centre is**

   (a) (a, b) **(b)** (b, a)

   (c) $\left(\frac{a}{2}, \frac{b}{2}\right)$ **(d)** $\left(\frac{b}{2}, -\frac{a}{2}\right)$

2. **If one end of a diameter of the circle** $x^2 + y^2 - 4x - 6y + 11 = 0$ **be (3, 4), then the other end is**

   (a) (0, 0) **(b) (1, 1)**

   (c) (1, 2) **(d) (2, 1)**

3. **The equation of the circle passing through the origin and cutting intercepts of length 3 and 4 units from the positive axes, is**

   (a) $x^2 + y^2 + 6x + 8y + 1 = 0$

   (b) $x^2 + y^2 - 6x - 8y = 0$

   (c) $x^2 + y^2 + 3x + 4y = 0$

   (d) $x^2 + y^2 - 3x - 4y = 0$

4. **If the vertices of a triangle be** $(2, -2)$, $(-1, -1)$ **and (5, 2), then the equation of its circumcircle is**

   (a) $x^2 + y^2 + 3x + 3y + 8 = 0$

   (b) $x^2 + y^2 - 3x - 3y - 8 = 0$

   (c) $x^2 + y^2 - 3x + 3y + 8 = 0$

   (d) None of these

5. **The equation of the circle having centre** $(1, -2)$ **and passing through the point of intersection of lines** $3x + y = 14$, $2x + 5y = 18$ **is**

   (a) $x^2 + y^2 - 2x + 4y - 20 = 0$

   (b) $x^2 + y^2 - 2x - 4y - 20 = 0$

   (c) $x^2 + y^2 + 2x - 4y - 20 = 0$

   (d) $x^2 + y^2 + 2x + 4y - 20 = 0$

6. **For all values of θ, the locus of the point of intersection of the lines $x\cos\theta + y\sin\theta = a$ and $x\sin\theta - y\cos\theta = b$ is**

   (a) An ellipse **(b) A circle**

   (c) A parabola **(d) A hyperbola**

7. **The lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units. The equation of the circle is**

   (a) $x^2 + y^2 + 2x - 2y = 62$ **(b)** $x^2 + y^2 - 2x + 2y = 47$

   (c) $x^2 + y^2 + 2x - 2y = 47$ **(d)** $x^2 + y^2 - 2x + 2y = 62$

8. **The locus of the centre of the circle which cuts a chord of length 2*a* from the positive *x*-axis and passes through a point on positive *y*-axis distant *b* from the origin is**

   (a) $x^2 + 2by = b^2 + a^2$ **(b)** $x^2 - 2by = b^2 + a^2$

   (c) $x^2 + 2by = a^2 - b^2$ **(d)** $x^2 - 2by = b^2 - a^2$

9. **The locus of the centre of the circle which cuts off intercepts of length $2a$ and $2b$ from *x*-axis and *y*-axis respectively, is**

   (a) $x + y = a + b$ **(b)** $x^2 + y^2 = a^2 + b^2$

   (c) $x^2 - y^2 = a^2 - b^2$ **(d)** $x^2 + y^2 = a^2 - b^2$

10. **A circle touches *x*-axis and cuts off a chord of length 2*l* from *y*-axis. The locus of the centre of the circle is**

    (a) A straight line **(b) A circle**

    (c) An ellipse **(d) A hyperbola**

11. **A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$, whose sides are parallel to the coordinate axes. One vertex of the square is**

    (a) $(1 + \sqrt{2}, -2)$ **(b)** $(1 - \sqrt{2}, -2)$

    (c) $(1, -2 + \sqrt{2})$ **(d) None of these**

12. **The equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at the point (5, 5), is**

    (a) $x^2 + y^2 - 18x - 16y - 120 = 0$ **(b)** $x^2 + y^2 - 18x - 16y + 120 = 0$

    (c) $x^2 + y^2 + 18x + 16y - 120 = 0$ **(d)** $x^2 + y^2 + 18x - 16y + 120 = 0$

13. The circle represented by the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ will be a point circle, if

(a) $g^2 + f^2 = c$ (b) $g^2 + f^2 > c$

(c) $g^2 + f^2 + c = 0$ (d) None of these

14. The number of circle having radius 5 and passing through the points (– 2, 0) and (4, 0) is

(a) One (b) Two

(c) Four (d) Infinite

15. The area of the circle whose centre is at (1, 2) and which passes through the point (4, 6) is

(a) 5π (b) 10π

(c) 25π (d) None of these

16. The equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$, is

(a) $x^2 + y^2 + 4x - 10y + 25 = 0$ (b) $x^2 + y^2 - 4x - 10y + 25 = 0$

(c) $x^2 + y^2 - 4x - 10y + 16 = 0$ (d) $x^2 + y^2 - 14y + 8 = 0$

17. The equation of the circle touching $x = 0, y = 0$ and $x = 4$ is

(a) $x^2 + y^2 - 4x - 4y + 16 = 0$ (b) $x^2 + y^2 - 8x - 8y + 16 = 0$

(c) $x^2 + y^2 + 4x + 4y + 4 = 0$ (d) $x^2 + y^2 - 4x - 4y + 4 = 0$

18. The equation of the circle passing through the points (0, 0), (0, *b*) and (*a*, *b*) is

(a) $x^2 + y^2 + ax + by = 0$ (b) $x^2 + y^2 - ax + by = 0$

(c) $x^2 + y^2 - ax - by = 0$ (d) $x^2 + y^2 + ax - by = 0$

19. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches *x*-axis, then

(a) $g = f$ (b) $g^2 = c$

(c) $f^2 = c$ (d) $g^2 + f^2 = c$

20. The equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ will represent a circle, if

(a) $a = b = 0$ and $c = 0$ (b) $f = g$ and $h = 0$

(c) $a = b \neq 0$ and $h = 0$ (d) $f = g$ and $c = 0$

21. A circle is concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has area double of its area. The equation of the circle is

(a) $x^2 + y^2 - 6x + 12y - 15 = 0$ (b) $x^2 + y^2 - 6x + 12y + 15 = 0$

(c) $x^2 + y^2 - 6x + 12y + 45 = 0$ (d) None of these

22. The equation of the circle with centre at (1, –2) and passing through the centre of the given circle $x^2+y^2+2y-3=0$, is

(a) $x^2+y^2-2x+4y+3=0$ (b) $x^2+y^2-2x+4y-3=0$

(c) $x^2+y^2+2x-4y-3=0$ (d) $x^2+y^2+2x-4y+3=0$

23. If the radius of the circle $x^2+y^2+2gx+2fy+c=0$ be *r*, then it will touch both the axes, if

(a) $g=f=r$ (b) $g=f=c=r$

(c) $g=f=\sqrt{c}=r$ (d) $g=f$ and $c^2=r$

24. If the lines $x+y=6$ and $x+2y=4$ be diameters of the circle whose diameter is 20, then the equation of the circle is

(a) $x^2+y^2-16x+4y-32=0$ (b) $x^2+y^2+16x+4y-32=0$

(c) $x^2+y^2+16x+4y+32=0$ (d) $x^2+y^2+16x-4y+32=0$

25. For the circle $x^2+y^2+3x+3y=0$, which of the following relations is true

(a) Centre lies on *x*-axis (b) Centre lies on *y*-axis

(c) Centre is at origin (d) Circle passes through origin

26. The equation of the circle passing through the point (2, 1) and touching *y*-axis at the origin is

(a) $x^2+y^2-5x=0$ (b) $2x^2+2y^2-5x=0$

(c) $x^2+y^2+5x=0$ (d) None of these

27. Equation of a circle whose centre is origin and radius is equal to the distance between the lines $x=1$ and $x=-1$ is

(a) $x^2+y^2=1$ (b) $x^2+y^2=\sqrt{2}$

(c) $x^2+y^2=4$ (d) $x^2+y^2=-4$

28. The centre and radius of the circle $2x^2+2y^2-x=0$ are

(a) $\left(\frac{1}{4},0\right)$ and $\frac{1}{4}$ (b) $\left(-\frac{1}{2},0\right)$ and $\frac{1}{2}$

(c) $\left(\frac{1}{2},0\right)$ and $\frac{1}{2}$ (d) $\left(0,-\frac{1}{4}\right)$ and $\frac{1}{4}$

29. **The equation of the circle which touches both the axes and whose radius is *a*, is**

(a) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ (b) $x^2 + y^2 + ax + ay - a^2 = 0$

(c) $x^2 + y^2 + 2ax + 2ay - a^2 = 0$ (d) $x^2 + y^2 - ax - ay + a^2 = 0$

30. **A circle which passes through origin and cuts intercepts on axes *a* and *b*, the equation of circle is**

(a) $x^2 + y^2 - ax - by = 0$ (b) $x^2 + y^2 + ax + by = 0$

(c) $x^2 + y^2 - ax + by = 0$ (d) $x^2 + y^2 + ax - by = 0$

31. **The radius of a circle which touches *y*-axis at (0,3) and cuts intercept of 8 units with *x*-axis, is**

(a) 3 (b) 2

(c) 5 (d) 8

32. **The equation of the circumcircle of the triangle formed by the lines** $y + \sqrt{3}x = 6, y - \sqrt{3}x = 6,$ **and** $y = 0$**, is**

(a) $x^2 + y^2 - 4y = 0$ (b) $x^2 + y^2 + 4x = 0$

(c) $x^2 + y^2 - 4y = 12$ (d) $x^2 + y^2 + 4x = 12$

33. **If the lines** $l_1x + m_1y + n_1 = 0$ **and** $l_2x + m_2y + n_2 = 0$ **cuts the axes at con-cyclic points, then**

(a) $l_1l_2 = m_1m_2$ (b) $l_1m_1 = l_2m_2$

(c) $l_1l_2 + m_1m_2 = 0$ (d) $l_1m_2 = l_2m_1$

34. **The equation of a circle with centre** $(-4, 3)$ **and touching the circle** $x^2 + y^2 = 1$**, is**

(a) $x^2 + y^2 + 8x - 6y + 9 = 0$ (b) $x^2 + y^2 + 8x + 6y - 11 = 0$

(c) $x^2 + y^2 + 8x + 6y - 9 = 0$ (d) None of these

35. **A line meets the coordinate axes in *A* and *B*. A circle is circumscribed about the triangle *OAB*. If *m* and *n* are the distance of the tangents to the circle at the points *A* and *B* respectively from the origin, the diameter of the circle is**

(a) $m(m + n)$ (b) $m + n$

(c) $n(m + n)$ (d) $\frac{1}{2}(m + n)$

36. **The equation to a circle whose centre lies at the point (–2, 1) and which touches the line** $3x-2y-6=0$ **at (4, 3), is**

