TANGENTS AND NORMALS

OBJECTIVE PROBLEMS

1. **For a curve y = f(x) if $\frac{dy}{dx} = 2x$ then the angle made by the tangent at (1,1) with $\overrightarrow{OX}$ is**

   1) $\pi/4$ 2) $\pi/3$

   3) $\tan^{-1} 2$ 4) $\tan^{-1} 1/2$

2. **The angle made by the tangent line at (1,3) on the curve y = 4x-x^2 with $\overrightarrow{OX}$ is**

   1) $\tan^{-1} 2$ 2) $\tan^{-1} 1/3$

   3) $\tan^{-1} 3$ 4) $\pi/4$

3. **Slope of the tangent line at x = 2 on $y = \frac{8}{4+x^2}$ is**

   1) 1 /2 2) -1/2 3) -2 4) None

4. **Slope of the normal line at (a, b) to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ is**

   1) /b/a 2) b/a 3)-a/b 4) a/b

5. **Slope of the normal line to the curve $xy^2 + yx^2 = 2$ at (1,-2) is**

   1) 5/8 2) -8/5 3) -1 4) None

6. **The slope of the normal line to the curve $x = a(t - \sin t), y = a(1 - \cos t)$ at a point 't' is**

   1) tan t/2 2) cot t/2 3) –cot t/2 4) –tan t/2

7. **Equation of the tangent line at (2,4) to the curve $y(x^2 - 1) = 6x$ is**

   1) $3x - 10y + 34 = 0$ 2) $10x - 3y = 32$

   3) $10x + 3y = 32$ 4) none

9. **Equation of the normal line to the curve $y(x+2) = 5$ at the point (1,5/3) is**

   1) 27x+15y = 2 2) 27x-15y = 2

   3) 15x-27y = 2 4) none

10. **Slope of the tangent to $y = \sqrt{4 - x^2}$ at the point where abscissa and ordinate are equal is**

    1) -1 2) 1 3) 0 4) $\sqrt{2}$

11. Length of the subnormal at (0,b) to $y = be^{-x/a}$ is

1) $y^2/|a|$ 2) a 3) $b^2|a|$ 4) y^2/a

12. A point which the tangent to the curve $y = x^2(x-2)^2$ is parallel to the x – axis is

1) (-1,9) 2) 2a 3) $|a|/2$ 4) none

13. A point at which the tangent to the curve $y = x^2(x-2)^2$ is parallel to the x-axis is

1) (-1,9)) 2) (1,-1) 3) (1,1) 4) (2,1)

14. Length of the sub tangent at (-a,a) on $x^2y^2 = a^4(a>0)$ is

1) 3a 2) a 3) 2a 4) 4a

15. Length of the subtangent at any point on $y^n = a^{n-1}x$ is

1) Proportional to abscissa

2) Proportional to ordinate

3) Length of the subnormal

4) None

16. The length of the subtangent at any point of $y = be^{x/a}$ is

1) Varies as the abscissa

2) Varies as the ordinate

3) Constant

4) Length of the subnormal

17. Equation of the tangent at (1,-1) to the curve $x^3 - xy^2 - 4x^2 - xy + 5x + 3y + 1 = 0$ is

1) x-1=0 2) x+1=0 3) y-1=0 4) y+1=0

18. The tangent line at $\left(\frac{a}{\sqrt{8}}, \frac{a}{\sqrt{8}}\right)$ to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is parallel to the line

1) x = -y 2) x = y 3) x = 0 4) y = 0

19. If ψ is the angle made by the tangent line at a point P(x, y) on the curve y = f(x) with $\overrightarrow{OX}$ then length of the normal is

1) $|y\sec\psi|$ 2) $|y\cos\psi|$

3) $|y\operatorname{cosec}\psi|$ 4) $|y\tan\psi|$

20. At any point on the curve y = f(x) if $m = \frac{dy}{dx}$ then $\frac{\text{length of the tangent}}{\text{length of the normal}} =$

1) 1/m 2) $1/|m|$ 3) m 4) $|m|$

21. At any point on the curve y = f(x) if $m = \frac{dy}{dx}$ **then** $\frac{\text{length of the subnormal}}{\text{length of the sub tangent}} =$

