PROBABILITY

OBJECTIVE PROBLEMS

1. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is

(a)
$$\frac{2}{11}$$
 (b) $\frac{3}{11}$

- (c) $\frac{4}{11}$ (d) 0
- 2. A coin is tossed and a dice is rolled. The probability that the coin shows the head and the dice shows 6 is
 - (a) $\frac{1}{8}$ (b) $\frac{1}{12}$
 - (c) $\frac{1}{2}$ (d) 1
- 3. *A* and *B* toss a coin alternatively, the first to show a head being the winner. If *A* starts the game, the chance of his winning is
 - (a) 5/8 (b) 1/2

(c)
$$1/3$$
 (d) $2/3$

4. One card is drawn from each of two ordinary packs of 52 cards. The probability that at least one of them is an ace of heart, is



- 5. From a book containing 100 pages, one page is selected randomly. The probability that the sum of the digits of the page number of the selected page is 11, is
 - (a) $\frac{2}{25}$ (b) $\frac{9}{100}$
 - (c) $\frac{11}{100}$ (d) None of these

- 6. The probability of getting at least one tail in 4 throws of a coin is
 - (a) $\frac{15}{16}$ (b) $\frac{1}{16}$
 - (c) $\frac{1}{4}$ (d) None of these
- 7. In a single throw of two dice, the probability of getting more than 7 is
 - (a) $\frac{7}{36}$ (b) $\frac{7}{12}$
 - (c) $\frac{5}{12}$ (d) $\frac{5}{36}$
- 8. From a pack of 52 cards two are drawn with replacement. The probability, that the first is a diamond and the second is a king, is

(a)
$$\frac{1}{26}$$
 (b) $\frac{17}{2704}$

- (c) $\frac{1}{52}$ (d) None of these
- 9. There are *n* letters and *n* addressed envelopes. The probability that all the letters are not kept in the right envelope, is

(a)
$$\frac{1}{n!}$$

- (c) $1 \frac{1}{n}$ (d) None of these
- 10. A problem of mathematics is given to three students whose chances of solving the problem are 1/3, 1/4 and 1/5 respectively. The probability that the question will be solved is

(a)
$$\frac{2}{3}$$
 (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

(b) $1 - \frac{1}{n!}$

- 11. The probability of happening an event A in one trial is 0.4. The probability that the event A happens at least once in three independent trials is
 - (a) 0.936 (b) 0.784
 - (c) 0.904 (d) 0.216

- 12. A card is drawn at random from a pack of 52 cards. The probability that the drawn card is a court card *i.e.* a jack, a queen or a king, is
- (a) $\frac{3}{52}$ (b) $\frac{3}{13}$ (c) $\frac{4}{13}$ (d) None of these
- 13. Three letters are to be sent to different persons and addresses on the three envelopes are also written. Without looking at the addresses, the probability that the letters go into the right envelope is equal to
 - (a) $\frac{1}{27}$ (b) $\frac{1}{9}$ (c) $\frac{4}{27}$ (d) $\frac{1}{6}$

(d) $\frac{1}{2}$

14. Three letters are to be sent to different persons and addresses on the three envelopes are also written. Without looking at the addresses, the probability that the letters go into the right envelope is equal to

(a)
$$\frac{1}{27}$$
 (b) $\frac{1}{9}$
(c) $\frac{4}{27}$ (d) $\frac{1}{6}$

15. Two dice are thrown simultaneously. What is the probability of obtaining a multiple of 2 on one of them and a multiple of 3 on the other

(a)
$$\frac{5}{36}$$
 (b) $\frac{11}{36}$

(c)
$$\frac{1}{6}$$

16. A box contains 10 good articles and 6 with defects. One article is chosen at random. What is the probability that it is either good or has a defect



- 17. The probabilities of winning the race by two athletes A and B are $\frac{1}{5}$ and $\frac{1}{4}$. The probability of winning by neither of them, is
 - (a) $\frac{3}{5}$ (b) $\frac{3}{4}$
 - (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