(a) $x^2+y^2+4x-2y-35=0$ (b) $x^2+y^2-4x+2y+35=0$

(c) $x^2+y^2+4x+2y+35=0$ (d) None of these

37. **The equation of circle whose diameter is the line joining the points (–4, 3) and (12, –1) is**

(a) $x^2+y^2+8x+2y+51=0$ (b) $x^2+y^2+8x-2y-51=0$

(c) $x^2+y^2+8x+2y-51=0$ (d) $x^2+y^2-8x-2y-51=0$

38. **If** (α,β) **is the centre of a circle passing through the origin, then its equation is**

(a) $x^2+y^2-\alpha x-\beta y=0$ (b) $x^2+y^2+2\alpha x+2\beta y=0$

(c) $x^2+y^2-2\alpha x-2\beta y=0$ (d) $x^2+y^2+\alpha x+\beta y=0$

39. **The locus of the centre of a circle which touches externally the circle** $x^2+y^2-6x-6y+14=0$ **and also touches the *y*-axis, is given by the equation**

(a) $x^2-6x-10y+14=0$ (b) $x^2-10x-6y+14=0$

(c) $y^2-6x-10y+14=0$ (d) $y^2-10x-6y+14=0$

40. **Area of the circle in which a chord of length** $\sqrt{2}$ **makes an angle** $\frac{\pi}{2}$ **at the centre is**

(a) $\frac{\pi}{2}$ (b) 2π

(c) π (d) $\frac{\pi}{4}$

41. **For** $ax^2+2hxy+3y^2+4x+8y-6=0$ **to represent a circle, one must have**

(a) $a=3,\ h=0$ (b) $a=1,\ h=0$

(c) $a=h=3$ (d) $a=h=0$

42. **Circles are drawn through the point (2, 0) to cut intercept of length 5 *units* on the *x*-axis. If their centres lie in the first quadrant, then their equation is**

(a) $x^2+y^2+9x+2fy+14=0$ (b) $3x^2+3y^2+27x-2fy+42=0$

(c) $x^2+y^2-9x+2fy+14=0$ (d) $x^2+y^2-2fy-9y+14=0$

43. **Equations to the circles which touch the lines** $3x-4y+1=0$**,** $4x+3y-7=0$ **and pass through (2, 3) are**

(a) $(x-2)^2+(y-8)^2=25$ (b) $5x^2+5y^2-12x-24y+31=0$

(c) Both (a) and (b) (d) None of these

44. **The equation of the circle in the first quadrant which touches each axis at a distance 5 from the origin is**

(a) $x^2 + y^2 + 5x + 5y + 25 = 0$ (b) $x^2 + y^2 - 10x - 10y + 25 = 0$

(c) $x^2 + y^2 - 5x - 5y + 25 = 0$ (d) $x^2 + y^2 + 10x + 10y + 25 = 0$

45. **The equation of circle whose centre lies on** $3x - y - 4 = 0$ **and** $x + 3y + 2 = 0$ **and has an area 154 square units is**

(a) $x^2 + y^2 - 2x + 2y - 47 = 0$ (b) $x^2 + y^2 - 2x + 2y + 47 = 0$

(c) $x^2 + y^2 + 2x - 2y - 47 = 0$ (d) None of these

46. **The equation of circle with centre (1, 2) and tangent** $x + y - 5 = 0$ **is**

(a) $x^2 + y^2 + 2x - 4y + 6 = 0$ (b) $x^2 + y^2 - 2x - 4y + 3 = 0$

(c) $x^2 + y^2 - 2x + 4y + 8 = 0$ (d) $x^2 + y^2 - 2x - 4y + 8 = 0$

47. **The equation of the circle of radius 5 and touching the coordinate axes in third quadrant is**

(a) $(x-5)^2 + (y+5)^2 = 25$ (b) $(x+4)^2 + (y+4)^2 = 25$

(c) $(x+6)^2 + (y+6)^2 = 25$ (d) $(x+5)^2 + (y+5)^2 = 25$

48. **If the lines** $2x + 3y + 1 = 0$ **and** $3x - y - 4 = 0$ **lie along diameters of a circle of circumference** 10π**, then the equation of the circle is**

(a) $x^2 + y^2 + 2x - 2y - 23 = 0$ (b) $x^2 + y^2 - 2x - 2y - 23 = 0$

(c) $x^2 + y^2 + 2x + 2y - 23 = 0$ (d) $x^2 + y^2 - 2x + 2y - 23 = 0$

49. **For what value of *k*, the points (0, 0), (1, 3), (2, 4) and (*k*, 3) are con-cyclic**

(a) 2 (b) 1

(c) 4 (d) 5

50. **If** $g^2 + f^2 = c$**, then the equation** $x^2 + y^2 + 2gx + 2fy + c = 0$ **will represent**

(a) A circle of radius *g* (b) A circle of radius *f*

(c) A circle of diameter $\sqrt{c}$ (d) A circle of radius 0

51. **A variable circle passes through the fixed point (2,0) and touches the *y*-axis . Then the locus of its centre is**

(a) A circle (b) An Ellipse

(c) A hyperbola (d) A parabola

52. The length of intercept, the circle $x^2 + y^2 + 10x - 6y + 9 = 0$ makes on the x-axis is

(a) 2 (b) 4

(c) 6 (d) 8

53. The centre of the circle $x = 2 + 3\cos\theta$, $y = 3\sin\theta - 1$ is

(a) (3, 3) (b) $(2, -1)$

(c) $(-2, 1)$ (d) $(-1, 2)$

54. The four distinct points (0, 0),(2, 0), (0, –2) and (k,–2)are con-cyclic, if k =

(a) –2 (b) 2

(c) 1 (d) 0

55. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points such that their abscissa x_1 and x_2 are the roots of the equation $x^2 + 2x - 3 = 0$ while the ordinates y_1 and y_2 are the roots of the equation $y^2 + 4y - 12 = 0$. The centre of the circle with PQ as diameter is

(a) $(-1, -2)$ (b) $(1, 2)$

(c) $(1, -2)$ (d) $(-1, 2)$

56. If one end of the diameter is (1, 1) and other end lies on the line $x + y = 3$, then locus of centre of circle is

(a) $x + y = 1$ (b) $2(x - y) = 5$ (c) $2x + 2y = 5$ (d) None of these

57. A circle is drawn to cut a chord of length 2a units along X-axis and to touch the Y-axis. The locus of the centre of the circle is

(a) $x^2 + y^2 = a^2$ (b) $x^2 - y^2 = a^2$

(c) $x + y = a^2$ (d) $x^2 - y^2 = 4a^2$

(e) $x^2 + y^2 = 4a^2$

58. If the length of tangent drawn from the point (5, 3) to the circle $x^2 + y^2 + 2x + ky + 17 = 0$ be 7, then k =

(a) 4 (b) – 4

(c) – 6 (d) 13/2

59. If OA and OB be the tangents to the circle $x^2+y^2-6x-8y+21=0$ drawn from the origin O, then AB =

(a) 11 (b) $\frac{4}{5}\sqrt{21}$

(c) $\sqrt{\frac{17}{3}}$ (d) None of these

60. The equations of the tangents to the circle $x^2+y^2=50$ at the points where the line $x+7=0$ meets it, are

(a) $7x \pm y+50=0$ (b) $7x \pm y-5=0$

(c) $y \pm 7x+5=0$ (d) $y \pm 7x-5=0$

61. The line $(x-a)\cos\alpha+(y-b)\sin\alpha=r$ will be a tangent to the circle $(x-a)^2+(y-b)^2=r^2$

(a) If $\alpha=30^o$ (b) If $\alpha=60^o$

(c) For all values of α (d) None of these

62. The equation of the normal to the circle $x^2+y^2=9$ at the point $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ is

(a) $x+y=0$ (b) $x-y=\frac{\sqrt{2}}{3}$

(c) $x-y=0$ (d) None of these

63. The equations of the tangents drawn from the point (0, 1) to the circle $x^2+y^2-2x+4y=0$ are

(a) $2x-y+1=0,\ x+2y-2=0$ (b) $2x-y+1=0,\ x+2y+2=0$

(c) $2x-y-1=0,\ x+2y-2=0$ (d) $2x-y-1=0,\ x+2y+2=0$

64. The equations of the tangents to the circle $x^2+y^2=36$ which are inclined at an angle of 45^o to the x-axis are

(a) $x+y=\pm\sqrt{6}$ (b) $x=y\pm 3\sqrt{2}$

(c) $y=x\pm 6\sqrt{2}$ (d) None of these

65. The length of tangent from the point (5, 1) to the circle $x^2+y^2+6x-4y-3=0$, is

(a) 81 (b) 29

(c) 7 (d) 21

66. If the line $lx+my=1$ be a tangent to the circle $x^2+y^2=a^2$, then the locus of the point (l, m) is

(a) A straight line (b) A Circle

(c) A parabola (d) An ellipse

67. The equations of the normals to the circle $x^2+y^2-8x-2y+12=0$ **at the points whose ordinate is –1, will be**

(a) $2x-y-7=0, 2x+y-9=0$ **(b)** $2x+y+7=0, 2x+y+9=0$

(c) $2x+y-7=0,\ 2x+y+9=0$ **(d)** $2x-y+7=0, 2x-y+9=0$

68. If the line $x=k$ **touches the circle** $x^2+y^2=9$ **, then the value of** ***k*** **is**

(a) **2 but not – 2** **(b)** **– 2 but not 2**

(c) **3** **(d)** **None of these**

69. If the ratio of the lengths of tangents drawn from the point (f,g) **to the given circle** $x^2+y^2=6$ **and** $x^2+y^2+3x+3y=0$ **be 2 : 1, then**

(a) $f^2+g^2+2g+2f+2=0$ **(b)** $f^2+g^2+4g+4f+4=0$

(c) $f^2+g^2+4g+4f+2=0$ **(d) None of these**

70. The line $y=mx+c$ **will be a normal to the circle with radius** ***r*** **and centre at (*a*, *b*), if**

(a) $a=mb+c$ **(b)** $b=ma+c$

(c) $r=ma-b+c$ **(d)** $r=ma-b$

71. The equation of the tangent at the point $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$ **of the circle** $x^2+y^2=\frac{a^2b^2}{a^2+b^2}$ **is**

(a) $\frac{x}{a}+\frac{y}{b}=1$ **(b)** $\frac{x}{a}+\frac{y}{b}+1=0$

(c) $\frac{x}{a}-\frac{y}{b}=1$ **(d)** $\frac{x}{a}-\frac{y}{b}+1=0$

72. Two tangents drawn from the origin to the circle $x^2+y^2+2gx+2fy+c=0$ **will be perpendicular to each other, if**

(a) $g^2+f^2=2c$ **(b)** $g=f=c^2$

(c) $g+f=c$ **(d)** **None of these**

73. The equation of circle which touches the axes of coordinates and the line $\frac{x}{3}+\frac{y}{4}=1$ **and whose centre lies in the first quadrant is** $x^2+y^2-2cx-2cy+c^2=0$ **, where** ***c*** **is**