1) Constant 2) $|m|$ 3) m^2 4) $1/m^2$

22. If the normal to the curve y = f(x) at (3,4) makes angle $3\pi/4$ **with** $\overrightarrow{OX}$ **then** $f'(3) =$

1) -1 2) 1 3) -3/4 4) 4/3

23. The length of subtangent, |ordinate of the point | and length of subnormal at a point on the curve y = f(x) are in

1) A.P. 2) G.P. 3) H.P. 4) None

24. For the parabola $y^2 = 4ax$ **the ratio of the length of subtangent to abscissa at any point is**

1) 1:1 2) 2:1 3) x:y 4) $x^2 : y$

25. The length of the normal at any point P(x, y) on the hyperbola $x^2 - y^2 = a^2$ **is**

1) a 2) $|a|$

3) OP where O is the origin 4) None

26. The length of the subtangent at any point on $y = ae^{x/b}, m, n > 0$ **is**

1) $|a|$ 2) a 3) $|b|$ 4) b

27. Length of the subtangent at (x_1, y_1) **on** $x^n y^m = a^{m+n} m, n > 0$ **is**

1) $\frac{n}{m}|x_1|$ 2) $\frac{n}{m}x_1$ 3) $\frac{m}{n}|x_1|$ 4) $\frac{n}{m}|y_1|$

28. If the gradient to the curve xy+ax+by=0 at (1,1) is 2 then (a,b) =

1) (-2,1) 2) (1,-2) 3) (-1,2) 4) (1,2)

29. If the tangent line at (x, y) to the curve $y = x^3 - 12x + 8$ **is parallel to the X-axis then x =**

1) ± 2 2) ± 1 3) 3 4) None

30. The points on the curve y = sinx where the tangent lines are parallel to X- axis are given by

1) $x = n\pi, n \in Z$ 2) $x = 2n\pi, n \in Z$

3) $x = (2n+1)n/2, n \in Z$ 4) None

31. The distance from the origin to the normal at x = 0 to the curve $y = e^{2x} + x^2$ **is**

1) $\sqrt{5}$ 2) $1/\sqrt{5}$ 3) $2/\sqrt{5}$ 4) 1

32. **If at a point (x_1, y_1) on the curve y = f(x) if lengths of sub tangent and subnormal are equal, then length of normal is**

1) $(1/2)y_1$ 2) $2y_1$ 3) $-\sqrt{2y_1}$ 4) $\sqrt{2}|y_1|$

33. **If $ax+by+c=0$ is normal to the curve xy = 1, then**

1) a>0, b<0 2) a>0, b>0 3) a<0,b<0 4) none

34. **The curves $y=x^3+x+1$ and $2y=x^3+5x$ touch each other at the point**

1) (1,3) 2))(-1,-1) 3) (0,1) 4) (-2,-9)

35. **If the curves $y=ax^3-3ax+4a$ and $y=12x+4a$ touch at x = 0 then a =**

1) -3 2) 3 3) -4 4) 4

36. **The common tangent line to the parabola $y^2=2x$ and circle $x^2+y^2-4x=0$ at which they touch each other , is**

1) x-axis 2) A bisector of coordinate axes

3) y-axis 4) None

37. **Area of the triangle formed by the tangent, normal at (1,1) on the curve $\sqrt{x}+\sqrt{y}=2$ and the x-axis is**

1) 1 sq. Unit 2) sq. Units

3) 1 /2 sq. Units 4)4 sq. Units

38. **Area of the triangle formed by the tangent at (x_1, y_1) to $xy=a^2$ and the coordinate axes is**

1) x_1y_1 2) $2|x_1y_1|$ 3) $1/2|x_1y_1|$ 4) a^2

39. **The curves $x^3-3xy^2=-2, 3x^2y-y^3=2$ cut each other at an angle**

1) 0 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/2$

40. **The x-intercept made by the tangent at 't' on the curve $x=a\cos^3 t, y=a\sin^3 t$ is**

1) a 2) a sin t 3) a cos t 4) a tan t

41. **Equation of the normal line at 'θ' to the curve $x=a(\theta=\sin\theta), y=a(1-\cos\theta)$ is**

