SYSTEM OF CIRCLES

OBJECTIVES

1. **A circle passes through (0, 0) and (1, 0) and touches the circle $x^2+y^2=9$, then the centre of circle is**

   (a) $\left(\frac{3}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},\frac{3}{2}\right)$

   (c) $\left(\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\pm\sqrt{2}\right)$

2. **The equation of the circle having its centre on the line $x+2y-3=0$ and passing through the points of intersection of the circles $x^2+y^2-2x-4y+1=0$ and $x^2+y^2-4x-2y+4=0$, is**

   (a) $x^2+y^2-6x+7=0$ (b) $x^2+y^2-3y+4=0$

   (c) $x^2+y^2-2x-2y+1=0$ (d) $x^2+y^2+2x-4y+4=0$

3. **The point of contact of the given circles $x^2+y^2-6x-6y+10=0$ and $x^2+y^2=2$, is**

   (a) (0, 0) (b) (1, 1)

   (c) (1, –1) (d) (–1, –1)

4. **Circles $x^2+y^2-2x-4y=0$ and $x^2+y^2-8y-4=0$**

   (a) Touch each other internally (b) Touch each other externally

   (c) Cuts each other at two points (d) None of these

5. **If the circles $x^2+y^2=a^2$ and $x^2+y^2-2gx+g^2-b^2=0$ touch each other externally, then**

   (a) $g=ab$ (b) $g^2=a^2+b^2$

   (c) $g^2=ab$ (d) $g=a+b$

6. **The radical centre of the circles $x^2+y^2+4x+6y=19, x^2+y^2=9$ and $x^2+y^2-2x-2y=5$ will be**

   (a) (1, 1) (b) (–1, 1) (c) (1, –1) (d) (0, 1)

7. **The equation of a circle passing through points of intersection of the circles $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ and point (1, 1) is**

   (a) $4x^2+4y^2-30x-10y-25=0$ (b) $4x^2+4y^2+30x-13y-25=0$

   (c) $4x^2+4y^2-17x-10y+25=0$ (d) None of these

8. **The equation of the circle which intersects circles** $x^2+y^2+x+2y+3=0$ **,** $x^2+y^2+2x+4y+5=0$ **and** $x^2+y^2-7x-8y-9=0$ **at right angle, will be**

(a) $x^2+y^2-4x-4y-3=0$ (b) $3(x^2+y^2)+4x-4y-3=0$

(c) $x^2+y^2+4x+4y-3=0$ (d) $3(x^2+y^2)+4(x+y)-3=0$

9. **Two given circles** $x^2+y^2+ax+by+c=0$ **and** $x^2+y^2+dx+ey+f=0$ **will intersect each other orthogonally, only when**

(a) $a+b+c=d+e+f$ (b) $ad+be=c+f$

(c) $ad+be=2c+2f$ (d) $2ad+2be=c+f$

10. **The locus of centre of a circle passing through (a, b) and cuts orthogonally to circle** $x^2+y^2=p^2$ **, is**

(a) $2ax+2by-(a^2+b^2+p^2)=0$ (b) $2ax+2by-(a^2-b^2+p^2)=0$

(c) $x^2+y^2-3ax-4by+(a^2+b^2-p^2)=0$ (d) $x^2+y^2-2ax-3by+(a^2-b^2-p^2)=0$

11. **The equation of the circle through the point of intersection of the circles** $x^2+y^2-8x-2y+7=0$ **,** $x^2+y^2-4x+10y+8=0$ **and (3, –3) is**

(a) $23x^2+23y^2-156x+38y+168=0$ (b) $23x^2+23y^2+156x+38y+168=0$

(c) $x^2+y^2+156x+38y+168=0$ (d) None of these

12. **The equation of line passing through the points of intersection of the circles** $3x^2+3y^2-2x+12y-9=0$ **and** $x^2+y^2+6x+2y-15=0$ **, is**

(a) $10x-3y-18=0$ (b) $10x+3y-18=0$

(c) $10x+3y+18=0$ (d) None of these

13. **The locus of the centres of the circles which touch externally the circles** $x^2+y^2=a^2$ **and** $x^2+y^2=4ax$ **, will be**

(a) $12x^2-4y^2-24ax+9a^2=0$ (b) $12x^2+4y^2-24ax+9a^2=0$

(c) $12x^2-4y^2+24ax+9a^2=0$ (d) $12x^2+4y^2+24ax+9a^2=0$

14. **If the circles of same radius a and centers at (2, 3) and (5, 6) cut orthogonally, then a =**

(a) 1 (b) 2

(c) 3 (d) 4

15. **Two circles** $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ **and** $S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ **cut each other orthogonally, then**

(a) $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ (b) $2g_1g_2 - 2f_1f_2 = c_1 + c_2$