18. A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is 1/4 and that of the woman's selection is 1/3. What is the probability that none of them will be selected

(a)
$$\frac{1}{2}$$
 (b) $\frac{1}{12}$

(c) $\frac{1}{4}$ (d) None of these

- **19.** The probability that an event will fail to happen is 0.05. The probability that the event will take place on 4 consecutive occasions is
 - (a) 0.00000625 (b) 0.18543125
 - $(c) 0.00001875 \qquad (d) 0.81450625$
- 20. A coin is tossed until a head appears or until the coin has been tossed five times. If a head does not occur on the first two tosses, then the probability that the coin will be tossed 5 times is

(a)
$$\frac{1}{2}$$
 (b) $\frac{3}{5}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

(b) $\frac{15}{16}$ (d) $\frac{3}{8}$

21. If the probabilities of boy and girl to be born are same, then in a 4 children family the probability of being at least one girl, is

(a)
$$\frac{14}{16}$$

(c)
$$\frac{1}{8}$$

22. A card is drawn at random from a pack of cards. What is the probability that the drawn card is neither a heart nor a king

(a)
$$\frac{4}{13}$$
 (b) $\frac{9}{13}$
(c) $\frac{1}{4}$ (d) $\frac{13}{26}$

- 23. Three persons work independently on a problem. If the respective probabilities that they will solve it are 1/3, 1/4 and 1/5, then the probability that none can solve it
 - (a) $\frac{2}{5}$ (b) $\frac{3}{5}$
 - (c) $\frac{1}{3}$ (d) None of these

- 24. Three persons work independently on a problem. If the respective probabilities that they will solve it are 1/3, 1/4 and 1/5, then the probability that none can solve it
 - (a) $\frac{2}{5}$ (b) $\frac{3}{5}$
 - (c) $\frac{1}{3}$ (d) None of these
- 25. If A and B are mutually exclusive events, then the value of P (A or B) is
 - (a) 0 (b) -1
 - (c) 1 (d) None of these
- 26. If A and B are mutually exclusive events, then the value of P (A or B) is
 - (a) 0 (b) -1
 - (c) 1 (d) None of these

27. For any event A

- (a) $P(A) + P(\overline{A}) = 0$ (b) $P(A) + P(\overline{A}) = 1$
- (C) P(A) > 1 (d) $P(\overline{A}) < 1$
- 28. The probability of *A*, *B*, *C* solving a problem are $\frac{1}{3}$, $\frac{2}{7}$, $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, the probability that exactly one of them will solve it, is
 - (a) $\frac{25}{168}$ (b) $\frac{25}{56}$
 - (c) $\frac{20}{168}$ (d)
- 29. A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected
 - (a) $\frac{1}{7}$ (b) $\frac{2}{7}$
 - (c) $\frac{3}{7}$ (d) None of these

30. A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept aside the first. This process is repeated till all the balls are drawn from the box. The probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red is

(a)
$$\frac{1}{1260}$$
 (b) $\frac{1}{7560}$

(c)
$$\frac{1}{126}$$
 (d) None of these

31. Seven chits are numbered 1 to 7. Three are drawn one by one with replacement. The probability that the least number on any selected chit is 5, is

(a) $1 - \left(\frac{2}{7}\right)^4$ (b) $4\left(\frac{2}{7}\right)^4$ (c) $\left(\frac{3}{7}\right)^3$ (d) None of these

- 32. The probability of India winning a test match against West Indies is $\frac{1}{2}$. Assuming independence from match to match, the probability that in a 5 match series India's second win occurs at the third test, is
 - (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$
- **33.** A locker can be opened by dialing a fixed three digit code (between 000 and 999). A stranger who does not know the code tries to open the locker by dialing three digits at random. The probability that the stranger succeeds at the *k*th trial is
 - (a) $\frac{k}{999}$ (b) $\frac{k}{1000}$ (c) $\frac{k-1}{1000}$ (d) None of these
- 34. The probabilities of a student getting I, II and III division in an examination are respectively $\frac{1}{10}, \frac{3}{5}$ and $\frac{1}{4}$. The probability that the student fails in the examination is
 - (a) $\frac{197}{200}$ (b) $\frac{27}{100}$
 - (c) $\frac{83}{100}$ (d) None of these

