TRIGONOMETRIC RATIOS

OBJECTIVES

1. **The equation** $\sec^2\theta = \frac{4xy}{(x+y)^2}$ **is only possible when**

(a) $x = y$ (b) $x < y$

(c) $x > y$ (d) None of these

2. $\tan 1°\tan 2°\tan 3°\tan 4°........ \tan 89° =$

(a) 1 (b) 0

(c) ∞ (d) 1/2

3. **If** $\tan\theta = \frac{-4}{3}$, **then** $\sin\theta =$

(a) – 4/5 but not 4/5 (b) – 4/5 or 4/5

(c) 4/5 but not – 4/5 (d) None of these

4. **If** $\tan\theta = -\frac{1}{\sqrt{10}}$ **and** θ **lies in the fourth quadrant, then** $\cos\theta =$

(a) $1/\sqrt{11}$ (b) $-1/\sqrt{11}$

(c) $\sqrt{\frac{10}{11}}$ (d) $-\sqrt{\frac{10}{11}}$

5. **Which of the following is correct**

(a) $\tan 1 > \tan 2$ (b) $\tan 1 = \tan 2$ (c) $\tan 1 < \tan 2$ (d) $\tan 1 = 1$

6. $(m+2)\sin\theta + (2m-1)\cos\theta = 2m+1$, **if**

(a) $\tan\theta = \frac{3}{4}$ (b) $\tan\theta = \frac{4}{3}$

(c) $\tan\theta = \frac{2m}{m^2+1}$ (d) None of these

7. If $\tan\theta + \sec\theta = e^x$, then $\cos\theta$ equals

(a) $\frac{(e^x + e^{-x})}{2}$ (b) $\frac{2}{(e^x + e^{-x})}$

(c) $\frac{(e^x - e^{-x})}{2}$ **(d)** $\cos\theta = \frac{2}{e^x + e^{-x}}$.

8. **If** $\tan\theta + \sin\theta = m$ **and** $\tan\theta - \sin\theta = n$, **then**

(a) $m^2 - n^2 = 4mn$ (b) $m^2 + n^2 = 4mn$

(c) $m^2 - n^2 = m^2 + n^2$ (d) $m^2 - n^2 = 4\sqrt{mn}$

9. If $(1+\sin A)(1+\sin B)(1+\sin C) = (1-\sin A)(1-\sin B)(1-\sin C)$, then each side is equal to

(a) $\pm \sin A \sin B \sin C$ (b) $\pm \cos A \cos B \cos C$

(c) $\pm \sin A \cos B \cos C$ (d) $\pm \cos A \sin B \sin C$

10. **If** $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, **then** $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

(a) 3 (b) 2

(c) 1 (d) 0

11. **The value of** $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$ **is**

(a) 2 (b) 0

(c) 4 (d) 6

12. $\cos 1° + \cos 2° + \cos 3° + ... + \cos 180° =$

(a) 0 (b) 1

(c) – 1 (d) 2

13. **If** $\pi < \alpha < \frac{3\pi}{2}$**, then** $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} + \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} =$

(a) $\frac{2}{\sin\alpha}$ (b) $-\frac{2}{\sin\alpha}$

(c) $\frac{1}{\sin\alpha}$ (d) $-\frac{1}{\sin\alpha}$

14. **Given that** $\pi < \alpha < \frac{3\pi}{2}$, **then the expression** $\sqrt{(4\sin^4\alpha + \sin^2 2\alpha)} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$ **is equal to**

(a) 2 (b) $2 + 4\sin\alpha$

(c) $-2 - 4\sin\alpha$ (d) None of these

15. **The value of** $\cos(270° + \theta)\cos(90° - \theta) - \sin(270° - \theta)\cos\theta$ **is**

(a) 0 (b) –1

(c) 1/2 (d) 1

16. **If angle** θ **be divided into two parts such that the tangent of one part is** k **times the tangent of the other and** ϕ **is their difference, then** $\sin\theta =$