(c) $2g_1g_2 + 2f_1f_2 = c_1 - c_2$ (d) $2g_1g_2 - 2f_1f_2 = c_1 - c_2$

16. **Circles** $x^2 + y^2 + 2gx + 2fy = 0$ **and** $x^2 + y^2 + 2g'x + 2f'y = 0$ **touch externally, if**

(a) $f'g = g'f$ (b) $fg = f'g'$

(c) $f'g' + fg = 0$ (d) $f'g + g'f = 0$

17. **Consider the circles** $x^2 + (y-1)^2 = 9, (x-1)^2 + y^2 = 25$**. They are such that**

(a) These circles touch each other

(b) One of these circles lies entirely inside the other

(c) Each of these circles lies outside the other

(d) They intersect in two points

18. **One of the limit point of the coaxial system of circles containing** $x^2 + y^2 - 6x - 6y + 4 = 0$**,** $x^2 + y^2 - 2x - 4y + 3 = 0$ **is**

(a) $(-1, 1)$ (b) $(-1, 2)$

(c) $(-2, 1)$ (d) $(-2, 2)$

19. **The equation of circle which passes through the point (1,1) and intersect the given circles** $x^2 + y^2 + 2x + 4y + 6 = 0$ **and** $x^2 + y^2 + 4x + 6y + 2 = 0$ **orthogonally, is**

(a) $x^2 + y^2 + 16x + 12y + 2 = 0$ (b) $x^2 + y^2 - 16x - 12y - 2 = 0$

(c) $x^2 + y^2 - 16x + 12y + 2 = 0$ (d) None of these

20. **Locus of the point, the difference of the squares of lengths of tangents drawn from which to two given circles is constant, is**

(a) Circle (b) Parabola

(c) Straight line (d) None of these

21. **The locus of centre of the circle which cuts the circles** $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ **and** $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ **orthogonally is**

(a) An ellipse (b) The radical axis of the given circles

(c) A conic (d) Another circle

22. **P, Q and R are the centres and** r_1, r_2, r_3 **are the radii respectively of three co-axial circles, then** $QRr_1^2 + RPr_2^2 + PQr_3^2$ **is equal to**

(a) $PQ.QR.RP$ (b) $-PQ.QR.RP$ (c) $PQ^2.QR^2.RP^2$ (d) None of these

23. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45^o at the major segment of the circle then value of m is

(a) 2 (b) – 2

(c) – 1 (d) None of these

24. The radical axis of two circles and the line joining their centres are

(a) Parallel (b) Perpendicular

(c) Neither parallel, nor perpendicular (d) Intersecting, but not fully perpendicular

25. Any circle through the point of intersection of the lines $x + \sqrt{3}y = 1$ and $\sqrt{3}x - y = 2$ if intersects these lines at points P and Q, then the angle subtended by the arc PQ at its centre is

(a) 180^o (b) 90^o

(c) 120^o (d) Depends on centre and radius

26. If the straight line $y = mx$ is outside the circle $x^2 + y^2 - 20y + 90 = 0$, then

(a) $m > 3$ (b) $m < 3$

(c) $|m| > 3$ (d) $|m| < 3$

27. The points of intersection of circles $x^2 + y^2 = 2ax$ and $x^2 + y^2 = 2by$ are

(a) (0, 0), (a, b) (b)(0, 0), $\left(\frac{2ab^2}{a^2 + b^2}, \frac{2ba^2}{a^2 + b^2}\right)$

(c) (0, 0), $\left(\frac{a^2 + b^2}{a^2}, \frac{a^2 + b^2}{b^2}\right)$ (d)None of these

28. If circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other, then

(a) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

(c) $\frac{1}{a} + \frac{1}{b} = c^2$ (d) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$

29. If two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(a) $2 < r < 8$ (b) $r = 2$

(c) $r < 2$ (d) $r > 2$

30. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$, if

(a) $2g'(g - g') + 2f'(f - f') = c - c'$ (b) $g'(g - g') + f'(f - f') = c - c'$

(c) $f(g - g') + g(f - f') = c - c'$ (d) None of these

31. The locus of the centre of a circle which cuts orthogonally the circle $x^2+y^2-20x+4=0$ and which touches $x=2$ is

(a) $y^2=16x+4$ (b) $x^2=16y$

(c) $x^2=16y+4$ (d) $y^2=16x$

32. The equation of a circle that intersects the circle $x^2+y^2+14x+6y+2=0$ orthogonally and whose centre is (0, 2) is

(a) $x^2+y^2-4y-6=0$ (b) $x^2+y^2+4y-14=0$

(c) $x^2+y^2+4y+14=0$ (d) $x^2+y^2-4y-14=0$

33. The equation of the circle which passes through the intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ and whose centre lies on $13x+30y=0$ is

