RANDOM VARIABLES

OBJECTIVE PROBLEMS

1. A random variable X has the following probability distribution

$X = (x_i)$	1	2	3	4
$P(X=x_i)$	k	2k	3k	4k

then k =

1) 10 2) 1/10 3) 0 4) 1/7

2. A random variable X has the following probability distribution

$X = (x_i)$	-2	-1	0	1	2	3
$P(X = x_i)$	0.1	k	0.2	2k	0.3	k

then k =

1) 1 2) 0.1 3) 0.01 4) 0.001

3. A random variable X has its range = {0, 1, 2} and the probabilities are given by $P(x=0)=3k^3$, $P(X=1) = 4k-10k^2$ $P(x=2) = 5k-1$ where k is constant. Then k =

1) 3 2) 1/3 3) 1 4) 1/10

4 A die is tossed twice. Getting a number greater than 4 is considered a success. For the probability distribution of the number of successes, the mean and variance are

1) 2/3, 2/3 2) 2/3, 4/9 3) 4/9, 1/3 4) 2/3, 2/9

5. The distribution of X is given below

X:	**1**	**2**	**3**	**4**
P(X=x)	**1/10**	**2/10**	**3/10**	**4/10**

then the standard deviation of X is

1) 0.25 2) 0.5 3) 1 4) 0.75

6. . The range of a random variable x=[1, 2, 3, ...] and the probabilities are given by $P(x=k)=\frac{c^k}{k!}$ (k=1, 2, 3, ...) then c =

1) $\log_e 2$ 2) $\log_e 3$ 3) $\log_e 4$ 4) $\log_e 5$

7. The probability of getting 2 heads and 3 tails when a coin is tossed 5 times is

1) 5/32 2) 5/16 3) 5/8 4) 3/7

8. 8 coins are tossed at a time, 256 times. The expected frequency of getting one head is

1) ${}^8c_4$ 2) ${}^8c_3$ 3) ${}^8c_1$ 4) ${}^8c_2$

9. A random variable X has its range = [1, 2, 3, ...] and the probability distribution of X is given by $P(X=x_n) = \frac{k(n+1)}{2^n}$, n=(1, 2, 3, ...) then k =

1) 3 2) 1/3 3) 1 4) 1/2

10. The probability of happening of an event in an experiment is 0.4. The probability of happening of the event at least once, if the experiment is repeated 3 times under similar conditions is

1) 89/125 2) 98/125 3) 87/125 4) 91/125

11. One hundred identical coins, each with probability, P of showing up heads are tossed. If 0<P< 1 and the probability of heads showing on 50 coins is equal to that of the heads showing on 51 coins, then the value of P is

1) ${}^{11}c_5$ 2) ${}^{11}c_5(0.4)^5$ 3) $\frac{51}{101}$ 4) 1/2

12. The binomial distribution for which mean = 6 and variance = 2, is

(a) $\left(\frac{2}{3}+\frac{1}{3}\right)^6$ (b) $\left(\frac{2}{3}+\frac{1}{3}\right)^9$

(c) $\left(\frac{1}{3}+\frac{2}{3}\right)^6$ (d) $\left(\frac{1}{3}+\frac{2}{3}\right)^9$

13. If the probability that a student is not a swimmer is 1/5, then the probability that out of 5 students one is swimmer is

(a) ${}^5C_1\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$ (b) ${}^5C_1\frac{4}{5}\left(\frac{1}{5}\right)^4$ (c) $\frac{4}{5}\left(\frac{1}{5}\right)^4$ (d) None of these

14. A biased coin with probability $p, 0 < p < 1,$ of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is $\frac{2}{5}$, then $p =$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) None of these

15. If a dice is thrown 7 times, then the probability of obtaining 5 exactly 4 times is

(a) ${}^7C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^3$ (b) ${}^7C_4\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^4$

(c) $\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^3$ (d) $\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^4$

16. If *X* follows a binomial distribution with parameters $n = 6$ and *p*. If $9P(X = 4) = P(X = 2),$ then $p =$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 1