- 35. A six faced dice is so biased that it is twice as likely to show an even number as an odd number when thrown. It is thrown twice. The probability that the sum of two numbers thrown is even, is
 - (a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
- **36.** For independent events A_1, A_2, \dots, A_n , $P(A_i) = \frac{1}{i+1}$, $i = 1, 2, \dots, n$. Then the probability that none

of the event will occur, is

- (a) $\frac{n}{n+1}$ (b) $\frac{n-1}{n+1}$ (c) $\frac{1}{n+1}$ (d) None of these
- 37. The probability that a teacher will give an unannounced test during any class meeting is 1/5. If a student is absent twice, then the probability that the student will miss at least one test is

(a)
$$\frac{4}{5}$$
 (b) $\frac{2}{5}$

- (c) $\frac{7}{5}$ (d) $\frac{9}{25}$
- 38. A bag contains 3 red and 7 black balls, two balls are taken out at random, without replacement. If the first ball taken out is red, then what is the probability that the second taken out ball is also red
 - (a) $\frac{1}{10}$ (b) $\frac{1}{15}$ (c) $\frac{3}{10}$ (d) $\frac{2}{21}$
- **39.** The corners of regular tetrahedrons are numbered 1, 2, 3, 4. Three tetrahedrons are tossed. The probability that the sum of upward corners will be 5 is



- 40. If a coin be tossed *n* times then probability that the head comes odd times is
 - (a) $\frac{1}{2}$ (b) $\frac{1}{2^n}$
 - (c) $\frac{1}{2^{n-1}}$ (d) None of these

41. Suppose that a die (with faces marked 1 to 6) is loaded in such a manner that for K = 1, 2, 3..., 6, the probability of the face marked K turning up when die is tossed is proportional to K. The probability of the event that the outcome of a toss of the die will be an even number is equal to

(a)
$$\frac{1}{2}$$
 (b) $\frac{4}{7}$

(c)
$$\frac{2}{5}$$
 (d) $\frac{1}{21}$

- 42. A binary number is made up of 16 bits. The probability of an incorrect bit appearing is *p* and the errors in different bits are independent of one another. The probability of forming an incorrect number is
 - (a) $\frac{p}{16}$ (b) p^{16}

(c)
$${}^{16}C_1p^{16}$$
 (d) $1-(1-p)^{16}$

- 43. In a college, 25% of the boys and 10% of the girls offer Mathematics. The girls constitute 60% of the total number of students. If a student is selected at random and is found to be studying Mathematics, the probability that the student is a girl, is
 - (a) $\frac{1}{6}$ (b) $\frac{3}{8}$
 - (c) $\frac{5}{8}$
- 44. The probability that a marksman will hit a target is given as 1/5. Then his probability of at least one hit in 10 shots, is
 - (a) $1 \left(\frac{4}{5}\right)^{10}$ (b) $\frac{1}{5^{10}}$ (c) $1 \frac{1}{5^{10}}$ (d) None of these

(d) $\frac{5}{6}$

- 45. A bag x contains 3 white balls and 2 black balls and another bag y contains 2 white balls and 4 black balls. A bag and a ball out of it are picked at random. The probability that the ball is white, is
 - (a) 3/5 (b) 7/15
 - (c) 1/2 (d) None of these