(a) $x^2+y^2+30x-13y-25=0$ (b) $4x^2+4y^2+30x-13y-25=0$

(c) $2x^2+2y^2+30x-13y-25=0$ (d) $x^2+y^2+30x-13y+25=0$

34. If the circle $x^2+y^2+6x-2y+k=0$ bisects the circumference of the circle $x^2+y^2+2x-6y-15=0$, then k =

(a) 21 (b) – 21

(c) 23 (d) – 23

35. If P is a point such that the ratio of the squares of the lengths of the tangents from P to the circles

$x^2+y^2+2x-4y-20=0$ and $x^2+y^2-4x+2y-44=0$ is 2 : 3, then the locus of P is a circle with centre

(a) (7, – 8) (b) (– 7, 8)

(c) (7, 8) (d) (– 7, – 8)

36. If d is the distance between the centres of two circles, r_1, r_2 are their radii and $d=r_1+r_2$, then

(a) The circles touch each other externally

(b) The circles touch each other internally

(c) The circles cut each other

(d) The circles are disjoint

37. If the circles $x^2+y^2=4, x^2+y^2-10x+\lambda=0$ touch externally, then λ is equal to

(a) –16 (b) 9

(c) 16 (d) 25

38. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a=0$ passes through P and Q for

(a) Infinitely many values of a

(b) Exactly two values of a

(c) Exactly one value of a

(d) No value of a

39. The radical axis of the pair of circle $x^2+y^2=144$ and $x^2+y^2-15x+12y=0$ is

(a) $15x-12y=0$ (b) $3x-2y=12$

(c) $5x-4y=48$ (d) None of these

40. The equation of radical axis of the circles $x^2+y^2+x-y+2=0$ and $3x^2+3y^2-4x-12=0$, is

(a) $2x^2+2y^2-5x+y-14=0$ (b) $7x-3y+18=0$

(c) $5x-y+14=0$ (d) None of these

41. If the circles $x^2+y^2-2ax+c=0$ and $x^2+y^2+2by+2\lambda=0$ intersect orthogonally, then the value of λ is

(a) c (b) – c

(c) 0 (d) None of these

42. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

(a) A hyperbola (b) A parabola

(c) An ellipse (d) A circle

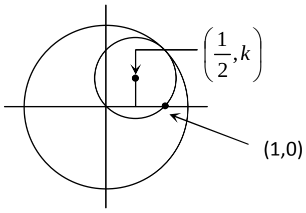
43. The number of common tangents to the circles $x^2+y^2=4$ and $x^2+y^2-6x-8y=24$ is

(a) 0 (b) 1 (c) 3 (d) 4

SYSTEM OF CIRCLES

HINTS AND SOLUTIONS

1. (d) Radius of the circle $r = \frac{3}{2}$



$\frac{1}{4} + k^2 = \frac{9}{4} \Rightarrow k = \pm\sqrt{2}$

Hence centre is $\left(\frac{1}{2}, \pm\sqrt{2}\right)$

2. (a) Required circle will be $S_1 + \lambda S_2 = 0$, $\lambda \neq -1$

i.e., $x^2 + y^2 - 2x - 4y + 1 + \lambda(x^2 + y^2 - 4x - 2y + 4) = 0$

$\Rightarrow x^2 + y^2 - 2\frac{(1+2\lambda)}{1+\lambda}x - 2\frac{(2+\lambda)}{1+\lambda}y + \frac{1+4\lambda}{1+\lambda} = 0$

Its centre $\left(\frac{1+2\lambda}{1+\lambda}, \frac{2+\lambda}{1+\lambda}\right)$ lies on $x + 2y - 3 = 0$

$\therefore \quad \lambda = -2$.

$\therefore$ The circle is $x^2 + y^2 - 6x + 7 = 0$

3. (b) $x^2 + y^2 - 6x - 6y + 10 = 0$ ....(i)

$x^2 + y^2 = 2$ ....(ii)

By verification ans.(1,1)

4. (a) $C_1(1, 2), C_2(0, 4), R_1 = \sqrt{5}, R_2 = 2\sqrt{5}$

$C_1C_2 = \sqrt{5}$ and $C_1C_2 = |R_2 - R_1|$

Hence circles touch internally.

5. (d) According to the condition,

$\sqrt{(g-0)^2+(0-0)^2}=a+b \Rightarrow g=a+b$.

6. (a) Radical axes are

$4x+6y=10$ or $2x+3y=5$ ....(i)

$2x+2y=4$ or $x+y=2$ ....(ii)

Point of intersection of (i) and (ii) is (1, 1).