17. If three dice are thrown together, then the probability of getting 5 on at least one of them is

(a) $\frac{125}{216}$ (b) $\frac{215}{216}$

(c) $\frac{1}{216}$ (d) $\frac{91}{216}$

18. In a box containing 100 eggs, 10 eggs are rotten. The probability that out of a sample of 5 eggs none is rotten if the sampling is with replacement is

(a) $\left(\frac{1}{10}\right)^5$ (b) $\left(\frac{1}{5}\right)^5$

(c) $\left(\frac{9}{5}\right)^5$ (d) $\left(\frac{9}{10}\right)^5$

19. A man makes attempts to hit the target. The probability of hitting the target is $\frac{3}{5}$. Then the probability that *A* hit the target exactly 2 times in 5 attempts, is

(a) $\frac{144}{625}$ (b) $\frac{72}{3125}$

(c) $\frac{216}{625}$ (d) None of these

20. **A dice is thrown ten times. If getting even number is considered as a success, then the probability of four successes is**

(a) $^{10}C_4\left(\frac{1}{2}\right)^4$ (b) $^{10}C_4\left(\frac{1}{2}\right)^6$

(c) $^{10}C_4\left(\frac{1}{2}\right)^8$ (d) $^{10}C_6\left(\frac{1}{2}\right)^{10}$

21. **The probability of a bomb hitting a bridge is $\frac{1}{2}$ and two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9, is**

(a) 8 (b) 7

(c) 6 (d) 9

22. **In a box of 10 electric bulbs, two are defective. Two bulbs are selected at random one after the other from the box. The first bulb after selection being put back in the box before making the second selection. The probability that both the bulbs are without defect is**

(a) $\frac{9}{25}$ (b) $\frac{16}{25}$

(c) $\frac{4}{5}$ (d) $\frac{8}{25}$

23. **The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. What is the probability that out of 5 such bulbs none will fuse after 150 days of use**

(a) $1-\left(\frac{19}{20}\right)^5$ (b) $\left(\frac{19}{20}\right)^5$

(c) $\left(\frac{3}{4}\right)^5$ (d) $90\left(\frac{1}{4}\right)^5$

24. **In a binomial distribution the probability of getting a success is 1/4 and standard deviation is 3, then its mean is**

(a) 6 (b) 8

(c) 12 (d) 10

25. **If there are *n* independent trials, *p* and *q* the probability of success and failure respectively, then probability of exactly *r* successes**

(a) $^nC_{n+r}p^rq^{n-r}$ (b) $^nC_rp^{r-1}q^{r+1}$

(c) $^nC_rq^{n-r}p^r$ (d) $^nC_rp^{r+1}q^{r-1}$

26. The probability that a man can hit a target is $\frac{3}{4}$. He tries 5 times. The probability that he will hit the target at least three times is

(a) $\frac{291}{364}$ (b) $\frac{371}{464}$

(c) $\frac{471}{502}$ (d) $\frac{459}{512}$

27. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$

(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

28. If X has binomial distribution with mean *np* and variance *npq*, then $\frac{P(X=k)}{P(X=k-1)}$ is

(a) $\frac{n-k}{k-1}.\frac{p}{q}$ (b) $\frac{n-k+1}{k}.\frac{p}{q}$

(c) $\frac{n+1}{k}.\frac{q}{p}$ (d) $\frac{n-1}{k+1}.\frac{q}{p}$

29. If X follows a binomial distribution with parameters $n=6$ and p and $4(P(X=4))=P(X=2)$, then $p=$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

30. The value of C for which $P(X=k)=Ck^2$ can serve as the probability function of a random variable X that takes 0, 1, 2, 3, 4 is

(a) $\frac{1}{30}$ (b) $\frac{1}{10}$

(c) $\frac{1}{3}$ (d) $\frac{1}{15}$

31. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

(a) $\frac{28}{256}$ (b) $\frac{219}{256}$

(c) $\frac{128}{256}$ (d) $\frac{37}{256}$

32. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to that of getting 9 heads, then the probability of getting 3 heads is