1) $x\cos\theta/2+y\sin\theta/2=a\theta\cos\theta/2+2a\sin\theta/2$

2) $x\cos\theta/2-y\sin\theta/2=a\theta\sin\theta/2$

3) $x\sin\theta/2-y\cos\theta/2=2a\sin\theta/2$

4) None

42. **Equation of the normal at x = 0 to the curve** $y=(1+x)^y+\sin^{-1}(\sin^2 x)$ **is**

1) x – y = 1 2) x + y = 1 3) x +y = 0 4) x + y = 2

43 **If the normal line to the curve** $x^3 = y^2$ **at** $(m^2, -m^3)$ **is** $y = 3mx - 4m^3$ **then** m^2

1) 3 /4 2) 9 /2 3) 2 /9 4) 2 /3

44. **The y-intercept of the tangent at any point of the curve** $ax^{-2} + by^{-2} = 1$ **is proportional to**

1) Cube of the abscissa 2) Cube of the ordinate

3) Square of the ordinate 4) None

45. **Length of the tangent to the curve** $x = a(\cos t + \log\tan t/2)$, **y = a sin t at any point 't' on it.**

1) Varies as abscissa

2) Varies as ordinate

3) Varies as length of normal

4) Constant

46. **At the point (a, a) on** $y^2 = \frac{x^3}{2a - x}$ **length of the sub tangent =**

1) Length of the subnormal

2) Square of the length of S.N

3) Twice the subnormal

4) None

47. **If the relation between sub-normal 'SN' and sub-tangent 'ST' at any point on the curve** $by^2 = (x+a)^3$ **is p(SN) = q(ST)^2 then p/q =**

a) 8/27a b) 8/27 c) 8/27b d) 8b/27

48. **The condition that the two curves** $x = y^2$, $xy = k$ **cut orthogonally is**

a) $2k^2 = 1$ b) $8k^2 = 1$ c) $8k^3 = 1$ d) $2k^3 = 1$

49. **The condition that the two curves** $y^2 = 4ax$, $xy = c^2$ **cut orthogonally is**

a) $c^2 = 16a^2$ b) $c^2 = 32a^2$ c) $c^4 = 16a^4$ d) $c^4 = 32a^4$

50. **If the tangent at 'P' on the curve** $x^m y^n = a^{m+n}$ **meets the axes at A and B then AP : PB =**

a) m : n b) n : m c) 1 : 1 d) 1 : 2

51. **If the tangent at the point (at^2, at^3) on the curve $ay^2 = x^3$ meets the curve again at**

a) $\left(\frac{at^2}{4}, -\frac{at^3}{8}\right)$ b) $\left(\frac{at^2}{4}, 8at\right)$ c) $\left(\frac{at^2}{2}, 2at^2\right)$ d) $(at^2, 2at)$

52. **If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts α and β on the coordinate axes, where $\alpha^2 + \beta^2 = 61$ then the value of |a| is**

a) 16 b) 28 c) 30 d) 31

TANGENTS AND NORMALS

HINTS AND SOLUTIONS

1. (3)

$$\left.\frac{dy}{dx}\right]_{(1,1)} = 2 = \tan\psi$$

2. (1)

$\left.\frac{dy}{dx}\right]_{(1,3)} = 4 - 2(1) = 2 = \tan\psi$ where ψ is the angle made by the tangent with $\overline{OX}$.

3. (1)

$$\left.\frac{dy}{dx}\right]_{x=2} = -\left.\frac{-8\times 2x}{(4+x^2)^2}\right]_{x=2} = -\frac{1}{2}$$

4. (4)

Differentiating and putting x = a, y = b;

$$n\left(\frac{a}{b}\right)^{n-1}\frac{1}{a} + n\left(\frac{b}{b}\right)^{n-1}\left(\frac{1}{b}\right)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{b}{a}$$

$\Rightarrow$ Slope of normal = $\frac{a}{b}$

5. (4)

Differentiating and putting x = 1, y = –2;

$$(1)2(-2)\frac{dy}{dx}+(-2)^2+\frac{dy}{dx}(1)^2+(-2)2(1)=0$$

$$\Rightarrow \frac{dy}{dx}=0$$

6. (4)

$\frac{dy}{dt}=a(1-\cos t)$ and $\frac{dy}{dt}=a(\sin t)\Rightarrow\frac{dy}{dx}=\cot\frac{t}{2}$.