7. (b) Required equation is

$(x^2+y^2+13x-3y)+\lambda(2x^2+2y^2+4x-7y-25)=0$

Which passes through (1, 1), so $\lambda=\frac{1}{2}$.

Hence required equation is

$4x^2+4y^2+30x-13y-25=0$.

8. (d) Verification.

9. (c) From Condition for orthogonal intersection, $\frac{ad}{2}+\frac{be}{2}=c+f$

10. (a) Let equation of circle be $x^2+y^2+2gx+2fy+c=0$ with $x^2+y^2=p^2$ cutting orthogonally, we get

$0+0=+c-p^2$ or $c=p^2$ and passes through (a, b), we get

$a^2+b^2+2ga+2fb+p^2=0$ Or

$2ax+2by-(a^2+b^2+p^2)=0$

Required locus as centre $(-g,-f)$ is changed to (x, y).

11. (a) Equation of circle is

$(x^2+y^2-8x-2y+7)+\lambda(x^2+y^2-4x+10y+8)=0$

Also point $(3,-3)$ lies on the above equation.

$\Rightarrow \lambda=\frac{7}{16}$

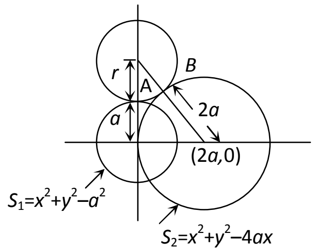
Hence required equation is

$23x^2+23y^2-156x+38y+168=0$.

12. (a) Common chord $=S_1-S_2$

$10x-3y-18=0$.

13. (a) Let $C \equiv (h, k)$, radius $= r$



Co-ordinates of $A \equiv \left[\frac{ah}{a+r}, \frac{ak}{a+r}\right]$

Co-ordinates of $B \equiv \left[\frac{2ar+2ah}{2a+r}, \frac{2ak}{2a+r}\right]$

Putting co-ordinates of A and B in S_1, S_2 respectively and eliminating r, we get the locus

$12x^2 - 4y^2 - 24ax + 9a^2 = 0$.

14. (c) $(C_1C_2)^2 = r_1^2 + r_2^2 \Rightarrow 2a^2 = 18 \Rightarrow a = 3$.

15. (a) concept.

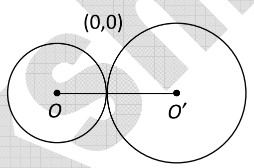
16. (a) $OA + O'A = OO'$

$\sqrt{g^2 + f^2} + \sqrt{f'^2 + g'^2} = \sqrt{(g'-g)^2 + (f'-f)^2}$

$\Rightarrow g^2 + f^2 + f'^2 + g'^2 + 2\sqrt{g^2 + f^2} \times \sqrt{f'^2 + g'^2}$

$= (g'-g)^2 + (f'-f)^2$

$\Rightarrow 2\sqrt{g^2 + f^2}\sqrt{f'^2 + g'^2} = -2(gg' + ff')$



$\Rightarrow g^2 f'^2 + f^2 g'^2 = 2gg'ff'$. $\therefore (gf' - fg')^2 = 0 \Rightarrow gf' = fg'$.

17. (b) Centres and radii of the given circles are

Centres: $C_1(0, 1), C_2(1, 0)$; Radii : $r_1 = 3, r_2 = 5$.

Clearly, $C_1C_2 = \sqrt{2} < (r_2 - r_1)$. Therefore one circle lies entirely inside the other.

18. (a) The equation of radical axis is $S_1 - S_2 = 0$ i.e., $4x + 2y - 1 = 0$.

$\therefore$ The equation of circle of co-axial system can be taken as $(x^2 + y^2 - 6x - 6y + 4) + \lambda(4x + 2y - 1) = 0$

Or $x^2 + y^2 - (6 - 4\lambda)x - (6 - 2\lambda)y + (4 - \lambda) = 0$ ….(i)

Whose centre is $C(3-2\lambda, 3-\lambda)$ and radius is

$r=\sqrt{(3-2\lambda)^2+(3-\lambda)^2-(4-\lambda)}$

If $r=0$, then we get $\lambda=2$ or $7/5$.

Putting the co-ordinates of C, the limit points are (–1, 1) and $\left(\frac{1}{5},\frac{8}{5}\right)$. One of these limit points is given in (a).

19. (c) Let equation of circle be $x^2+y^2+2gx+2fy+c=0$.

As it intersects orthogonally the given circles, we have $2g+4f=6+c$ and $4g+6f=2+c$.

As it passes through (1, 1), we have $2g+2f=-2-c$

From these, we get g, f and c as –8, 6, 2 respectively and hence equation of circle as

$x^2+y^2-16x+12y+2=0$.