(a) $\frac{35}{2^{12}}$ (b) $\frac{35}{2^{14}}$ (c) $\frac{7}{2^{12}}$ (d) None of these

33. The mean and variance of a random variable *X* having a binomial distribution are 4 and 2 respectively, then $P(X=1)$ is

(a) 1/32 (b) 1/16

(c) 1/8 (d) 1/4

34. A coin is tossed *n* times. The probability of getting head at least once is greater than 0.8, then the least value of *n* is

(a) 2 (b) 3

(c) 4 (d) 5

35. The probability that a student is not a swimmer is 1/5. What is the probability that out of 5 students, 4 are swimmers

(a) ${}^5C_4\left(\frac{4}{5}\right)^4\frac{1}{5}$ (b) $\left(\frac{4}{5}\right)^4\frac{1}{5}$

(c) ${}^5C_1\frac{1}{5}\left(\frac{4}{5}\right)^4\times{}^5C_4$ (d) None of these

36. A coin is tossed 2*n* times. The chance that the number of times one gets head is not equal to the number of times one gets tail is

(a) $\frac{(2n!)}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$ (b) $1-\frac{(2n!)}{(n!)^2}$

(c) $1-\frac{(2n!)}{(n!)^2}.\frac{1}{4^n}$ (d) None of these

37. If a random variable X follows Poisson distribution such that P(x=0)=P(x=1) = k, then k =

1) e 2) 1/e 3) e^2 4) $1/e^2$

38. If X is a Poisson variable such that P(x=1)=P(x=2), then P(X=4) =

1) $\frac{2}{3}e^{-2}$ 2) $\frac{3}{2}e^{-2}$ 3) e^{-2} 4) $\frac{1}{3}e^{-2}$

39. **Cycle tyres supplied in lots of 10 and there is a chance of 1 in 500 types to be defective. Using Poisson's distribution theory, the approximate number of lots containing no defective tyre in a consignment of 10,000 lots is**

1) 9980 2) 9998 3) 9802 4) 9982

40. **In a Poisson distribution the probability P(x=0) is twice the probability P(x = 1). Then the mean of the distribution is**

1) 1/3 2) 1/2 3) 1/4 4) 1/5

41. **For a Poison distribution, if P(1) = P(2), then P(0) =**

1) e 2) e^{-1} 3) e^{-2} 4) e^{-4}

42. **The number of traffic accidents of a city in each month is assumed to be Poisson variate with the parameter λ= 3. Then the probability that there will be at least one accident in a certain month is**

1) $\frac{e^3-1}{e^3}$ 2) $\frac{1}{e^3}$ 3) $\frac{1}{e}$ 4) $\frac{1}{e^2}$

43. **A machine is known to produce 10% defective items by applying Poisson's approximation, the probability that a random sample of size 20, will contain exactly 3 defective items is**

1) $\frac{4}{3.e^2}$ 2) $\frac{8}{3e^2}$ 3) $\frac{1}{4e^2}$ 4) $\frac{3}{4e^2}$

44. **The probability that a bomb dropped from a plane strikes the target is 1/5. The probability that out of 6 bombs dropped at least 2 bombs strike the target is**

1) 0.435 2) 0.345 3) 0.534 4) 0.452

45. **A's chance of winning a single game against B is 2/3. The probability that, in a series of 5 games with B, A wins atleast 3 games is**

1) 64/81 2) 65/81 3) 67/81 4) 66/81

46. A man takes a step forward with probability 0.4 and backwards with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is

1) ${}_{11}C_5\,(.24)^5$ 2) ${}_{11}C_6\,(.24)^6$ 3) ${}_{11}C_4(.24)^4$ 4) ${}_{11}C_7(.24)^4$

47. An irregular six faced die is thrown, the probability that in 10 throws it give 5 even numbers is twice the probability that it will give 4 even numbers. Then the probability that in 10 throws it give no even number is