7. (3)

Equation of the tangent line is

$$y-4=\left[\frac{(3)6-12(4)}{3^2}\right](x-2)$$

i.e., 10x + 3y = 32.

8. (2)

Equation of the normal line is

$$x-1=-\left[\frac{-5}{(1+2)^2}\right](y-\frac{5}{3})$$

i.e., 27x – 15y = 2.

9. (1)

x = y and $y=\sqrt{4-x^2}\Rightarrow x=\pm\sqrt{2}$

$$\Rightarrow \text{point}=(\sqrt{2},\sqrt{2})$$

Slope of tangent = $\left.\frac{dy}{dx}\right](\sqrt{2},\sqrt{2})=-1$.

10. (3)

$$\frac{dy}{dx}=\frac{-1}{a}b\cdot e^{-x/a}$$

$\therefore$ Length of S.N. at (0, b) = $\left|b\cdot\frac{-b}{a}\right|=\frac{b^2}{|a|}$

11. (3)

$$\frac{dy}{dx} = 2x(x-2)^2 + x^2 \cdot 2(x-2)$$
$$= 2x(x-2)(2x-2).$$

Slope = 0 $\Rightarrow$ x = 0 or 1 or 2.

12. (2)

Taking logarithms and differentiating;

$$\frac{2}{x} + \frac{2}{y}y' = 0 \Rightarrow \frac{y'}{y} = -\frac{1}{x}$$
$$\Rightarrow \frac{y}{y'} = -x = a$$

13. (1)

$$y^n = a^{n-1}x \Rightarrow \frac{dy}{dx} = \frac{y}{nx}.$$

$\therefore$ Length of S.T. = $\left|y \times \frac{nx}{y}\right| = |n||x| \propto |x|$

14. (3)

Length of S.T. = $\left|y \div \frac{b}{a}y\right| = \left|\frac{a}{b}\right|$ = constant

$$\left(\frac{dy}{dx} = b \cdot \frac{1}{a}e^{x/a} = \frac{b}{a}y\right)$$

15. (4)

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{-(3x^2 - y^2 - 8x - y + 5)}{(-2xy - x + 3)}$$

$\Rightarrow$ m = 0. (Putting x = 1, y = –1)

16. (1)

$$x^{2/3} + y^{2/3} = a^{2/3}$$
$$\Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$$
$$\Rightarrow \left.\frac{dy}{dx}\right]_{(a/\sqrt{8},\, a/\sqrt{8})} = -1$$

17. (1)

We have $\tan\psi = \frac{dy}{dx}$

$\therefore$ Length of the normal =

$$\left|y\sqrt{1+\tan^2\psi}\right| = |y\sec\psi|$$

18. (2)

$$\frac{\text{Length of tangent}}{\text{Length of normal}} = \left|\frac{1}{(dy/dx)}\right| = \frac{1}{|m|}$$

19. (3)

$$\frac{\text{Length of S.N.}}{\text{Length of S.T.}} = \left(\frac{dy}{dx}\right)^2 = m^2$$

20. (2)

Slope of tangent = $f'(x)$

$\Rightarrow$slope of normal = $\frac{-1}{f'(x)} = \tan\frac{3\pi}{4}$

$\Rightarrow \frac{-1}{f'(x) = -1} \Rightarrow f'(x) = 1$

21. (2)

Length of S.T., |ordinate of the point|, length of S.N.

$= |y/(dy/dx)|, |y|, \left|y\frac{dy}{dx}\right|$ are in G.P. with C.R. = $\left|\frac{dy}{dx}\right| = m$.