(a) $\frac{k+1}{k-1}\sin\phi$ (b) $\frac{k-1}{k+1}\sin\phi$

(c) $\frac{2k-1}{2k+1}\sin\phi$ (d) None of these

17. **If** $\sin\theta + \operatorname{cosec}\theta = 2$, **the value of** $\sin^{10}\theta + \operatorname{cosec}^{10}\theta$ **is**

(a) 10 (b) 2^{10}

(c) 2^9 (d) 2

18. **If** $\sin(\alpha-\beta)=\frac{1}{2}$ **and** $\cos(\alpha+\beta)=\frac{1}{2}$, **where** α **and** β **are positive acute angles, then**

(a) $\alpha=45°, \beta=15°$ (b) $\alpha=15°, \beta=45°$

(c) $\alpha=60°, \beta=15°$ (d) None of these

19. **If** $\sin x+\sin y=3(\cos y-\cos x)$, **then the value of** $\frac{\sin 3x}{\sin 3y}$ **is**

(a) 1 (b) – 1

(c) 0 (d) None of these

20. **If** $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$, **then** $\cos\theta+\sin\theta$ **is equal to**

(a) $\sqrt{2}\cos\theta$ (b) $\sqrt{2}\sin\theta$

(c) $2\cos\theta$ (d) $-\sqrt{2}\cos\theta$

21. **If** $x=\sec\phi-\tan\phi, y=\operatorname{cosec}\phi+\cot\phi$, **then**

(a) $x=\frac{y+1}{y-1}$ (b) $x=\frac{y-1}{y+1}$

(c) $y=\frac{1-x}{1+x}$ (d) None of these

22. **If** $\tan\theta+\sin\theta=m$ **and** $\tan\theta-\sin\theta=n$, **then**

(a) $m^2-n^2=4mn$ (b) $m^2+n^2=4mn$

(c) $m^2-n^2=m^2+n^2$ (d) $m^2-n^2=4\sqrt{mn}$

23. If $(\sec\alpha+\tan\alpha)(\sec\beta+\tan\beta)(\sec\gamma+\tan\gamma)$

$=\tan\alpha\tan\beta\tan\gamma$, then $(\sec\alpha-\tan\alpha)(\sec\beta-\tan\beta)$

$(\sec\gamma-\tan\gamma)=$

(a) $\cot\alpha\cot\beta\cot\gamma$ (b) $\tan\alpha\tan\beta\tan\gamma$

(c) $\cot\alpha+\cot\beta+\cot\gamma$ (d) $\tan\alpha+\tan\beta+\tan\gamma$

24. **The value of** $\sin 10°+\sin 20°+\sin 30°+...+\sin 360°$ **is**

(a) 1 (b) 0

(c) – 1 (d) None of these

25. **The value of** $\cos y\cos\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-y\right)\cos x+\sin y\cos\left(\frac{\pi}{2}-x\right)+\cos x\sin\left(\frac{\pi}{2}-y\right)$ **is zero, if**

(a) $x=0$ (b) $y=0$

(c) $x=y$ (d) $x=n\pi-\frac{\pi}{4}+y, (n\in I)$

26. $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}=$

(a) 1 (b) – 1

(c) 0 (d) 2

27. **If θ lies in the second quadrant, then the value of** $\sqrt{\left(\frac{1-\sin\theta}{1+\sin\theta}\right)}+\sqrt{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)}$

(a) $2\sec\theta$ (b) $-2\sec\theta$

(c) $2\mathrm{cosec}\,\theta$ (d) None of these

28. **If** $\cos\theta=\frac{1}{2}\left(x+\frac{1}{x}\right)$**, then** $\frac{1}{2}\left(x^2+\frac{1}{x^2}\right)=$

(a) $\sin 2\theta$ (b) $\cos 2\theta$

(c) $\tan 2\theta$ (d) $\sec 2\theta$

29. **The value of** $e^{\log_{10}\tan 1°+\log_{10}\tan 2°+\log_{10}\tan 3°+..........+\log_{10}\tan 89°}$ **is**