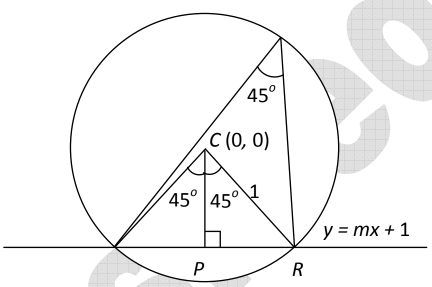
20. (c) $t_1^2-t_2^2=k\Rightarrow S_1-S_2=k\Rightarrow 1^{st}$ degree equation which represents a straight line parallel to radical axis $S_1-S_2=0$.

21. (b) Radical axis of the given circles.

22. (b) As $\Sigma g_1^2(g_2-g_3)=-(g_1-g_2)(g_2-g_3)(g_3-g_1)$.

23. (c) Given circle is $x^2+y^2=1$

$C(0,0)$ and radius = 1 and chord is $y=mx+1$ $\cos 45^o=\frac{CP}{CR}$



$CP=$ Perpendicular distance from (0,0) to chord $y=mx+1$

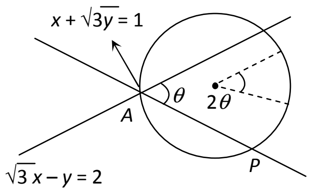
$CP=\frac{1}{\sqrt{m^2+1}}$ (CR = radius = 1)

$\cos 45^\circ=\frac{1/\sqrt{m^2+1}}{1}\Rightarrow\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{m^2+1}}$

$m^2+1=2\implies m=\pm 1$.

24. (b) We know that the radical axis of two circles is the locus of a point which moves in such a way that the lengths of the tangents drawn from it to the two circles are equal. It is a line perpendicular to the line joining the centres of two circles.

25. (a) Let the point of intersection of two lines is A.



$\therefore$ The angle subtended by PQ on centre C

= Two times the angle subtended by PQ on point A.

For $x+\sqrt{3}y=1$, $m_1=\frac{-1}{\sqrt{3}}$ and For $\sqrt{3}x-y=2$, $m_2=\sqrt{3}$

$\because m_1\times m_2=\frac{-1}{\sqrt{3}}\times\sqrt{3}=-1$, $\therefore \angle A=90^o$

$\therefore$ The angle subtended by arc PQ at its centre $=2\times 90^o=180^o$

26. (d) If the straight line y = mx is outside the given circle then distance from centre of circle > radius of circle $\frac{10}{\sqrt{1+m^2}}>\sqrt{10}$

$\Rightarrow (1+m^2)<10 \Rightarrow m^2<9 \Rightarrow |m|<3.$

27. (b) Given circles are $x^2+y^2=2ax$ …..(i)

and $x^2+y^2=2by$ …..(ii)

(i) – (ii) $\Rightarrow$ $0=2(ax-by) \Rightarrow y=\frac{a}{b}x$

From (i), $x^2+\frac{a^2}{b^2}x^2=2ax \Rightarrow x\left\{\left(1+\frac{a^2}{b^2}\right)x-2a\right\}=0$

$\Rightarrow x=0, \frac{2ab^2}{a^2+b^2}$

For $x=0$, $y=0$ and for $x=\frac{2ab^2}{a^2+b^2}$, $y=\frac{2a^2b}{a^2+b^2}$

$\therefore$ The points of intersection are (0, 0) and $\left(\frac{2ab^2}{a^2+b^2},\frac{2a^2b}{a^2+b^2}\right)$.

28. (d) $C_1(-a,0)$; $C_2(0,-b)$; $R_1(\sqrt{a^2-c})$;

$R_2(\sqrt{b^2-c})$

$C_1C_2=\sqrt{a^2+b^2}$

Since they touch each other, therefore

$\sqrt{a^2 - c} + \sqrt{b^2 - c} = \sqrt{a^2 + b^2}$

$\Rightarrow a^2b^2 - b^2c - a^2c = 0$

Multiply by $\frac{1}{a^2b^2c}$, we get $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$.

29. (a) When two circles intersect each other, then

Difference between their radii<Distance between centers $\Rightarrow r - 3 < 5 \Rightarrow r = 8$

.....(i)

Sum of their radii > Distance between centres .....(ii)

$\Rightarrow r + 3 > 5 \Rightarrow r > 2$

Hence by (i) and (ii), $2 < r < 8$.

30. (a) Common chord $S_1 - S_2 = 0$ passes through the centre of $S_2 = 0$.