(a) **1** **(b)** **2**

(c) **3** **(d)** **6**

74. **A tangent to the circle $x^2+y^2=5$ at the point (1,–2)..... the circle $x^2+y^2-8x+6y+20=0$**

(a) **Touches** (b) **Cuts at real points**

(c) **Cuts at imaginary points** (d) **None of these**

75. **Square of the length of the tangent drawn from the point (α,β) to the circle $ax^2+ay^2=r^2$ is**

(a) $a\alpha^2+a\beta^2-r^2$ (b) $\alpha^2+\beta^2-\frac{r^2}{a}$

(c) $\alpha^2+\beta^2+\frac{r^2}{a}$ (d) $\alpha^2+\beta^2-r^2$

76. **The number of common tangents to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$ is**

(a) 1 (b) 2

(c) 3 (d) 4

77. **The area of triangle formed by the tangent, normal drawn at $(1,\sqrt{3})$ to the circle $x^2+y^2=4$ and positive *x*-axis, is**

(a) $2\sqrt{3}$ (b) $\sqrt{3}$

(c) $4\sqrt{3}$ (d) **None of these**

78. **Line $y=x+a\sqrt{2}$ is a tangent to the circle $x^2+y^2=a^2$ at**

(a) $\left(\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$ (b) $\left(-\frac{a}{\sqrt{2}},-\frac{a}{\sqrt{2}}\right)$

(c) $\left(\frac{a}{\sqrt{2}},-\frac{a}{\sqrt{2}}\right)$ (d) $\left(-\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$

79. **Length of the tangent from (x_1,y_1) to the circle $x^2+y^2+2gx+2fy+c=0$ is**

(a) $(x_1^2+y_1^2+2gx_1+2fy_1+c)^{1/2}$ (b) $(x_1^2+y_1^2)^{1/2}$

(c) $[(x_1+g)^2+(y_1+f)^2]^{1/2}$ (d) **None of these**

80. **The points of intersection of the line $4x-3y-10=0$ and the circle $x^2+y^2-2x+4y-20=0$ are**

(a) $(-2,-6),(4,2)$ (b) $(2,6),(-4,-2)$

(c) $(-2,6),(-4,2)$ (d) **None of these**

81. **The angle between the tangents from (α,β) to the circle $x^2+y^2=a^2$, is**

(a) $\tan^{-1}\left(\frac{a}{\sqrt{\alpha^2+\beta^2-a^2}}\right)$ (b) $\tan^{-1}\left(\frac{\sqrt{\alpha^2+\beta^2-a^2}}{a}\right)$

(c) $2\tan^{-1}\left(\frac{a}{\sqrt{\alpha^2+\beta^2-a^2}}\right)$ (d) **None of these**

82. The gradient of the tangent line at the point $(a\cos\alpha, a\sin\alpha)$ **to the circle** $x^2+y^2=a^2$**, is**

(a) $\tan\alpha$ **(b)** $\tan(\pi-\alpha)$

(c) $\cot\alpha$ **(d)** $-\cot\alpha$

83. The line $y=mx+c$ **intersects the circle** $x^2+y^2=r^2$ **at two real distinct points, if**

(a) $-r\sqrt{1+m^2}<c\le 0$ **(b)** $0\le c<r\sqrt{1+m^2}$

(c) (a) and (b) both **(d)** $-c\sqrt{1-m^2}<r$

84. If ***OA*** **and** ***OB*** **are the tangents from the origin to the circle** $x^2+y^2+2gx+2fy+c=0$ **and** ***C*** **is the centre of the circle, the area of the quadrilateral** $OACB$ **is**

(a) $\frac{1}{2}\sqrt{c(g^2+f^2-c)}$ **(b)** $\sqrt{c(g^2+f^2-c)}$

(c) $c\sqrt{g^2+f^2-c}$ **(d)** $\frac{\sqrt{g^2+f^2-c}}{c}$

85. If a circle passes through the points of intersection of the coordinate axis with the lines $\lambda x-y+1=0$ **and** $x-2y+3=0$**, then the value of** λ **is**

(a) 1 **(b) 2**

(c) 3 **(d) 4**

86. The equation of the tangent to the circle $x^2+y^2-2x-4y-4=0$ **which is perpendicular to** $3x-4y-1=0$**, is**

(a) $4x+3y-5=0$ **(b)** $4x+3y+25=0$

(c) $4x-3y+5=0$ **(d)** $4x+3y-25=0$

87. Given the circles $x^2+y^2-4x-5=0$ **and** $x^2+y^2+6x-2y+6=0$**. Let** ***P*** **be a point** (α,β) **such that the tangents from** ***P*** **to both the circles are equal, then**

(a) $2\alpha+10\beta+11=0$ **(b)** $2\alpha-10\beta+11=0$

(c) $10\alpha-2\beta+11=0$ **(d)** $10\alpha+2\beta+11=0$

88. If $a>2b>0$ **then the positive value of** ***m*** **for which** $y=mx-b\sqrt{1+m^2}$ **is a common tangent to** $x^2+y^2=b^2$ **and** $(x-a)^2+y^2=b^2$**, is**

(a) $\frac{2b}{\sqrt{a^2-4b^2}}$ **(b)** $\frac{\sqrt{a^2-4b^2}}{2b}$

(c) $\frac{2b}{a-2b}$ **(d)** $\frac{b}{a-2b}$

89. **If a circle, whose centre is (–1, 1) touches the straight line $x+2y+12=0$, then the coordinates of the point of contact are**

(a) $\left(\frac{-7}{2},-4\right)$ (b) $\left(\frac{-18}{5},\frac{-21}{5}\right)$

(c) (2,–7) (d) (–2, –5)

90. **The tangent at *P*, any point on the circle $x^2+y^2=4$, meets the coordinate axes in *A* and *B*, then**

(a) Length of *AB* is constant

(b) *PA* and *PB* are always equal

(c) The locus of the mid point of *AB* is $x^2+y^2=x^2y^2$

(d) None of these

91. **If the circle $(x-h)^2+(y-k)^2=r^2$ touches the curve $y=x^2+1$ at a point (1, 2), then the possible locations of the points (*h*, *k*) are given by**

(a) $hk=5/2$ (b) $h+2k=5$

(c) $h^2-4k^2=5$ (d) $k^2=h^2+1$

92. **The line $ax+by+c=0$ is a normal to the circle $x^2+y^2=r^2$. The portion of the line $ax+by+c=0$ intercepted by this circle is of length**

(a) *r* (b) r^2

(c) 2*r* (d) $\sqrt{r}$

93. **The gradient of the normal at the point (–2, –3) on the circle $x^2+y^2+2x+4y+3=0$ is**

(a) 1 (b) –1

(c) $\frac{3}{2}$ (d) $\frac{1}{2}$

94. **A circle with centre (*a*, *b*) passes through the origin. The equation of the tangent to the circle at the origin is**

(a) $ax-by=0$ (b) $ax+by=0$

(c) $bx-ay=0$ (d) $bx+ay=0$

95. **If $\frac{x}{\alpha}+\frac{y}{\beta}=1$ touches the circle $x^2+y^2=a^2$, then point $(1/\alpha,1/\beta)$ lies on a/an**

(a) Straight line (b) Circle

(c) Parabola (d) Ellipse

96. Assertion (a) : The circle $x^2+y^2=1$ has exactly two tangents parallel to the x-axis

Reason (R) : $\frac{dy}{dx}=0$ on the circle exactly at the point $(0,\pm 1)$. Of these statements

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

97. If $5x-12y+10=0$ and $12y-5x+16=0$ are two tangents to a circle, then the radius of the circle is

(a) 1 (b) 2

(c) 4 (d) 6

98. The square of the length of the tangent from (3, –4) on the circle $x^2+y^2-4x-6y+3=0$ is

(a) 20 (b) 30

(c) 40 (d) 50

99. The locus of a point which moves so that the ratio of the length of the tangents to the circles $x^2+y^2+4x+3=0$ and $x^2+y^2-6x+5=0$ is 2:3 is

(a) $5x^2+5y^2-60x+7=0$ (b) $5x^2+5y^2+60x-7=0$

(c) $5x^2+5y^2-60x-7=0$ (d) $5x^2+5y^2+60x+7=0$

100. The distance between the chords of contact of the tangents to the circle $x^2+y^2+2gx+2fy+c=0$ from the origin and the point (g, f) is

(a) $\frac{1}{2}\left(\frac{g^2+f^2-c}{\sqrt{g^2+f^2}}\right)$ (b) $\left(\frac{g^2+f^2-c}{\sqrt{g^2+f^2}}\right)$

(c) $\frac{1}{2}\left(\frac{g^2+f^2-c}{g^2+f^2}\right)$ (d) None of these

101. If the middle point of a chord of the circle $x^2+y^2+x-y-1=0$ be (1, 1), then the length of the chord is

(a) 4 (b) 2

(c) 5 (d) None of these

102. Locus of the middle points of the chords of the circle $x^2+y^2=a^2$ which are parallel to $y=2x$ will be

(a) A circle with radius *a* (b) A straight line with slope $-\frac{1}{2}$

(c) A circle will centre (0, 0) (d) A straight line with slope – 2

103. The equation of the common chord of the circles $(x-a)^2+(y-b)^2=c^2$ and $(x-b)^2+(y-a)^2=c^2$ is

(a) $x-y=0$ (b) $x+y=0$

(c) $x+y=a^2+b^2$ (d) $x-y=a^2-b^2$

104. The co-ordinates of pole of line $lx+my+n=0$ with respect to circles $x^2+y^2=1$, is

(a) $\left(\frac{l}{n},\frac{m}{n}\right)$ (b) $\left(-\frac{l}{n},-\frac{m}{n}\right)$

(c) $\left(\frac{l}{n},-\frac{m}{n}\right)$ (d) $\left(-\frac{l}{n},\frac{m}{n}\right)$

105. The length of the chord intercepted by the circle $x^2+y^2=r^2$ on the line $\frac{x}{a}+\frac{y}{b}=1$ is

(a) $\sqrt{\frac{r^2(a^2+b^2)-a^2b^2}{a^2+b^2}}$ (b) $2\sqrt{\frac{r^2(a^2+b^2)-a^2b^2}{a^2+b^2}}$

(c) $2\frac{\sqrt{r^2(a^2+b^2)-a^2b^2}}{a^2+b^2}$ (d) None of these

106. A line $lx+my+n=0$ meets the circle $x^2+y^2=a^2$ at the points *P* and *Q*. The tangents drawn at the points *P* and *Q* meet at *R*, then the coordinates of *R* is