(a) 0 (b) e

(c) $1/e$ (d) None of these

30. **If** $\sin x+\sin^2 x=1$**, then the value of** $\cos^{12}x+3\cos^{10}x+3\cos^8 x+\cos^6 x-2$ **is equal to**

(a) 0 (b) 1

(c) – 1 (d) 2

31. **If** $\cos x+\cos^2 x=1$**, then the value of** $\sin^2 x+\sin^4 x$ **is**

(a) 1 (b) – 1

(c) 0 (d) 2

32. **$(\sec\theta - \cos\theta)^2 + (\mathrm{cosec}\theta - \sin\theta)^2 - \tan^2\theta - \cot^2\theta =$**

1) -1 2) 1 3) 7 4) - 7

33. If $\frac{\sin^4 A}{a}+\frac{\cos^4 A}{b}=\frac{1}{a+b}$, then $\frac{\sin^8 A}{a^3}+\frac{\cos^8 A}{b^3}=$

1) $\frac{2}{(a+b)^3}$ 2) $\frac{1}{(a+b)^3}$ 3) $\frac{1}{(a+b)^2}$ 4) $\frac{1}{(a-b)^2}$

34. **The value of (1+cotθ-cosecθ)(1+tanθ+secθ) is**

1) 2 2) -2 3) 3 4) -3

35. **If (1-cosA)(1-cosB)(1-cosC) = sinAsinBsinC, then (1+cosA)(1+cosB)(1+cosC) =**

1) cosA cosB cosC 2) sinA sinB sinC

3) - cosA cosB cosC 4) $\sin^2 A \sin^2 B \sin^2 C$

36. **If tanθ = 3/4 and θ is not in the first quadrant, then** $\dfrac{\sin\left(\frac{\pi}{2}+\theta\right)-\cot(\pi-\theta)}{\tan\left(\frac{3\pi}{2}-\theta\right)-\cos\left(\frac{3\pi}{2}+\theta\right)}=$

1) 0 2) 1 3) 8/29 4) 29/8

37. **If $7\sin^2\theta + 3\cos^2\theta = 4$, then tanθ =**

1) $\pm 1/\sqrt{3}$ 2) ± 1 3) $\pm\sqrt{3}$ 4) 1/3

38. **If a cosθ - b sinθ = c, then a sinθ + b cosθ =**

1) $\pm\sqrt{a^2+b^2-c^2}$ 2) $\pm\sqrt{a^2-b^2-c^2}$

3) $\pm\sqrt{c^2-a^2-b^2}$ 4) $\pm\sqrt{c^2+a^2+b^2}$

39. **If 5 cosθ + 7 sinθ = 7, then $(7\cos\theta - 5\sin\theta)^2$ =**

1) 25 2) 49 3) 24 4) -49

40. **If secθ + tanθ = 1/5 then sinθ =**

1) - 5/13 2) –12/13 3) 12/13 4) 5/13

41. $\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}=$

1) 1 2) 2 3) 4 4) 3

42. **If $\tan 20^0 = k$, then** $\dfrac{\tan 250^0+\tan 340^0}{\tan 20^0-\tan 110^0}=$

1) $\dfrac{1-k^2}{1+k^2}$ 2) $\dfrac{1+k^2}{1-k^2}$ 3) $\dfrac{2k}{1-k^2}$ 4) $\dfrac{1-k^2}{2k}$

43. **Minimum value of $\sec^2\theta + \mathrm{cosec}^2\theta$ is**

1) 1 2) 2 3) 3 4) 4

44. **Minimum value of $\tan\theta + \cot\theta$ in $\left(0, \frac{\pi}{2}\right)$ is**

1) 1 2) 2 3) 3 4) 4

TRIGONOMETRIC RATIOS

HINTS AND SOLUTIONS

1. (a) $\cos^2\theta \le 1$

$\sec^2\theta = \frac{4xy}{(x+y)^2} \ge 1 \Rightarrow 4xy \ge (x+y)^2 \Rightarrow (x-y)^2 \le 0$

2. (a) $\tan 1^\circ \tan 2^\circ .... \tan 89^\circ$

$= (\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ)...... = 1\times1\times1.... = 1.$

3. (b) $\text{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\sin^2\theta = \frac{1}{\text{cosec}^2\theta} = \frac{16}{25} \Rightarrow \sin\theta = \pm\frac{4}{5},$

4. (c) $\tan\theta = -\frac{1}{\sqrt{10}}$, Therefore θ is in IV quadrant. So $\cos\theta = +ve$.