31. (d) Let the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ .....(i)

It cuts the circle $x^2 + y^2 - 20x + 4 = 0$ orthogonally

$\therefore 2(-10g + 0 \times f) = c + 4 \Rightarrow -20g = c + 4$ .....(ii)

Circle (i) touches the line $x = 2$, $\therefore x + 0y - 2 = 0$

$\therefore \left|\frac{-g + 0 - 2}{\sqrt{1^2 + 0^2}}\right| = \sqrt{g^2 + f^2 - c}$

$\Rightarrow (g + 2)^2 = g^2 + f^2 - c \Rightarrow 4g + 4 = f^2 - c$ .....(iii)

Eliminating c from (ii) and (iii), we get

$-16g - 4 = f^2 - 4 \Rightarrow f^2 + 16g = 0$

Hence the locus of $(-g, -f)$ is $y^2 - 16x = 0$.

32. (d) In circle, $x^2 + y^2 + 14x + 6y + 2 = 0$

$g = 7, f = 3, c = 2$

Centre of circle $(-g, -f) = (0, 2)$, (Given)

For orthogonally intersection, $2gg' + 2ff' = c + c'$

$0 - 12 = 2 + c' \Rightarrow c' = -14$

Put the values, in equation $x^2 + y^2 + 2g'x + 2f'x + c' = 0$.

$\Rightarrow x^2 + y^2 + 0 - 4y - 14 = 0 \Rightarrow x^2 + y^2 - 4y - 14 = 0$.

33. (b) The equation of required circle is $S_1 + \lambda S_2 = 0$.

$\Rightarrow x^2(1+\lambda) + y^2(1+\lambda) + x(2+13\lambda) - y\left(\frac{7}{2} + 3\lambda\right) - \frac{25}{2} = 0$

Centre = $\left(\frac{-(2+13\lambda)}{2}, \frac{\frac{7}{2}+3\lambda}{2}\right)$

$\because$ Centre lies on $13x + 30y = 0$

$\Rightarrow -13\left(\frac{2+13\lambda}{2}\right) + 30\left(\frac{\frac{7}{2}+3\lambda}{2}\right) = 0 \Rightarrow \lambda = 1$.

Hence the equation of required circle is $4x^2 + 4y^2 + 30x - 13y - 25 = 0$.

34. (d) $2g_2(g_1 - g_2) + 2f_2(f_1 - f_2) = c_1 - c_2$

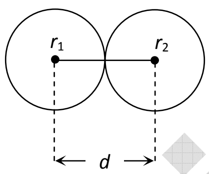
$2(1)\ (3-1) + 2(-3)\ (-1+3) = k + 15$

$4 - 12 = k + 15$ or $-8 = k + 15 \Rightarrow k = -23$.

35. (b) $\frac{x^2 + y^2 + 2x - 4y - 20}{x^2 + y^2 - 4x + 2y - 44} = \frac{2}{3}$

$\Rightarrow x^2 + y^2 + 14x - 16y + 28 = 0$, $\therefore$ Centre $= (-7, 8)$.

36. (a)



Clearly, circles touch each other externally.

37. (a) Circles $x^2 + y^2 = 4$, $x^2 + y^2 - 10x + \lambda = 0$ touch externally

$\therefore C_1C_2 = r_1 + r_2$

$\Rightarrow C_1(0, 0)$ and $C_2 = (5, 0)$

$r_1 = 2$ and $r_2 = \sqrt{25 + \lambda}$

$\therefore \sqrt{(5-0)^2 + 0} = 2 + \sqrt{25 + \lambda}$

$\Rightarrow 5 - 2 = \sqrt{25 + \lambda} \Rightarrow 3 = \sqrt{25 + \lambda}$

$\Rightarrow 9 = 25 + \lambda \Rightarrow \lambda = -16$

38. (d) Equation of line PQ (i.e., common chord) is $5ax + (c-d)y + a + 1 = 0$ .....(i)

Also given equation of line PQ is

$5x + by - a = 0$ .....(ii)

Therefore $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$; As $\frac{a+1}{-a} = a$

$\Rightarrow a^2 + a + 1 = 0$

Therefore no real value of a exists, (as D<0).

39. (c)The radical axis of the circle $S_1 = 0$ and $S_2 = 0$ is $S_1 - S_2 = 0$

$\therefore (x^2 + y^2 - 144) - (x^2 + y^2 - 15x + 12y) = 0$

$\Rightarrow 15x - 12y - 144 = 0 \Rightarrow 5x - 4y = 48$.

40. (b) Radical axis is $S_1 - S_2$

$S_1 \equiv x^2 + y^2 + x - y + 2 = 0$

$S_2 = x^2 + y^2 - \frac{4}{3}x - 4 = 0$

$\Rightarrow S_1 - S_2 = \frac{7}{3}x - y + 6 = 0$

Or $7x - 3y + 18 = 0$.