(a) $\left(\frac{a^2l}{n},\frac{a^2m}{n}\right)$ (b) $\left(\frac{-a^2l}{n},\frac{-a^2m}{n}\right)$

(c) $\left(\frac{a^2n}{l},\frac{a^2n}{m}\right)$ (d) None of these

107. The length of the common chord of the circles $x^2+y^2+2x+3y+1=0$ and $x^2+y^2+4x+3y+2=0$ is

(a) $9/2$ (b) $2\sqrt{2}$

(c) $3\sqrt{2}$ (d) $3/2$

108. Length of the common chord of the circles $x^2+y^2+5x+7y+9=0$ and $x^2+y^2+7x+5y+9=0$ is

(a) 9 (b) 8 (c) 7 (d) 6

109. If polar of a circle $x^2+y^2=a^2$ with respect to (x',y') is $Ax+By+C=0$, then its pole will be

(a) $\left(\frac{a^2A}{-C},\frac{a^2B}{-C}\right)$ **(b)** $\left(\frac{a^2A}{C},\frac{a^2B}{C}\right)$

(c) $\left(\frac{a^2C}{A},\frac{a^2C}{B}\right)$ **(d)** $\left(\frac{a^2C}{-A},\frac{a^2C}{-B}\right)$

110. If the circle $x^2+y^2=a^2$ cuts off a chord of length 2*b* from the line $y=mx+c$, then

(a) $(1-m^2)(a^2+b^2)=c^2$ **(b)** $(1+m^2)(a^2-b^2)=c^2$

(c) $(1-m^2)(a^2-b^2)=c^2$ **(d) None of these**

111. The radius of the circle, having centre at (2,1) whose one of the chord is a diameter of the circle $x^2+y^2-2x-6y+6=0$ is

(a) 1 **(b) 2**

(c) 3 **(d)** $\sqrt{3}$

112. The intercept on the line $y=x$ by the circle $x^2+y^2-2x=0$ is *AB*, equation of the circle on *AB* as a diameter is

(a) $x^2+y^2+x-y=0$ **(b)** $x^2+y^2-x+y=0$

(c) $x^2+y^2+x+y=0$ **(d)** $x^2+y^2-x-y=0$

113. A line through (0,0) cuts the circle $x^2+y^2-2ax=0$ at *A* and *B*, then locus of the centre of the circle drawn on *AB* as a diameter is

(a) $x^2+y^2-2ay=0$ **(b)** $x^2+y^2+ay=0$

(c) $x^2+y^2+ax=0$ **(d)** $x^2+y^2-ax=0$

114. From the origin chords are drawn to the circle $(x-1)^2+y^2=1$. The equation of the locus of the middle points of these chords is

(a) $x^2+y^2-3x=0$ **(b)** $x^2+y^2-3y=0$

(c) $x^2+y^2-x=0$ **(d)** $x^2+y^2-y=0$

115. If the line $x-2y=k$ cuts off a chord of length 2 from the circle $x^2+y^2=3$, then *k* =

(a) 0 **(b)** ± 1 **(c)** $\pm\sqrt{10}$ **(d) None of these**

116. The equation of the chord of the circle $x^2+y^2=a^2$ having (x_1,y_1) as its mid-point is

(a) $xy_1+yx_1=a^2$ **(b)** $x_1+y_1=a$

(c) $xx_1+yy_1=x_1^2+y_1^2$ **(d)** $xx_1+yy_1=a^2$

117. The equation of the circle with origin as centre passing the vertices of an equilateral triangle whose median is of length 3*a* is

(a) $x^2+y^2=9a^2$ **(b)** $x^2+y^2=16a^2$

(c) $x^2+y^2=a^2$ **(d) None of these**

118. If $\left(m_i,\frac{1}{m_i}\right), i=1,2,3,4$ are con-cyclic points, then the value of $m_1.m_2.m_3.m_4$ is

(a) 1 **(b) – 1**

(c) 0 **(d) None of these**

119. Tangents are drawn from the point (4, 3) to the circle $x^2+y^2=9$. The area of the triangle formed by them and the line joining their points of contact is

(a) $\frac{24}{25}$ **(b)** $\frac{64}{25}$

(c) $\frac{192}{25}$ **(d)** $\frac{192}{5}$

120. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x+y=1$. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1

(a) $x+y=0$ **(b)** $x-y=0$

(c) $x-7y=0$ **(d)** $x-7y=0$

121. The two points *A* and *B* in a plane are such that for all points *P* lies on circle satisfied $\frac{PA}{PB}=k$, then *k* will not be equal to

(a) 0 **(b) 1**

(c) 2 **(d) None of these**

122. A circle is inscribed in an equilateral triangle of side *a*, the area of any square inscribed in the circle is

(a) $\frac{a^2}{3}$ **(b)** $\frac{2a^2}{3}$ **(c)** $\frac{a^2}{6}$ **(d)** $\frac{a^2}{12}$

123. The area of the triangle formed by joining the origin to the points of intersection of the line $x\sqrt{5}+2y=3\sqrt{5}$ and circle $x^2+y^2=10$ is

(a) 3 **(b)** 4

(c) 5 **(d)** 6

124. The abscissae of *A* and *B* are the roots of the equation $x^2+2ax-b^2=0$ and their ordinates are the roots of the equation $y^2+2py-q^2=0$. The equation of the circle with *AB* as diameter

(a) $x^2+y^2+2ax+2py-b^2-q^2=0$

(b) $x^2+y^2+2ax+py-b^2-q^2=0$

(c) $x^2+y^2+2ax+2py+b^2+q^2=0$

(d) None of these

125. Let *PQ* and *RS* be tangents at the extremeties of the diameter *PR* of a circle of radius *r*. If *PS* and *RQ* intersect at a point *X* on the circumference of the circle, then 2*r* equals

(a) $\sqrt{PQ.RS}$ **(b)** $\frac{PQ+RS}{2}$

(c) $\frac{2PQ.\ RS}{PQ+RS}$ **(d)** $\sqrt{\frac{PQ^2+RS^2}{2}}$

126. Let *AB* be a chord of the circle $x^2+y^2=r^2$ subtending a right angle at the centre. Then the locus of the centroid of the ΔPAB as *P* moves on the circle is

(a) A parabola **(b)** A circle

(c) An ellipse **(d)** A pair of straight lines

127. If two distinct chords, drawn from the point (*p*, *q*) on the circle $x^2+y^2=px+qy$, (where $pq\neq 0$) are bisected by the *x*-axis, then

(a) $p^2=q^2$ **(b)** $p^2=8q^2$

(c) $p^2<8q^2$ **(d)** $p^2>8q^2$

128. If a straight line through $C(-\sqrt{8},\sqrt{8})$ making an angle of $135°$ with the *x*-axis cuts the circle $x=5\cos\theta, y=5\sin\theta$ at points *A* and *B*, then the length of *AB* is

(a) 3 **(b)** 7

(c) 10 **(d)** None of these

129. A chord *AB* drawn from the point $A(0,3)$ on circle $x^2+4x+(y-3)^2=0$ meets to *M* in such a way that $AM=2AB$, then the locus of point *M* will be

(a) Straight line **(b) Circle**

(c) Parabola **(d) None of these**

130. If a circle passes through the point (*a*, *b*) and cuts the circle $x^2+y^2=4$ orthogonally, then locus of its centre is

(a) $2ax-2by-(a^2+b^2+4)=0$

(b) $2ax+2by-(a^2+b^2+4)=0$

(c) $2ax-2by+(a^2+b^2+4)=0$

(d) $2ax+2by+(a^2+b^2+4)=0$

131. The locus of centre of the circle which touches the circle $x^2+(y-1)^2=1$ externally and also touches *x*-axis is

(a) $\{(x,y):x^2+(y-1)^2=4\}\cup\{(x,y):y<0\}$

(b) $\{(x,y):x^2=4y\}\cup\{(0,y):y<0\}$

(c) $\{(x,y):x^2=y\}\cup\{(0,y):y<0\}$

(d) $\{(x,y):x^2=4y\}\cup\{(x,y):y<0\}$

132. The tangents are drawn from the point (4, 5) to the circle $x^2+y^2-4x-2y-11=0$. The area of quadrilateral formed by these tangents and radii, is

(a) 15 *sq. units* **(b) 75 *sq. units*** **(c) 8 *sq. units*** **(d) 4 *sq. units***

CIRCLES

HINTS AND SOLUTIONS

1. **(c) Let the equation of circle be** $x^2 + y^2 + 2gx + 2fy + c = 0$. **Now on passing through the points, we get three equations.**

$c = 0$ ....(i)

$a^2 + 2ga + c = 0$ ....(ii)

$b^2 + 2fb + c = 0$ ....(iii)

On solving them, we get $g = -\frac{a}{2}, f = -\frac{b}{2}$

Hence the centre is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

2. **(c) Centre is (2, 3). One end is (3, 4).**

P_2 **divides the join of** P_1 **and** ***O*** **in ratio of 2 : 1.**

Hence P_2 **is** $\left(\frac{4-3}{2-1}, \frac{6-4}{2-1}\right) \equiv (1, 2)$.

3. **(d) Obviously the centre of the circle is** $\left(\frac{3}{2}, 2\right)$.

Therefore, the equation of circle is

$$\left(x - \frac{3}{2}\right)^2 + (y-2)^2 = \left(\frac{5}{2}\right)^2 \Rightarrow x^2 + y^2 - 3x - 4y = 0.$$

4. **(b) Let us find the equation of family of circles through** $(2, -2)$ **and** $(-1, -1)$.

i.e. $(x-2)(x+1) + (y+2)(y+1) + \lambda\left(\frac{y+2}{-2+1} - \frac{x-2}{2+1}\right) = 0$

Now for point (5, 2) to lie on it, we should have λ **given by**

$$3 \,.\, 6 + 4 \,.\, 3 + \lambda\left(\frac{4}{-1} - 1\right) = 0 \Rightarrow \lambda = \frac{30}{5} = 6$$

Hence equation is

$$(x-2)(x+1) + (y+2)(y+1) + 6\left(\frac{y+2}{-1} - \frac{x-2}{3}\right) = 0$$

Or $x^2 + y^2 - 3x - 3y - 8 = 0$.

5. (a) The point of intersection of $3x+y-14=0$ and $2x+5y-18=0$ are

$$x=\frac{-18+70}{15-2},\ y=\frac{-28+54}{13}\Rightarrow x=4,\ y=2$$

i.e., point is (4, 2).

Therefore radius is $\sqrt{(9)+(16)}=5$ and equation is $x^2+y^2-2x+4y-20=0$.