5. (b) $\sin 1 > \sin 1^\circ$

6. (b) Squaring the given relation and putting $\tan\theta = t,$

$(m+2)^2 t^2 + 2(m+2)(2m-1)t + (2m-1)^2 = (2m+1)^2(1+t^2)$

$\Rightarrow 3(1-m^2)t^2 + (4m^2+6m-4)t - 8m = 0$

$\Rightarrow (3t-4)[(1-m^2)t + 2m] = 0$,

7. (b) $\tan\theta + \sec\theta = e^x$ .....(i)

$\therefore \sec\theta - \tan\theta = e^{-x}$ .....(ii)

Adding $2\sec\theta = e^x + e^{-x} \Rightarrow \cos\theta = \frac{2}{e^x + e^{-x}}$.

8. (d) $(m+n) = 2\tan\theta,\ m-n = 2\sin\theta$

$\therefore m^2 - n^2 = 4\tan\theta . \sin\theta$ ..... (i)

$4\sqrt{mn} = 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sin\theta . \tan\theta$ ..... (ii)

From (i) and (ii), $m^2 - n^2 = 4\sqrt{mn}$.

9. (b) $(1-\sin^2 A)(1-\sin^2 B)(1-\sin^2 C)$

$= (1-\sin A)^2(1-\sin B)^2(1-\sin C)^2$

$\Longrightarrow (1-\sin A)(1-\sin B)(1-\sin C)=\pm\cos A\cos B\cos C$

Similarly, $(1+\sin A)(1+\sin B)(1+\sin C)=\pm\cos A\cos B\cos C$.

10. (d) $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$

$\Rightarrow \sin\theta_1=\sin\theta_2=\sin\theta_3=1$, $\quad (\because -1\le\sin x\le 1)$

$\Rightarrow \theta_1=\theta_2=\theta_3=\frac{\pi}{2}\Rightarrow \cos\theta_1+\cos\theta_2+\cos\theta_3=0$.

11. (b) $(\sin^2\theta+\cos^2\theta)^3=(1)^3$

$\Rightarrow \sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta=1$

and $\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta=1$

Both gives,

$2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$.

12. (c) $(\cos 1^\circ+\cos 179^\circ)+(\cos 2^\circ+\cos 178^\circ)+....$

$+(\cos 89^\circ+\cos 91^\circ)+\cos 90^\circ+\cos 180^\circ=-1$.

13. (b) $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}+\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\frac{1-\cos\alpha+1+\cos\alpha}{\sqrt{1-\cos^2\alpha}}$

$=\frac{2}{\pm\sin\alpha}=\frac{2}{-\sin\alpha}, \left(\text{since } \pi<\alpha<\frac{3\pi}{2}\right).$

14. (a) α is in third quadrant $\sqrt{(4\sin^4\alpha+\sin^2 2\alpha)}+4\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

15. (d) $\cos(270+\theta)\cos(90-\theta)-\sin(270-\theta)\cos\theta$

$=\sin\theta.\sin\theta+\cos\theta.\cos\theta=1$.

16. (a) Let $A+B=\theta$ and $A-B=\phi$.

Then $\tan A=k\tan B$ or $\frac{k}{1}=\frac{\tan A}{\tan B}=\frac{\sin A\cos B}{\cos A\sin B}$

Applying componendo and dividendo

17. (d) We have,

$\sin\theta+\operatorname{cosec}\theta=2\Rightarrow \sin^2\theta+1=2\sin\theta$

$\Rightarrow \sin^2\theta-2\sin\theta+1=0$

$\Rightarrow (\sin\theta-1)^2=0\Rightarrow \sin\theta=1$

Required value of $\sin^{10}\theta+\operatorname{cosec}^{10}\theta=(1)^{10}+\frac{1}{(1)^{10}}=2$.