22. (2)

$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$

$\Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow y/\frac{dy}{dx} = \frac{y^2}{2a}$

$S.T.: x = \frac{y^2}{2a} : x = \frac{4ax}{2a} : x = 2:1$

23. (3)

$\frac{dy}{dx} = \frac{x}{y}$ and length of normal =

$$\left|y\sqrt{1+\frac{x^2}{y^2}}\right| = \sqrt{x^2+y^2} = OP$$

24. (3)

Length of S.T. = $\left|y \div \frac{1}{b} y\right| = |b|$

25. (3)

Taking logarithms and differentiating:

$$\frac{dy}{dx} = \frac{-ny}{mx}$$

Length of S.T. = $\left|y_1 \times \frac{-mx_1}{ny_1}\right| = \frac{m}{n}|x_1|$

26. (2)

(1, 1) $\in$ curve $\Rightarrow$ a + b = –1

Differentiating and putting x = 1, y = 1

$$(1)\frac{dy}{dx} + 1 + a + b\frac{dy}{dx} = 0$$

$$\Rightarrow 2 = \frac{dy}{dx} = -\frac{(a+1)}{b+1} \Rightarrow a + 2b = -3$$

27. (1)

$$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 12 = 0 \Rightarrow x = \pm 2.$$

28. (3)

$$\frac{dy}{dx} = 0 \Rightarrow \cos x = 0$$

$$\Rightarrow x = (2n+1)\frac{\pi}{2} \text{ where } n \in Z.$$

29. (2)

x = 0, $y = e^{2x} + x^2 \Rightarrow y = 1.$

$$\frac{dy}{dx} = 2e^{2x} + 2x.$$

Equation of normal is y – 1 = 2x.

30. (4)

$\left|y/\frac{dy}{dx}\right| = \left|y\frac{dy}{dx}\right| \Rightarrow \frac{dy}{dx} = 1$ at any point (x, y)

Length of normal = $|y_1|\sqrt{1+m^2} = |y_1|\sqrt{2}$

31. (1)

A point on the curve xy = 1 is (t, 1/t) ($t \neq 0$)

$xy = 1 \Rightarrow y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{-1}{x^2}$

Slope of the normal at t = $t^2 = -\frac{a}{b} > 0$

$\Rightarrow$a, b of opposite sign.

32. (1)

By substitution we see that the point (1, 3) lies on both the curves.

$y = x^3 + x + 1$

$\Rightarrow \frac{dy}{dx} = 3x^2 + 1 \Rightarrow \left.\frac{dy}{dx}\right]_{(1,3)} = 4$

$2y = x^3 + 5x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(3x^2 + 5) \Rightarrow \left.\frac{dy}{dx}\right]_{(1,3)} = 4$

33. (3)

Putting x = 0, y = 4a,

$m_1 = [3ax^2 - 3a]_{r=0} = -3a$, $m_2 = 12$

$\therefore m_1 = m_2 \Rightarrow -3a = 12$

34. (3)

$y^2 = 2x$ parabola and $(x-2)^2 + (y-0)^2 = 2^2$

Circle touch each other and touch y-axis at the origin.

35. (1)

$\left.\frac{dy}{dx}\right]_{(1,1)} = -1$ and area of the triangle

$$= \frac{y^2(1+y_1^2)}{2|y_1|} = \frac{1^2(1+1)^2}{2(1)} = 1 \text{ sq.unit}$$

36. (2)

Equation of the tangent is $xy_1 + x_1y = 2a^2$

$(x_1, y_1) \in$ curve $\Rightarrow x_1y_1 = a^2$.

Area of the triangle

$$= \frac{1}{2}|\text{x-intercept} \times \text{y-intercept}|$$

$$= \frac{1}{2}\left|\frac{2a^2}{y_1} \times \frac{2a^2}{x_1}\right|$$

$$= 2a^2 = 2|x_1y_1|$$

37. (4)

Let $P(x_1, y_1)$ be a point of intersection.

$$x^3 - 3xy^2 = -1$$

$$\Rightarrow 3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

$$3x^2y - y^3 = 2$$

$$\Rightarrow 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} + 6xy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}$$

$\therefore$ At every point of intersection product of the slopes = –1.

38. (3)

Equation of tangent is $\frac{x}{a\cos t} + \frac{y}{a\sin t} = 1$.