6. (b) The point of intersection is

$$x=a\cos\theta+b\sin\theta$$

$$y=a\sin\theta-b\cos\theta.$$

Therefore, $x^2+y^2=a^2+b^2$.

Obviously, it is equation of a circle.

7. (b) Centre of circle = Point of intersection of diameters = (1, –1)

Now area $=154\Rightarrow \pi r^2=154\Rightarrow r=7$

Hence the equation of required circle is

$$(x-1)^2+(y+1)^2=7^2\Rightarrow x^2+y^2-2x+2y=47.$$

8. (c) Here $2\sqrt{g^2-c}=2a\Rightarrow g^2-a^2-c=0$ .....(i)

and it passes through (0, *b*), therefore

$b^2+2fb+c=0$ ….(ii)

On adding (i) and (ii), we get $g^2+2fb=a^2-b^2$

Hence locus is $x^2+2by=a^2-b^2$.

9. (c) $2\sqrt{g^2-c}=2a$ ….(i)

$2\sqrt{f^2-c}=2b$ ….(ii)

On squaring (i) and (ii) and then subtracting (ii) from (i), we get $g^2-f^2=a^2-b^2$.

Hence the locus is $x^2-y^2=a^2-b^2$.

10. (d) If the circle $x^2+y^2+2gx+2fy+c=0$ touches the *x*-axis,

then $-f=\sqrt{g^2+f^2-c}\Rightarrow g^2=c$ .....(i)

and cuts a chord of length 2*l* from *y*-axis

$\Rightarrow 2\sqrt{f^2-c}=2l\Rightarrow f^2-c=l^2$ ….(ii)

Subtracting (i) from (ii), we get $f^2-g^2=l^2$.

Hence the locus is $y^2 - x^2 = l^2$, which is obviously a hyperbola.

11. **(d)** Obviously the centre of the given circle is $(1, -2)$. Since the sides of square are parallel to the axes, therefore, first three alternates cannot be vertices of square because in first two (*a* and *b*) $y = -2$ and in (c) $x = 1$, which passes through centre $(1, -2)$ but it is not possible. Hence answer (d) is correct.

12. (b) Let the centre of the required circle be (x_1, y_1) and the centre of given circle is (1, 2). Since radii of both circles are same, therefore, point of contact (5, 5) is the mid point of the line joining the centres of both circles. Hence $x_1 = 9$ and $y_1 = 8$. Hence the required equation is $(x-9)^2 + (y-8)^2 = 25$

13. **(a) Using condition of point circle**

$R = \sqrt{g^2 + f^2 - c} = 0 \Rightarrow g^2 + f^2 = c$ **.**

14. **(b)** Two, centre of each lying on the perpendicular bisector of the join of the two points.

15. (c) Obviously radius $= \sqrt{(1-4)^2 + (2-6)^2} = 5$

Hence the area is given by $\pi r^2 = 25\pi \ sq.\ units$.

16. (b) First find the centre. Let centre be (*h*, *k*), then

$\sqrt{(h-2)^2 + (k-3)^2} = \sqrt{(h-4)^2 + (k-5)^2}$ **….(i)**

and $k - 4h + 3 = 0$ **….(ii)**

From (i), we get $-4h - 6k + 8h + 10k = 16 + 25 - 4 - 9$

Or $4h + 4k - 28 = 0$ **or** $h + k - 7 = 0$ **….(iii)**

From (iii) and (ii), we get (*h*, *k*) as (2, 5). Hence centre is (2, 5) and radius is 2. Now find the equation of circle.

17. **(d)** $(x-2)^2 + (y-2)^2 = 4$

$x^2 + 4 - 4x + y^2 + 4 - 4y = 4$

18. **(c) Equation of circle passing through (0, 0) is**

$x^2 + y^2 + 2gx + 2fy = 0$ **….(i)**

Also, circle (i) is passing through (0, *b*) and (*a*, *b*)

$\therefore f = -\frac{b}{2}$ **and** $a^2 + b^2 + 2ag + 2\left(-\frac{b}{2}\right) b = 0$

$\Rightarrow g = -\frac{a}{2}$

Hence the equations of circle is, $x^2 + y^2 - ax - by = 0$ **.**

19. **(b)** Touches x-axis, hence radius = ordinate of centre. Hence $\sqrt{g^2+f^2-c}=(-f)$ or $g^2=c$.

20. **(c)** It is a fundamental concept.

21. **(a)** Equation of circle concentric to given circle is $x^2+y^2-6x+12y+k=0$ ....(i)

Radius of circle (i) $=\sqrt{2}$ (radius of given circle)

$\Rightarrow \sqrt{9+36-k}=\sqrt{2}\sqrt{9+36-15}$

$\Rightarrow 45-k=60 \Rightarrow k=-15$

Hence the required equation of circle is

$x^2+y^2-6x+12y-15=0$.

22. **(a)** According to the question, the required circle passes through (0,–1). Therefore, the radius is the distance between the points (0, –1) and (1, –2) *i.e.*, $\sqrt{2}$.

Hence the equation is $(x-1)^2+(y+2)^2=(\sqrt{2})^2$

$\Rightarrow x^2+y^2-2x+4y+3=0$

23. **(c)** Conditions are $g=f=r$ and $\sqrt{g^2+f^2-c}=r \Rightarrow g=\sqrt{c}$.

24. **(a)** Here $r=10$ (radius)

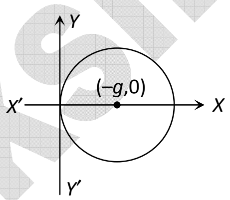
Centre will be the point of intersection of the diameters, *i.e.* (8, –2). Hence required equation is

$(x-8)^2+(y+2)^2=10^2 \Rightarrow x^2+y^2-16x+4y-32=0$.

25. **(d)** If $c=0$; circle passes through origin.

26. **(b)** We have the equation of circle

$x^2+y^2+2gx+2fy+c=0$



But it passes through (0, 0) and (2, 1), then

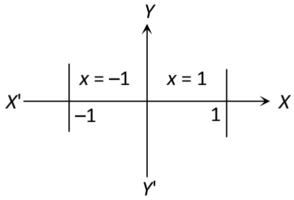
$c=0$ ....(i)

$5+4g+2f=0$ ....(ii)

Also $\sqrt{g^2+f^2-c}=|g| \Rightarrow f=0$ $\{\because c=0\}$

From (ii), $g=-\frac{5}{4}$

27. (c) Radius = 2, *C* (0, 0)



Equation of circle can be found from this.

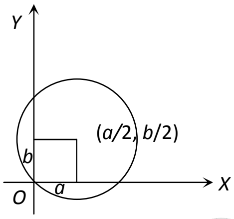
28. (a) The circle is $x^2+y^2-\frac{1}{2}x=0$.

Centre $(-g,-f)=\left(\frac{1}{4},0\right)$ **and** $R=\sqrt{\frac{1}{16}+0-0}=\frac{1}{4}$.

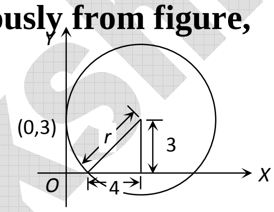
29. (a) Required equation is $(x-a)^2+(y-a)^2=a^2$

$\Rightarrow x^2+y^2-2ax-2ay+a^2=0$.

30. (a) Centre is $\left(\frac{a}{2},\frac{b}{2}\right)$ **and radius** $=\sqrt{\frac{a^2+b^2}{4}}$



31. (c) Obviously from figure,



Radius is $r=\sqrt{4^2+3^2}=5$.

32. (c) Solving $y=0$ **and** $y+\sqrt{3}x=6$**, we get** $(2\sqrt{3},0)$**, only option (c) satisfies the co-ordinate.**

33. (a) Line $l_1x+m_1y+n_1=0$ **cuts *x* and *y*-axes in** $A\left(-\frac{n_1}{l_1},0\right)$, $B\left(0,-\frac{n_1}{m_1}\right)$ **and line** $l_2x+m_2y+n_2=0$ **cuts axes in** $C\left(-\frac{n_2}{l_2},0\right)$, $D\left(0,\frac{-n_2}{m_2}\right)$.

So AC and BD are chords along x and y-axes intersecting at origin O. Since A, B, C, D are concyclic, so $OA.OC = OB.OD$

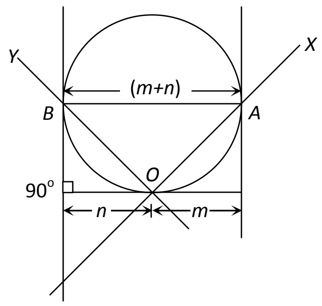
Or $\left|\left(-\frac{n_1}{l_1}\right)\left(-\frac{n_2}{l_2}\right)\right| = \left|\left(-\frac{n_1}{m_1}\right)\left(-\frac{n_2}{m_2}\right)\right|$

Or $|l_1 l_2| = |m_1 m_2|$

So $l_1 l_2 = m_1 m_2$ is correct among the given choices, which is given in (a).

34. (a) verification

35. (b) It is clear from the figure that diameter is $m+n$.



36. (a) Centre (–2, 1), radius $= \sqrt{36+4} = \sqrt{40}$

Hence equation of circle is $x^2 + y^2 + 4x - 2y - 35 = 0$.

37. (d) $(x+4)(x-12)+(y-3)(y+1)=0$

38. (c) Radius = Distance from origin $= \sqrt{\alpha^2 + \beta^2}$

$\therefore\ (x-\alpha)^2 + (y-\beta)^2 = \alpha^2 + \beta^2$

$\Rightarrow x^2 + y^2 - 2\alpha x - 2\beta y = 0$.

39. (d) Let the centre be (h, k), then radius = h

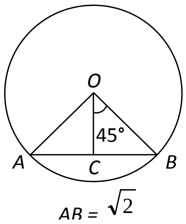
Also $CC_1 = R_1 + R_2$

or $\sqrt{(h-3)^2 + (k-3)^2} = h + \sqrt{9+9-14}$

$\Rightarrow (h-3)^2 + (k-3)^2 = h^2 + 4 + 4h$

$\Rightarrow k^2 - 10h - 6k + 14 = 0$ or $y^2 - 10x - 6y + 14 = 0$.

40. (c) Let AB be the chord of length $\sqrt{2}$, O be centre of the circle and let OC be the perpendicular from O on AB. Then



$AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

In $\Delta\, OBC, OB = BC \operatorname{cosec} 45^o = \frac{1}{\sqrt{2}}.\sqrt{2} = 1$

$\therefore$ Area of the circle $= \pi(OB)^2 = \pi$.

41. (a) It is obvious.