18. (a) $\sin(\alpha-\beta)=\frac{1}{2}=\sin 30^\circ \Rightarrow \alpha-\beta=30^\circ$ .....(i)

and $\cos(\alpha+\beta)=\frac{1}{2}\Rightarrow \alpha+\beta=60^\circ$ .....(ii)

Solving (i) and (ii), we get $\alpha=45^\circ$ and $\beta=15^\circ$.

19. (b) Standard problem.

20. (a) $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$

$\Rightarrow \cos\theta=(\sqrt{2}+1)\sin\theta \Rightarrow (\sqrt{2}-1)\cos\theta=\sin\theta$

$\Rightarrow \sqrt{2}\cos\theta-\cos\theta=\sin\theta \Rightarrow \sin\theta+\cos\theta=\sqrt{2}\cos\theta.$

21. (b) Standard problem

22. (d) $(m+n)=2\tan\theta,\ m-n=2\sin\theta$

$\therefore\ m^2-n^2=4\tan\theta.\sin\theta$ ..... (i)

$4\sqrt{mn}=4\sqrt{\tan^2\theta-\sin^2\theta}=4\sin\theta.\tan\theta$ ..... (ii)

From (i) and (ii), $m^2-n^2=4\sqrt{mn}$.

23. (a) $(\sec\alpha+\tan\alpha)(\sec\beta+\tan\beta)(\sec\gamma+\tan\gamma)$

$=\tan\alpha\tan\beta\tan\gamma$ ... (i)

Let $x=(\sec\alpha-\tan\alpha)(\sec\beta-\tan\beta)(\sec\gamma-\tan\gamma)$ ... (ii)

Multiply (i) and (ii)

24. (b) Since $\sin 190^\circ=-\sin 10^\circ,\ \sin 200^\circ=-\sin 20^\circ,$

$\sin 210^\circ=-\sin 30^\circ,\ \sin 360^\circ=\sin 180^\circ=0$

25. (d) The expression is equal to

$\sin(x-y)+\cos(x-y)=\sqrt{2}\left\{\sin\left(\frac{\pi}{4}+x-y\right)\right\},$

Which is zero, if $\sin\left(\frac{\pi}{4}+x-y\right)=0$

i.e., $\frac{\pi}{4}+x-y=n\pi(n\in I)\Rightarrow x=n\pi-\frac{\pi}{4}+y$.

26. (d) $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}$

$=\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{\pi}{8}=2\left(\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}\right)=2\times 1=2$.

27. (b)

28. (b) $\cos\theta = \frac{1}{2}\left(x+\frac{1}{x}\right) \Rightarrow x+\frac{1}{x} = 2\cos\theta$

We know that $x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$

$= (2\cos\theta)^2 - 2 = 4\cos^2\theta - 2 = 2\cos 2\theta$

$\therefore \frac{1}{2}\left(x^2+\frac{1}{x^2}\right) = \frac{1}{2}\times 2\cos 2\theta = \cos 2\theta$

29. (d) $e^{\log_{10}\tan 1^o + \log_{10}\tan 2^o + \log_{10}\tan 3^o + ...+\log_{10}\tan 89^o}$

$= e^{\log_{10}(\tan 1^o \tan 2^o \tan 3^o.....\tan 89^o)} = e^{\log_{10} 1} = e^o = 1$

30. (c) Standard problem

31. (a) $\cos x + \cos^2 x = 1 \Rightarrow \cos x = \sin^2 x$

$\therefore \sin^2 x + \sin^4 x = \cos x + \cos^2 x = 1.$

32.(b)

33.(b)

34.(a)

35.(b)

36.(c)

37.(a)

38.(a)

39.(a)

40.(B)

41.(b)

42.(a)

43(d)

44. (b)