41. (d) Condition for circle intersects orthogonally,

$2(g_1 g_2 + f_1 f_2) = c_1 + c_2 \Rightarrow 0 = c + 2\lambda \Rightarrow \lambda = -\frac{c}{2}$

42. (b) Let centre $\equiv (h, k)$; As $C_1C_2 = r_1 + r_2$,(Given)

$\Rightarrow \sqrt{(h-0)^2 + (k-3)^2} = |k+2|$

$\Rightarrow h^2 = 5(2k - 1)$

Hence locus, $x^2 = 5(2y - 1)$, which is parabola.

43. (b) Circles $S_1 \equiv x^2 + y^2 = 2^2$,

$S_2 \equiv (x-3)^2 + (y-4)^2 = 7^2$

$\therefore$ Centres $C_1 = (0, 0)$, $C_2 = (3, 4)$

and radii $r_1 = 2, r_2 = 7$ $\therefore C_1C_2 = 5$, $r_2 - r_1 = 5$

i.e., circles touch internally. Hence there is only one common tangent.

SYSTEM OF CIRCLES

PRACTICE EXERCISE

1. The distance from (1, 2) to the R.A. of the circles $x^2 + y^2+6x-16=0$,

$x^2+y^2-2x+6y- 6=0$ is

1) 1 2) 2 3) 3 4) 7/5

2. The radical centre of the circles $x^2 + y^2 = 9$, $x^2 + y^2 +4x+6y-19=0$,

$x^2+y^2-2x-2y- 5=0$ is

1) (0, 0) 2) (0, 1) 3) (1, 0) 4) (1, 1)

3. The centre of the circle orthogonal to $x^2+y^2+4y+1= 0$,
$x^2+y^2+6x+y–8 = 0$, $x^2+y^2–4x-4y+37=0$ is

1) (1,1) 2) (2,2) 3) (3,3) 4) (0,0)

4. The radius of the circle which cuts orthogonally circles $x^2 + y^2 - 4x - 2y + 6 = 0$,

$x^2 + y^2 - 2x + 6y = 0$, $x^2 + y^2 - 12x + 12y + 30 = 0$ is

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) $\sqrt{6}$ 4) 2

5. The equation of the circle passing through (0, 0) and cutting orthogonally the circles $x^2+y^2+6x-15= 0$, $x^2 + y^2 - 8y - 10 = 0$ is

1) $x^2 + y^2 - 10x + 5y = 0$ 2) $2x^2 + 2y^2 - 10x + 5y = 0$

3) $x^2 + y^2 + 10x + 5y = 0$ 4) $2x^2 + 2y^2 + 10x + 5y = 0$

6. The equation of the circle passing through the origin and the points of intersection of $x^2+y^2 = 4$, $x^2 + y^2 - 2x - 4y + 4 = 0$ is

1) $x^2 + y^2 + x + 2y = 0$ 2) $x^2 + y^2 - x -2y = 0$

3) $x^2 + y^2 + x - 2y = 0$ 4) $x^2 + y^2 - x + 2y = 0$

7. The equation of the circle passing through the origin and the points of intersection of the two circles $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 4x - 2y - 4 = 0$ is

1) $x^2 + y^2 + 28x + 18y = 0$ 2) $x^2 + y^2 - 28x - 18y = 0$

3) $x^2 + y^2 - 14x - 9y = 0$ 4) $x^2 + y^2 + 14x + 9y = 0$

8. The equation of the circle passing through the points of intersection of $x^2+y^2-2x-2=0$, $x^2 + y^2 + 5x - 6y + 4 = 0$ and cutting orthogonally the circle $x^2+y^2 = 4$ is

1) $x^2 + y^2 - 5x - 6y + 4 = 0$ 2) $x^2 + y^2 + 5x + 6y + 4 = 0$

3) $x^2 + y^2 - 5x + 6y - 4 = 0$ 4) $x^2 + y^2 + 5x - 6y + 4 = 0$

9. The equation of the circle whose diameter is the common chord of the circles $x^2+y^2+2x+3y+1= 0$, $x^2 + y^2 + 4x + 3y + 2 = 0$ is

1) $x^2 + y^2 + x + 3y + 1 = 0$ 2) $x^2 + y^2 + 2x + 6y + 1 = 0$

3) $2x^2 + 2y^2 + 2x + 6y + 1 = 0$ 4) $2x^2 + 2y^2 + 2x + 6y + 3 = 0$

10. The line $2x+3y=1$ cuts the circle $x^2 + y^2 = 4$ in A and B. The equation of the circle on AB as diameter is

1) $13x^2 + 13y^2 - 4x - 6y - 50 = 0$ 2) $13x^2 + 13y^2 - 4x - 6y - 52 = 0$

3) $13x^2 + 13y^2 + 4x + 6y + 50 = 0$ 4) $13(x^2+y^2)+4x+6y+52=0$

11. 3 circles with centres A, B, C intersect orthogonally the radical centre of the three circles is