42. (c) The circle g, f, c passes through (2, 0)

$\therefore\ 4 + 4g + c = 0$ ....(i)

Intercept on x-axis is $2\sqrt{(g^2 - c)} = 5$

$\therefore 4(g^2 + 4g + 4) = 25$ by (i)

Or $(2g + 9)(2g - 1) = 0 \Rightarrow g = -\frac{9}{2}, \frac{1}{2}$

Since centre $(-g, -f)$ lies in 1st quadrant, we choose $g = -\frac{9}{2}$ so that $-g = \frac{9}{2}$ (positive).

$\therefore c = 14$, (from (i)).

43. (c) Both the circles given in option (a) and (b) satisfy the given conditions.

44. (b) The centre of the circle which touches each axis in first quadrant at a distance 5, will be (5, 5) and radius will be 5.

$\therefore (x - h)^2 + (y - k)^2 = a^2 \Rightarrow (x - 5)^2 + (y - 5)^2 = (5)^2$

$\Rightarrow x^2 + y^2 - 10x - 10y + 25 = 0$.

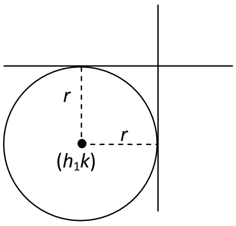
45. (a) Centre is $(1, -1)$ (point of intersection of two given lines) and $\pi r^2 = 154 \Rightarrow r = 7$

$\therefore$ Equation of required circle is $(x - 1)^2 + (y + 1)^2 = 49$

$\Rightarrow\ x^2 + y^2 - 2x + 2y - 47 = 0.$

46. (b) $\because$ **Radius of circle = perpendicular distance of tangent from the centre of circle**

47. (d) **Since circle touches the co-ordinate axes in III quadrant.**



$\therefore$ **Radius** $= -h = -k$ **. Hence** $h = k = -5$

$\therefore$ **Equation of circle is** $(x+5)^2 + (y+5)^2 = 25$.

48. (d) **According to question two diameters of the circle are** $2x + 3y + 1 = 0$ **and** $3x - y - 4 = 0$

Solving, we get $x = 1, y = -1$

$\therefore$ **Centre of the circle is (1, – 1)**

Given $2\pi r = 10\pi \Rightarrow r = 5$

$\therefore$ **Required circle is** $(x-1)^2 + (y+1)^2 = 5^2$

or $x^2 + y^2 - 2x + 2y - 23 = 0$ **.**

49. (b) **The equation of circle through points (0, 0), (1, 3) and (2, 4) is**

$x^2 + y^2 - 10x = 0$

50. (d) **Radius of given circle** $= \sqrt{g^2 + f^2 - c}$

$g^2 + f^2 = c$ **(given),** $\therefore$ **Radius = 0.**

51. (d) **Suppose the centre of circle be** (h, k)**. Since it touches the** y - axis **,** $\therefore$ **radius of circle =** ***h***

Now $(h-2)^2 + k^2 = h^2 \Rightarrow h^2 + 4 - 4h + k^2 = h^2$

$\Rightarrow k^2 = 4h - 4$ **. Hence the locus of centre is** $y^2 = 4x - 4$ **, which is a parabola.**

52. (d) **Comparing the given equation with** $x^2 + y^2 + 2gx + 2fy + c = 0$ **, we get** $g = 5$

$\therefore$ **Length of intercept on** ***x*****-axis =** $2\sqrt{g^2 - c}$

$= 2\sqrt{(5)^2 - 9}$ = **8.**

53. (b) $x = 2 + 3\cos\theta, y = 3\sin\theta - 1$

$x^2 + y^2 = 4 + 9\cos^2\theta + 12\cos\theta + 9\sin^2\theta + 1 - 6\sin\theta$

$= 14 + 12\cos\theta - 6\sin\theta$

$= 4(2 + 3\cos\theta) - 2(3\sin\theta - 1) + 4$

$\Rightarrow x^2 + y^2 = 4x - 2y + 4$

$\Rightarrow (x^2 - 4x + 4) + (y^2 + 2y + 1) = 9$

$\Rightarrow (x-2)^2 + (y+1)^2 = 9$ **, ∴ centre is** $(2,-1)$**.**

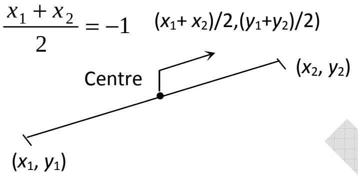
54. (b) The equation of circle passing through $(0,0), (2,0)$ **and (0, – 2) is** $x^2 + y^2 - 2x + 2y = 0$**. If it passes through** $(k,-2)$**, then** $k^2 + 4 - 2k - 4 = 0 \Rightarrow k = 0, 2$

$\because (0,-2)$ **is already a point on circle** $\therefore$ ***k* = 2.**

55. (a) x_1, x_2 **are roots of** $x^2 + 2x + 3 = 0$

$\Rightarrow x_1 + x_2 = -2$

$\therefore \frac{x_1 + x_2}{2} = -1$



y_1, y_2 **are roots of** $y^2 + 4y - 12 = 0$

$\Rightarrow y_1 + y_2 = -4 \Rightarrow \frac{y_1 + y_2}{2} = -2$

Centre of circle $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (-1,-2)$**.**

56. (c) The other end is $(t, 3-t)$

So the equation of the variable circle is

$(x-1)(x-t) + (y-1)(y-3+t) = 0$

Or $x^2 + y^2 - (1+t)x - (4-t)y + 3 = 0$

∴ The centre (α, β) **is given by**

$\alpha = \frac{1+t}{2}, \beta = \frac{4-t}{2}$

$\Rightarrow \ 2\alpha + 2\beta = 5$

Hence, the locus is $2x + 2y = 5$.

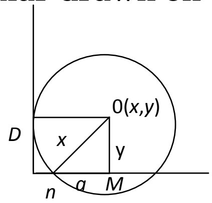
57. (b) Since the perpendicular drawn on chord from $O(x, y)$ **bisects the chord.**

$NM = a \quad OM = y$

$(ON)^2 = (OM)^2 + (ON)^2$

$x^2 = y^2 + a^2$

$x^2 - y^2 = a^2$



58. (b) According to the condition,

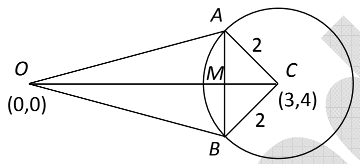
$\sqrt{(5)^2 + (3)^2 + 2(5) + k(3) + 17} = 7$

$\Rightarrow 61 + 3k = 49 \Rightarrow k = -4$.

59. (b) Here the equation of *AB* (chord of contact) is

$0 + 0 - 3(x + 0) - 4(y + 0) + 21 = 0$

$\Rightarrow 3x + 4y - 21 = 0$ **….(i)**



***CM* = perpendicular distance from (3, 4) to line (i) is**

$\frac{3 \times 3 + 4 \times 4 - 21}{\sqrt{9+16}} = \frac{4}{5}$

$AM = \sqrt{AC^2 - CM^2} = \sqrt{4 - \frac{16}{25}} = \frac{2}{5}\sqrt{21}$

$\therefore \ AB = 2AM = \frac{4}{5}\sqrt{21}$.

60. (a) Points where $x + 7 = 0$ **meets the circle** $x^2 + y^2 = 50$ **are** $(-7, 1)$ **and** $(-7, -1)$**. Hence equations of tangents at these points are** $-7x \pm y = 50$ **or** $7x \pm y + 50 = 0$ **.**

61. (c) According to the condition of tangency

$r = \frac{a\cos\alpha + b\sin\alpha - (a\cos\alpha + b\sin\alpha) - r}{\sqrt{\cos^2\alpha + \sin^2\alpha}}$

$\Rightarrow r = |-r| \Rightarrow r = r$.

62. **(c) We know that the equation of normal to the circle** $x^2 + y^2 = a^2$ **at the point** (x_1, y_1) **is** $\frac{x}{x_1} - \frac{y}{y_1} = 0$.

Therefore , $\frac{x}{1/\sqrt{2}} - \frac{y}{1/\sqrt{2}} = 0 \Rightarrow x - y = 0$.

63. **(a) Required equations are given by** $SS_1 = T^2$

$\Rightarrow (x^2 + y^2 - 2x + 4y)(1+4) = \{y - 1(x) + 2(y+1)\}^2$

$\Rightarrow 2x^2 - 2y^2 - 3x + 4y + 3xy - 2 = 0$

$\Rightarrow (2x - y + 1)(x + 2y - 2) = 0$.

64. **(c)** $y = mx + c$ **is a tangent, if** $c = \pm a\sqrt{1+m^2}$ **, where** $m = \tan 45^o = 1$

$\therefore$ **The equation is** $y = x \pm 6\sqrt{2}$.

65. **(c)** Length of tangent is given by $L_T = \sqrt{S_1} = \sqrt{49} = 7$.

66. (b) If the line $lx + my - 1 = 0$ touches the circle $x^2 + y^2 = a^2$, then applying the condition of tangency, we have $\pm \frac{l.0 + m.0 - 1}{\sqrt{l^2 + m^2}} = a$

On squaring and simplifying, we get the required locus $x^2 + y^2 = \frac{1}{a^2}$. Hence it is a circle.

67. **(a) The abscissa of point is found by substituting the ordinates and solving for abscissa.**

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x = \frac{8 \pm \sqrt{64 - 60}}{2} = \frac{8 \pm 2}{2} = 5$ **or 3**

***i.e.*, points are** $(5, -1)$ **and (3, –1).**

Normal is given by, $\frac{x-5}{5-4} = \frac{y+1}{-1-1} \Rightarrow 2x + y - 9 = 0$

and $\frac{x-3}{3-4} = \frac{y+1}{-1-1} \Rightarrow 2x - y - 7 = 0$.

68. **(c)** $k = 3,$ **as perpendicular from centre on line = radius.**

69. **(c) According to the condition,** $\frac{f^2 + g^2 - 6}{f^2 + g^2 + 3f + 3g} = \frac{4}{1} \Rightarrow f^2 + g^2 + 4f + 4g + 2 = 0$.

70. **(b) Normal passes through centre, therefore** $b = ma + c$.

71. **(a) From formula of tangent at a point,**

$x\left(\frac{ab^2}{a^2+b^2}\right) + y\left(\frac{a^2b}{a^2+b^2}\right) = \frac{a^2b^2}{a^2+b^2} \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$.

72. (a) The equation of tangents will be

$c(x^2 + y^2 + 2gx + 2fy + c) = (gx + fy + c)^2$

These tangents are perpendicular, hence the coefficients of x^2 **+ coefficients of** $y^2 = 0$

$\Rightarrow c - g^2 + c - f^2 = 0 \Rightarrow f^2 + g^2 = 2c$.