39. (1)

$$\frac{dy}{dx} = \frac{a(\sin\theta)}{a(1+\cos\theta)} = \tan\frac{\theta}{2}$$

$\therefore$ Equation of the normal line:

$$x - a(\theta + \sin\theta) = -\tan\frac{\theta}{2}[y - a(1 - \cos\theta)]$$

i.e., $x\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2} = a\left[\theta\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2}\right]$

40. (2)

$x = 0 \Rightarrow y = 1 \Rightarrow$ the point is (0, 1)

$$\frac{dy}{dx} = (1+x)^y\left[\frac{y}{1+x} + \log(1+x)\frac{dy}{dx}\right] + \frac{2\sin x\cos x}{\sqrt{1-\sin^4 x}}$$

$$\frac{dy}{dx}_{(0,1)} = 1\left[\frac{1}{1+0} + 0\right] + 0 = 1$$

Equation of the normal is $y - 1 = -1(x - 0)$.

41. (3)

$y^2 = x^3$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y}$ and $\left.\frac{dy}{dx}\right]_{(m^2,-m^3)} = -\frac{3m}{2}$

$\therefore \frac{2}{3m}$ = slope of normal = 3m

$\Rightarrow m^2 = \frac{2}{9}$

42. (2)

$$\frac{a}{x^2} + \frac{b}{y^2} = 1 \Rightarrow \frac{dy}{dx} = \frac{-ay^3}{bx^3}$$

$(x_1, y_1) \in$ curve $\Rightarrow \frac{a}{x_1^2} + \frac{b}{y_1^2} = 1$

Equation of the tangent at a point (x_1, y_1) is

$$y - y_1 = \frac{-ay_1^3}{bx_1^3}(x - x_1)$$

i.e., $\frac{x}{bx_1^3} + \frac{y}{ay_1^3} = \frac{1}{bx_1^2} + \frac{1}{ay_1^2}$

Y-intercept = $\frac{ay_1^3}{ab}\left(\frac{a}{x_1^2} + \frac{b}{y_1^2}\right) = \frac{1}{b}y_1^3$.

43. (4)

$$\frac{dy}{dx} = \frac{a\cos t}{a\left(-\sin t + \left(\frac{1}{2}\right)\frac{1}{\tan(t/2)}\sec^2\left(\frac{t}{2}\right)\right)}$$

$$= \frac{\cos t}{(-\sin t + (1/\sin t))} = \tan t$$

Length of tangent = $\left|\frac{a\sin t}{\tan t}\right|\sqrt{1+\tan^2 t} = |a|$

44. (4)

By log differentiation : $\frac{2}{y}y' = \frac{3}{x} + \frac{1}{2a-x}$

At the point (a, a): $\frac{y'}{y} = \frac{2}{a}$ and $\text{S.T.} = \frac{|a|}{2}$

45. (2)

$$x^{m+n} = a^{m-n}y^{2n}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(m+n)y}{2nx}$$

$$(\text{S.T}) = \left[y \times \frac{2nx}{(m+n)y}\right] = \left(\frac{2n}{m+n}\right)\cdot x,$$

$$(\text{S.N.}) = y \times \frac{(m+n)y}{2nx} = \left(\frac{m+n}{2n}\right)\frac{y^2}{x}$$

46. (2)

$$y - e^{xy} + x = 0 \Rightarrow \frac{dx}{dy} = \frac{-f_y}{f_x} = \frac{-(1-xe^{xy})}{-ye^{xy}+1}$$

By substitution we find $\frac{dx}{dy} = 0$ for the point (1, 0).

$\Rightarrow$tangent is vertical.

47. (4)

48. (1)

$$\frac{\partial f}{\partial x}\cdot\frac{\partial g}{\partial x}+\frac{\partial f}{\partial y}\cdot\frac{\partial g}{\partial y}=0$$

$$\Rightarrow y-2xy=0\Rightarrow x=1/2$$

$$xy=k, x=\frac{1}{2}\Rightarrow y=2k \text{ and}$$

$$x=y^2\Rightarrow 4k^2=\frac{1}{2}$$

49. (4)

50. (2)

51. (1)

52. (3)