1) In centre of ΔABC 2) Orthocentre of ΔABC

3) Circumcentre of ΔABC 4) Centroid of ΔABC

12. The centres of two circles are (a, c), (b, c) and their common chord is the y - axis. If the radius of one circle is r, the radius of the other circle is

1) $2\sqrt{r^2 + b^2 - a^2}$ 2) $\sqrt{r^2 + b^2 - a^2}$ 3) $\sqrt{r^2 + b^2 + a^2}$ 4) $\sqrt{r^2 + a^2 - b^2}$

13. The equations of four circles are $(x\pm a)^2 + (y\pm a)^2 = a^2$. The radius of a circle touching all the four circles externally is

1) $2\sqrt{2}a$ 2) $(\sqrt{2} + 1)a$ 3) $(\sqrt{2} - 1)a$ 4) $(2 + \sqrt{2})a$

14. The circle passing through the points of intersection of the circles $x^2+y^2 - 3x-6y+8 = 0$, $x^2+y^2 - 2x - 4y+4=0$ and touching the line $x+2y = 5$ is

1) $x^2 + y^2 -x - 2y = 0$ 2) $x^2 + y^2 = 4$ 3) $x^2 + y^2 + 4 = 0$ 4) $x^2 + y^2 + x+2y=0$

15. The locus of the centre of the circle of radius 2 which rolls on the outside the circle $x^2 + y^2 + 3x – 6y- 9=0$ is

1) $x^2 + y^2 + 3x – 6y + 5 =0$ 2) $x^2 + y^2 + 3x – 6y – 31 = 0$

3) $x^2 + y^2 + 3x – 6y + 21 = 0$ 4) $4(x^2 + y^2 +3x-6y)+29 = 0$

16. The equation to the circle whose diameter is the common chord of the circles

$(x - a)^2 + y^2 = a^2; x^2 + (y - b)^2 = b^2$ is

1) $x^2 + y^2 - (ax+by) = 0$ 2) $x^2 + y^2 - 2ab(ax+by)=0$

3) $x^2 + y^2 + 2ab(ax+by)=0$ 4) $(a^2 + b^2)(x^2 + y^2) - 2ab(bx + ay) = 0$

17. The equation of the circle passing through the points of intersection of

$x^2 + y^2 + 12x + 8y - 33 = 0$, $x^2 + y^2 = 5$ and touching the x-axis is

1) $x^2 + y^2 + 6x + 4y + 9 = 0$ 2) $x^2 + y^2 - 6x + 4y + 9 = 0$

3) $x^2 + y^2 - 6x - 4y + 9 = 0$ 4) None

18. $x = 1$ is the common chord of two circles intersecting orthogonally. If the equation of one of circle is $x^2 + y^2 = 4$, then the equation of the other circle is

1) $x^2 + y^2 + 8x + 4 = 0$ 2) $x^2 +y^2 + 8x - 4 = 0$

3) $x^2 + y^2 - 8x + 4 = 0$ 4) $x^2 + y^2 + 2x - 6y + 6 =0$

19. There are two circles whose equations are $x^2 + y^2 = 9$ and $x^2 + y^2 – 8x – 6y + n^2 = 0$, $n\in Z$. If the two circles have exactly two common tangents, then the number of possible values of n is

1) 2 2) 8 3) 9 4) 5

20. A rod PQ of length 2a slides with its ends on the axes. The locus of the circumcentre of ΔOPQ is

1) $x^2 + y^2 = 2a^2$ 2) $x^2 + y^2 = 4a^2$ 3) $x^2 + y^2 = 3a^2$ 4) $x^2 + y^2 = a^2$

21. The number of common tangents of the circles $x^2 + y^2 + 2gx + 8 = 0$ and $x^2 + y^2 + 2\mu y - 8 = 0$ is

1) 1 2) 2 3) 3 4) 4

22. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is

1) $x^2 + y^2 + x + y = 0$ 2) $x^2 + y^2 - x + y = 0$

3) $x^2 + y^2 - x - y = 0$ 4) $x^2 + y^2 + x - y = 0$

23. $x^2 + y^2 + px + b = 0$ and $x^2 + y^2 + qx + b = 0$ are two circles. If one circle entirely lies in another circle, then

1) $pq < 0, b > 0$ 2) $pq < 0, b < 0$ 3) $pq > 0, b > 0$ 4) $pq > 0, b < 0$

SYSTEM OF CIRCLES

PRACTICE TEST KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
4	4	3	2	2	2	2	4	3	1
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2	2	3	1	2	4	3	3	3	4
21	**22**	**23**							
2	3	3							