1) $\left(\frac{1}{4}\right)^{10}$ 2) $\left(\frac{1}{2}\right)^{10}$ 3) $\left(\frac{3}{8}\right)^{10}$ 4) $\left(\frac{1}{3}\right)^{10}$

48. 6 dice are thrown 729 times. The number of times one may expect at least 3 dice to show 4 or 5 is

1) 233 2) 332 3) 323 4) 234

49. In 256 sets of twelve tosses of a coin, the number of cases in which one can expect eight heads and four tails is

1) 25 2) 28 3) 31 4) 35

50. A product is supposed to contain 5% defective items. The probability that a sample of 8 items 6will contain less than 2 defective items is

1) $\left(\frac{19}{20}\right)^7\left(\frac{27}{20}\right)$ 2) $\left(\frac{19}{20}\right)^8$ 3) $\left(\frac{27}{20}\right)$ 4) $\left(\frac{19}{20}\right)^6\left(\frac{1}{20}\right)$

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1. (b) sum of prob. =1

2. (b) sum of prob. =1

3. (b)

4. (b) apply mean and variance formula

5. (c)

6. (b)

7. (b)

8. (c)

9. (b)

10. (b)

11. (c)

12. (d) $np = 6$

$npq = 2 \Rightarrow q = \frac{1}{3}, p = \frac{2}{3}$ and $n = 9.$

Hence the binomial distribution is $\left(\frac{1}{3}+\frac{2}{3}\right)^9$.

13. (b) Required probability = ${}^5C_1\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^4$

14. (b) Let X denotes the number of tosses required. Then $P(X = r) = (1-p)^{r-1}.\ p,$ for $r = 1, 2, 3......$

15. (a)Required probability $= {}^7C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^3.$

16. (c) $9.{}^6C_4 p^4 q^2 = {}^6C_2 p^2 q^4$

17. (d) Required probability

$$={}^3C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2+{}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)+{}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0=\frac{91}{216}.$$

18. (d) Let $P(\text{fresh egg})=\frac{90}{100}=\frac{9}{10}=p$

$P(\text{rotten egg})=\frac{10}{100}=\frac{1}{10}=q$; $n=5,\ r=5$

19. (a)Probability of success $(p)=\frac{3}{5}\Rightarrow q=1-p=\frac{2}{5}$

Hence the probability of 2 hits in 5 attempts

$$={}^5C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^3=\frac{144}{625}.$$

20. (d) Required probability

$$={}^{10}C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6={}^{10}C_4\left(\frac{1}{2}\right)^{10}={}^{10}C_6\left(\frac{1}{2}\right)^{10}.$$

21. (a)Let n be the least number of bombs required and X the number of bombs that hit the bridge.

Then X follows a binomial distribution with parameter n and $p=\frac{1}{2}$.

Now $P(X\geq 2)>0.9\Rightarrow 1-P(X<2)>0.9$

$\Rightarrow P(X=0)+P(X=1)<0.1$

$\Rightarrow {}^nC_0\left(\frac{1}{2}\right)^n+{}^nC_1\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right)<0.1\Rightarrow 10(n+1)<2^n$

This gives $n\geq 8$.

22. (b) Here P(without defected) $=\frac{8}{10}=\frac{4}{5}=p$

P(defected) $=\frac{2}{10}=\frac{1}{5}=q$ and $n=2,\ r=2$

23. (b) Here $p=\frac{19}{20},\ q=\frac{1}{20},\ n=5,\ r=5$

The required probability $={}^5C_5\left(\frac{19}{20}\right)^5.\left(\frac{1}{20}\right)^0=\left(\frac{19}{20}\right)^5.$

24. (c) $p=\frac{1}{4}\ q=\frac{3}{4}$

Mean = np

Standard deviation = $\sqrt{\text{Variance}}\Rightarrow$ Variance = 9

$\Rightarrow npq=9\Rightarrow n.\frac{1}{4}.\frac{3}{4}=9\Rightarrow n=48$

Mean $= np = \frac{1}{4}\times 48 = 12$.