73. (d) Its centre is of type (*c*, *c*) and radius is

$$\left|\frac{4c + 3c - 12}{5}\right| = \sqrt{c^2} \Rightarrow c = 6.$$

74. (a) Tangent is $x - 2y - 5 = 0$ **and points of intersection with circle** $x^2 + y^2 - 8x + 6y + 20 = 0$ **are given by**

$4y^2 + 25 + 20y + y^2 - 16y - 40 + 6y - 20 = 0$

$\Rightarrow 5y^2 + 10y + 5 = 0$

75. (b) Length of tangent is $\sqrt{S_1}$.

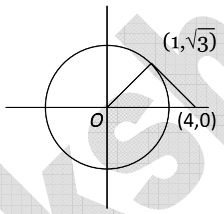
Equation of circle is $x^2 + y^2 - \frac{r^2}{a} = 0$

Hence $S_1 = \alpha^2 + \beta^2 - \frac{r^2}{a}$.

76. (c) Centres of circles are $C_1(2, 3)$ **and** $C_2(-3, -9)$ **and their radii are** $r_1 = 5$ **and** $r_2 = 8$.

Obviously $r_1 + r_2 = C_1C_2$ ***i.e.*, circles touch each other externally. Hence there are three common tangents.**

77. (a) $T \equiv x + \sqrt{3}y - 4 = 0$



Hence the required area $= \frac{1}{2} \times 4 \times \sqrt{3} = 2\sqrt{3}$.

78. (d) Suppose that the point be (*h*, *k*). Tangent at (*h*, *k*) is $hx + ky = a^2 \equiv x - y = -\sqrt{2}a$

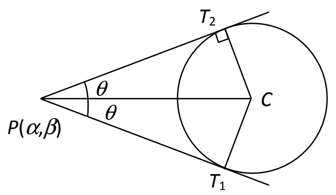
or $\frac{h}{1} = \frac{k}{-1} = \frac{a^2}{-\sqrt{2}a}$ **or** $h = -\frac{a}{\sqrt{2}}, k = \frac{a}{\sqrt{2}}$

Therefore, point of contact is $\left(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$.

79. (a) concept

80. (a) Substituting $x = \frac{3y+10}{4}$ in equation of circle, we get a quadratic in *y*. Solving, we get two values of *y* as 2 and – 6 from which we get value of *x*.

81. (c) $\tan\frac{\theta}{2} = \frac{CT_1}{PT_1} = \frac{a}{\sqrt{\alpha^2+\beta^2-a^2}}$



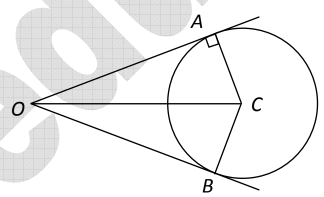
82. (d) Equation of a tangent at $(a\cos\alpha, a\sin\alpha)$ to the circle $x^2+y^2=a^2$ is $ax\cos\alpha + ay\sin\alpha = a^2$.

Hence its gradient is $-\frac{a\cos\alpha}{a\sin\alpha} = -\cot\alpha$.

83. (c) Substituting equation of line $y = mx + c$ in circle $x^2+y^2=r^2$

84. (b) Area of quadrilateral $= 2$ [area of ΔOAC]

$= 2.\frac{1}{2}OA\,.\,AC = \sqrt{S_1}.\sqrt{g^2+f^2-c}$



85. (b) Points of intersection with co-ordinate axes are $\left(-\frac{1}{\lambda},0\right)$, $(0,1)$ and $(-3,0), \left(0,\frac{3}{2}\right)$.

Equation of circle through (0, 1), (–3, 0) and $\left(0,\frac{3}{2}\right)$ is $x^2+y^2+\frac{7x}{2}-\frac{5y}{2}+\frac{3}{2}=0$.

86. (d) Tangent is of form $4x+3y+c=0$. From condition of tangency to the circle, we get $c=-25$.

Hence equation is $4x+3y-25=0$.

87. (c) Accordingly, $\alpha^2+\beta^2-4\alpha-5 = \alpha^2+\beta^2+6\alpha-2\beta+6$

$\Rightarrow 10\alpha - 2\beta + 11 = 0$.

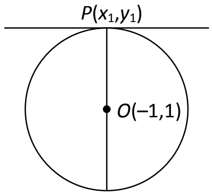
88. (a) Any tangent to $x^2+y^2=b^2$ is

$y = mx - b\sqrt{1+m^2}$. It touches $(x-a)^2+y^2=b^2$,

if $\frac{ma-b\sqrt{1+m^2}}{\sqrt{m^2+1}} = b$ or $ma = 2b\sqrt{1+m^2}$

or $m^2a^2 = 4b^2 + 4b^2m^2$, $\therefore m = \pm\frac{2b}{\sqrt{a^2-4b^2}}$.

89. (b) **Let point of contact be** $P(x_1, y_1)$.



90. (c) Let $P(x_1, y_1)$ **be a point on** $x^2 + y^2 = 4$

91. (b) Put point $(1,2)$ **in each option, only equation** $h + 2k = 5$ **satisfies. Hence option (b) is correct.**

92. (c) Length of intercepted part is diameter *i.e.*, 2*r*.

93. (a) The equation of tangent at point $(-2, -3)$ **to the circle** $x^2 + y^2 + 2x + 4y + 3 = 0$ **is,**

$-2x - 3y + 1(x - 2) + 2(y - 3) + 3 = 0$

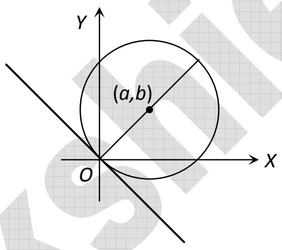
$\Rightarrow -2x - 3y + x - 2 + 2y - 6 + 3 = 0$

$\Rightarrow -x - y - 5 = 0 \Rightarrow x + y + 5 = 0$

or $y = -x - 5$ **; so,** $m = -1$

Hence, gradient of normal $= \frac{-1}{-1} = 1$.

94. (b) Obviously the slope of the tangent will be $-\left(\frac{1}{b/a}\right)$ ***i.e.,*** $-\frac{a}{b}$.



Hence the equation of the tangent is $y = -\frac{a}{b}x$

95. (b) $y = -\frac{\beta}{\alpha}x + \beta$ **touches the circle,**

$\therefore\ \beta^2 = a^2\left(1 + \frac{\beta^2}{\alpha^2}\right) \Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{1}{a^2}$

$\therefore$ **Locus of** $\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ **is** $x^2 + y^2 = \left(\frac{1}{a}\right)^2$.

96. (a) Both the sentences are true and *R* is the correct explanation of *A*, because for tangents which are parallel to *x*- axis, $\frac{dy}{dx}=0$.(a) Given tangents are

$5x-12y+10=0,\ 5x-12y-16=0$

Radius $=\frac{c_1-c_2}{2\sqrt{a^2+b^2}}=\frac{26}{2.13}=1$.

97. (c) Length of tangent

$=\sqrt{3^2+(-4)^2-4(3)-6(-4)+3}=\sqrt{40}$

$\therefore$ Square of length of tangent $=40$.

98. (d) Let the point be (x_1, y_1)

According to question, $\frac{\sqrt{x_1^2+y_1^2+4x_1+3}}{\sqrt{x_1^2+y_1^2-6x_1+5}}=\frac{2}{3}$

Squaring both sides, $\frac{x_1^2+y_1^2+4x_1+3}{x_1^2+y_1^2-6x_1+5}=\frac{4}{9}$

$\Rightarrow 9x_1+9y_1^2+36x_1+27=4x_1^2+4y_1^2-24x_1+20$

$\Rightarrow 5x_1^2+5y_1^2+60x_1+7=0$

Hence, locus is $5x^2+5y^2+60x+7=0$.

99. (a) Chord of contact from origin $\equiv gx+fy+c=0$

and from $(g, f)\equiv gx+fy+g(x+g)+f(y+f)+c=0$

Or $2gx+2fy+g^2+f^2+c=0$

$\therefore$ Distance $=\frac{\frac{g^2+f^2+c}{2}-c}{\sqrt{g^2+f^2}}=\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$.

101. (d) the point (1, 1) lies outside the circle, therefore no such chord exist.

102. (b) Since locus of middle point of all chords is the diameter, perpendicular to the chord.

103. (a) We know that the equation of common chord is $S_1-S_2=0$, where S_1 and S_2 are the equations of given circles, therefore

$(x-a)^2+(y-b)^2+c^2-(x-b)^2-(y-a)^2-c^2=0$

$\Rightarrow 2bx-2ax+2ay-2by=0 \qquad \Rightarrow 2(b-a)x-2(b-a)y=0 \Rightarrow x-y=0$.

104. (b) Let pole be (x_1, y_1) then polar

will be $xx_1+yy_1=1$ comparing with $lx+my+n=0 \Rightarrow x_1=-\frac{l}{n}, y_1=-\frac{m}{n}$.

105. (b) Length of chord

$= 2\{(\text{radius})^2 - (\text{length of} \perp \text{from centre to chord})^2\}^{1/2}$

$= 2\left\{r^2 - \left(\frac{-1}{\sqrt{(1/a^2)+(1/b^2)}}\right)^2\right\}^{1/2}$

$= 2\sqrt{\frac{r^2(a^2+b^2)-a^2b^2}{a^2+b^2}}$.

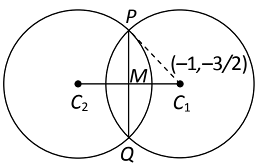
106. (b) Suppose point be (*h*, *k*). Equation of common chord of contact is

$hx + ky - a^2 = 0 \equiv lx + my + n = 0$

or $\frac{h}{l} = \frac{k}{m} = \frac{-a^2}{n}$ **or** $h = \frac{-a^2 l}{n}$, $k = \frac{-a^2 m}{n}$.

107. (b) Equation of common chord *PQ* is $2x + 1 = 0$

C_1M **= perpendicular distance of common chord from centre** $C_1 = \left|\frac{-2+1}{\sqrt{2^2}}\right| = \left|-\frac{1}{2}\right|$



Here; $C_1\left(-1, -\frac{3}{2}\right)$, $r_1 = \frac{3}{2} = C_1P$

PQ* = 2*PM $= 2\sqrt{C_1P^2 - C_1M^2} = 2\sqrt{\frac{9}{4} - \frac{1}{4}} = 2\sqrt{2}$.

108. (d) Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow 2x - 2y = 0$ ***i.e.,*** $x - y = 0$

$\because$ **Length of perpencicular drawn from** C_1 **to** $x - y = 0$ **is** $\frac{1}{\sqrt{2}}$

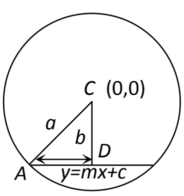
$\therefore$ **Length of common chord** $= 2\sqrt{\frac{19}{2} - \frac{1}{2}} = 6$.