25. (c) Concept.

26. (d) $p = \frac{3}{4} \Rightarrow q = \frac{1}{4}$ and $n = 5$

Probability $= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5$

27. (d) Probability of getting odd $p = \frac{3}{6} = \frac{1}{2}$.

Probability of getting others $q = \frac{3}{6} = \frac{1}{2}$

Variance $= npq = 5.\frac{1}{2}.\frac{1}{2} = \frac{5}{4}$.

28. (b) mean = np and variance = npq

$$\therefore \frac{P(X=k)}{P(X=k-1)} = \frac{{}^nC_k(p)^k(q)^{n-k}}{{}^nC_{k-1}(p)^{k-1}(q)^{n-k+1}} = \frac{{}^nC_k}{{}^nC_{k-1}}.\frac{p}{q}$$

$$\therefore \frac{P(X=k)}{P(X=k-1)} = \frac{n-k+1}{k}.\frac{p}{q}.$$

29. (d) $4P(X=4) = P(X=2) \Rightarrow 4.{}^6C_4p^4q^2 = {}^6C_2p^2q^4$

$\Rightarrow 4p^2 = q^2 \Rightarrow 4p^2 = (1-p)^2$

$\Rightarrow 3p^2 + 2p - 1 = 0 \Rightarrow p = \frac{1}{3}$.

30. (a) $\sum_{k=0}^{4} P(X=k) = 1 \Rightarrow \sum_{k=0}^{4} Ck^2 = 1$

$\Rightarrow C(1^2 + 2^2 + 3^2 + 4^2) = 1 \Rightarrow C = \frac{1}{30}$.

31. (a) $\left.\begin{matrix} np = 4 \\ npq = 2 \end{matrix}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$P(X=2) = {}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 = 28.\frac{1}{2^8} = \frac{28}{256}$.

32. (a)Let the coin be tossed n times

P(7 heads) $= {}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_7\left(\frac{1}{2}\right)^n$

and P(9 heads) $= {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9} = {}^nC_9\left(\frac{1}{2}\right)^n$

P(7 heads) $= P$(9 heads) $\Rightarrow {}^nC_7 = {}^nC_9 \Rightarrow n = 16$

$\therefore P$(3 heads) $= {}^{16}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{16-3} = {}^{16}C_3\left(\frac{1}{2}\right)^{16} = \frac{35}{2^{12}}$.

33. (a) $\left.\begin{matrix} np = 4 \\ npq = 2 \end{matrix}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$$P(X = 1) = {}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8.\frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}.$$

34. (b) Let X be the number of heads getting. X follows binomial distribution with parameters $n, p = 1/2$ Given that $P(X \geq 1) \geq 0.8$

$\Rightarrow 1 - P(X = 0) \geq 0.8 \Rightarrow P(X = 0) \leq 0.2$

$\Rightarrow {}^nC_0\,(1/2)^n \leq 0.2 \;\Rightarrow \frac{1}{2^n} \leq \frac{1}{5} \Rightarrow 2^n = 5.$

$\therefore$ The least value of n is 3.

35. (a)The probability that student is not swimmer

$p = \frac{1}{5}$ and probability that student is swimmer $q = \frac{4}{5}$.

$\therefore$ Probability that out of 5 students 4 are swimmer

$$= {}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^{5-4} = {}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right).$$

36. (c)The required probability

= 1 – Probability of equal number of heads and tails

$$= 1 - {}^{2n}C_n\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^{2n-n} = 1 - \frac{(2n)!}{n!n!}\left(\frac{1}{4}\right)^n = 1 - \frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}.$$

37. (b)

38. (a)

39. (c)

40. (b)

41. (c)

42. (a)

43. (a)

44. (b)

45. (a)

46. (a)

47. (c)

48. (a)

49. (c)

50. (a)