109. (a) Polar of the circle is $xx' + yy' = a^2$**, but it is given by** $Ax + By + C = 0$**, then** $\frac{x'}{A} = \frac{y'}{B} = \frac{a^2}{-C}$

Hence pole is $\left(\frac{a^2A}{-C}, \frac{a^2B}{-C}\right)$.

110. (b) We know $CD = \left|\frac{c}{\sqrt{1+m^2}}\right|$ **….(i)**

But according to figure,



$a^2 - b^2 = CD^2$

From (i) and (ii), we get $a^2 - b^2 = \frac{c^2}{(1+m^2)}$

$\Rightarrow (a^2 - b^2)(1+m^2) = c^2$.

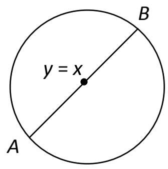
111. (c) The centre of the given circle is (1, 3) and radius is 2. So, *AB* is a diameter of the given circle has its midpoint as (1, 3). The radius of the required circle is 3.

112. (d) Given, circle is $x^2 + y^2 - 2x = 0$ **......(i)**

and line is $y = x$ **.....(ii)**

Puting $y = x$ **in (i),**

We get $2x^2 - 2x = 0 \Rightarrow x = 0, 1$



From (i), $y = 0$ **, 1**

Let $A = (0, 0)$**,** $B = (1, 1)$

Equation of required circle is

$(x-0)(x-1) + (y-0)(y-1) = 0$

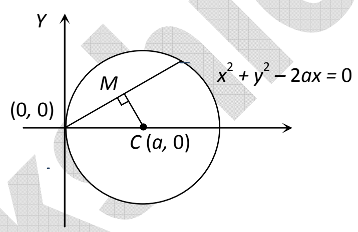
or $x^2 + y^2 - x - y = 0$.

113. (d) Let chord *AB* is $y = mx$ **.....(i)**

Equation of *CM*, $x + my = \lambda$

It is passing through (*a*, 0)

$\therefore \quad x + my = a$ **.....(ii)**



From (i) and (ii), $x + y.\frac{y}{x} = a \Rightarrow x^2 + y^2 = ax$

114. (c) The given circle is $x^2 + y^2 - 2x = 0$. **Let** (x_1, y_1) **be the middle point of any chord of this circle, than its equation is** $S_1 = T$.

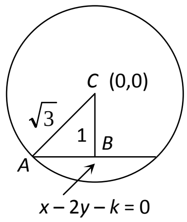
or $x_1^2 + y_1^2 - 2x_1 = xx_1 + yy_1 - (x + x_1)$

If it passes through (0, 0), then

$x_1^2 + y_1^2 - 2x_1 = -x_1 \Rightarrow x_1^2 + y_1^2 - x_1 = 0$

Hence the required locus of the given point (x_1, y_1) **is** $x^2 + y^2 - x = 0$.

115. (c) Obviously $BC=\sqrt{2}$



Hence, $\pm\frac{0-2.0-k}{\sqrt{1^2+(-2)^2}}=\sqrt{2}\Rightarrow k=\pm\sqrt{10}$.

116. (c) $T=S_1$ **is the equation of desired chord, hence**

$xx_1+yy_1-a^2=x_1^2+y_1^2-a^2\Rightarrow xx_1+yy_1=x_1^2+y_1^2$.

117. (d) Centre (0, 0), radius $=3a\times\frac{2}{3}=2a$.

Hence circle $x^2+y^2=4a^2$ **as centroid divides median in the ratio of** $2:1$.

118. (a) Let equation of circle be $x^2+y^2+2gx+2fy+c=0$. **If** $\left(m,\frac{1}{m}\right)$ **lies on this circle, then**

$m^2+\frac{1}{m^2}+2gm+2f\frac{1}{m}+c=0$

or $m^4+2gm^3+2fm+cm^2+1=0$

This is a fourth degree equation in ***m*** **having** m_1,m_2,m_3,m_4 **as its roots.**

Therefore, $m_1m_2m_3m_4=$ **product of roots** $=\frac{1}{1}=1$.

119. (c) Required area $=\frac{a}{h^2+k^2}(h^2+k^2-a^2)^{3/2}$

$=\frac{3}{4^2+3^2}(4^2+3^2-9)^{3/2}=\frac{192}{25}$.

120. (b) Let the equation of line passing through origin be $y=mx$. **Therefore**

$x^2+y^2-x+3y=0\Rightarrow x^2+m^2x^2-x+3mx=0$

$\Rightarrow x[x(1+m^2)-(1-3m)]=0$

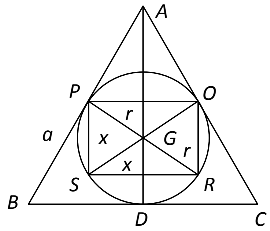
121. (b) Let $A=(a,0)$, $B=(-a,0)$, $P=(\alpha,\beta)$

$\therefore\ \frac{PA^2}{PB^2}=k^2\Rightarrow(\alpha-a)^2+\beta^2=k^2[(\alpha+a)^2+\beta^2]$

$\therefore$ **Locus is** $(x^2+y^2)(1-k^2)-2a(1+k^2)x+(1-k^2)a^2=0$

This is a circle for $k\neq 1$.

122. (c) If p be the altitude, then $p = a\sin 60^o = \frac{a}{2}\sqrt{3}$ **.**



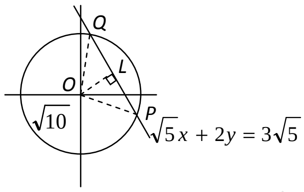
Since the traingle is equilateral, therefore centroid, orthocentre, circumcentre and incentre all coincide.

Hence, radius of the inscribed circle $= \frac{1}{3}p = \frac{a}{2\sqrt{3}} = r$ or diameter $= 2r = \frac{a}{\sqrt{3}}$.

Now if x be the side of the square inscribed, then angle in a semicircle being a right angle, hence

$$x^2 + x^2 = d^2 = 4r^2 \Rightarrow 2x^2 = \frac{a^2}{3}$$

123. (c) Length of perpendicular from origin to the line $x\sqrt{5} + 2y = 3\sqrt{5}$ **is**



$$OL = \frac{3\sqrt{5}}{\sqrt{(\sqrt{5})^2 + 2^2}} = \frac{3\sqrt{5}}{\sqrt{9}} = \sqrt{5}$$

Radius of the given circle $= \sqrt{10} = OQ = OP$

$$PQ = 2QL = 2\sqrt{OQ^2 - OL^2} = 2\sqrt{10-5} = 2\sqrt{5}$$

Thus area of $\Delta OPQ = \frac{1}{2} \times PQ \times OL = \frac{1}{2} \times 2\sqrt{5} \times \sqrt{5} = 5$ **.**

124. (a) Let $A(x_1, y_1)$ **and** $B(x_2, y_2)$ **,then**

$x_1 + x_2 = -2a$ **,** $x_1 x_2 = -b^2$

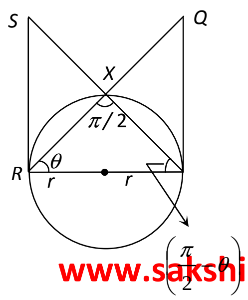
$y_1 + y_2 = -2p$ **,** $y_1 y_2 = -q^2$

Now find centre and radius and hence the equation of circle.

125. (a) $\tan\theta = \frac{PQ}{PR} = \frac{PQ}{2r}$

Also $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{RS}{2r}$

i.e., $\cot\theta = \frac{RS}{2r}$



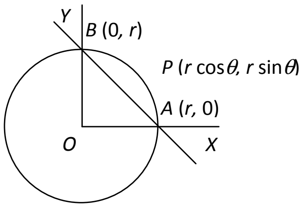
$\therefore\ \tan\theta.\cot\theta = \frac{PQ.RS}{4r^2}$

$\Rightarrow\ 4r^2 = PQ.RS \ \Rightarrow\ 2r = \sqrt{(PQ)(RS)}$.

126. (b) Let the centroid $=(\alpha, \beta)$

Then $\alpha = \frac{r + r\cos\theta}{3}, \quad \beta = \frac{r + r\sin\theta}{3}$

Or $\left(\alpha - \frac{r}{3}\right)^2 + \left(\beta - \frac{r}{3}\right)^2 = \frac{r^2}{9}$



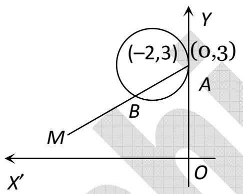
$\therefore$ **The locus is** $\left(x - \frac{r}{3}\right)^2 + \left(y - \frac{r}{3}\right)^2 = \left(\frac{r}{3}\right)^2$, **which is a circle.**

127. (d) Let (*h*, 0) be a point on *x*-axis, then the equation of chord whose mid-point is (*h*, 0) will be

$xh - \frac{1}{2}p(x+h) - \frac{1}{2}q(y+0) = h^2 - ph$

128. (c) Line *AB* is $x + y = 0$, **which is diameter of the circle** $x^2 + y^2 = 25$. **Its length** $= 2r = 10$.

129. (b) $\left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0$



$\Rightarrow \frac{h^2}{4} + \frac{8h}{4} + \frac{(k-3)^2}{4} = 0$

or $x^2 + y^2 + 8x - 6y + 9 = 0$, **which is a circle.**

130. (b) Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ **......(i)**

Circle (i) cuts circle $x^2 + y^2 - 4 = 0$ **orthogonally**

$\Rightarrow\ 2g.0 + 2f.0 = c - 4 \Rightarrow c = 4$

Since circle (i) passes through (*a, b*)

$\therefore a^2 + b^2 + 2ga + 2fb + 4 = 0$

$\therefore$ **Locus of centre** $(-g, -f)$ **is**

$2ax + 2by - (a^2 + b^2 + 4) = 0$.

131. (d) The given normals are $x - 3y = 0,\ x - 3 = 0$ **which intersect at centre whose co-ordinates are (3, 1). The given circle is** $C_1(3, -3)$ $r_1 = 1, C_2$ **is (3, 1) and** $r_2 = (?)$**. If the two circles touch externally, then** $C_1C_2 = r_1 + r_2 \Rightarrow 4 = 1 + r_2 \Rightarrow r_2 = 3$

$$\therefore (x-3)^2 + (y-1)^2 = 3^2 \text{ or } x^2 + y^2 - 6x - 2y + 1 = 0.$$

132. (c) Length of each tangent

$L^2 = (4)^2 + (5)^2 - (4 \times 4) - (2 \times 5) - 11$

$L = 2$

$r = \sqrt{2^2 + 1^2 - (-11)}$

$r = 4$

Area $= L \times r = 8$ ***sq. units*.**