- 46. A problem in Mathematics is given to three students *A*, *B*, *C* and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved is
 - (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$
- 47. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact
 - (a) $\frac{4}{5}$ (b) $\frac{1}{5}$ (c) $\frac{7}{20}$ (d) $\frac{3}{20}$
- 48. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is
 - (a) $\frac{8}{9}$ (b) $\frac{7}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$
- 49. In a throw of a dice the probability of getting one in even number of throw is
 - (a) $\frac{5}{36}$ (b) $\frac{5}{11}$ (c) $\frac{6}{11}$ (d) $\frac{1}{6}$
- 50. If any four numbers are selected and they are multiplied, then the probability that the last digit will be 1, 3, 5 or 7 is
 - (a) $\frac{4}{625}$ (b) $\frac{18}{625}$ (c) $\frac{16}{625}$ (d) None of these
- 51. Word 'UNIVERSITY' is arranged randomly. Then the probability that both 'I' does not come together, is
 - (a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

- 52. If Mohan has 3 tickets of a lottery containing 3 prizes and 9 blanks, then his chance of winning prize are
 - (a) $\frac{34}{55}$ (b) $\frac{21}{55}$ (c) $\frac{17}{55}$ (d) None of these
- 53. The letter of the word `ASSASSIN' are written down at random in a row. The probability that no two *S* occur together is
 - (a) $\frac{1}{35}$ (b) $\frac{1}{14}$ (c) $\frac{1}{15}$ (d) None of these
- 54. A box contains 25 tickets numbered 1, 2, 25. If two tickets are drawn at random then the probability that the product of their numbers is even, is
 - (a) $\frac{11}{50}$ (b) $\frac{13}{50}$
 - (c) $\frac{37}{50}$ (d) None of these
- 55. Six cards are drawn simultaneously from a pack of playing cards. What is the probability that 3 will be red and 3 black
 - (a) ${}^{26}C_6$ (b) $\frac{{}^{26}C_3}{{}^{52}C}$
 - (c) $\frac{{}^{2_6}C_3 \times {}^{2_6}C_3}{{}^{5_2}C_6}$ (d) $\frac{1}{2}$
- 56. Three mangoes and three apples are in a box. If two fruits are chosen at random, the probability that one is a mango and the other is an apple is
 - (a) $\frac{2}{3}$ (b) $\frac{3}{5}$
 - (c) $\frac{1}{3}$ (d) None of these
- 57. A word consists of 11 letters in which there are 7 consonants and 4 vowels. If 2 letters are chosen at random, then the probability that all of them are consonants, is
 - (a) $\frac{5}{11}$ (b) $\frac{21}{55}$ (c) $\frac{4}{11}$ (d) None of these

- 58. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, then the probability that 2 are white and 1 is red, is
 - (a) $\frac{5}{204}$ (b) $\frac{7}{102}$ (c) $\frac{3}{68}$ (d) $\frac{1}{13}$
- 59. There are *n* different objects 1, 2, 3,.....*n* distributed at random in *n* places marked 1, 2, 3,*n*. The probability that at least three of the objects occupy places corresponding to their number is
 - (a) $\frac{1}{6}$ (b) $\frac{5}{6}$
 - (c) $\frac{1}{3}$ (d) None of these
- 60. In a lottery 50 tickets are sold in which 14 are of prize. A man bought 2 tickets, then the probability that the man win the prize, is
 - (a) $\frac{17}{35}$ (b) $\frac{18}{35}$
 - (c) $\frac{72}{175}$ (d) $\frac{13}{175}$
- 61. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral, is equal to

(a)
$$\frac{1}{2}$$

(c)
$$\frac{1}{10}$$
 (d)

62. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral, is equal to

(a)
$$\frac{1}{2}$$
 (b) $\frac{1}{5}$
(c) $\frac{1}{10}$ (d) $\frac{1}{20}$

- 63. A bag contains 4 white and 3 red balls. Two draws of one ball each are made without replacement. Then the probability that both the balls are red is
 - (a) $\frac{1}{7}$ (b) $\frac{2}{7}$
 - (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

- 64. A box contains 10 mangoes out of which 4 are rotten. 2 mangoes are taken out together. If one of them is found to be good, the probability that the other is also good is
 - (a) $\frac{1}{3}$ (b) $\frac{8}{15}$
 - (c) $\frac{5}{18}$ (d) $\frac{2}{3}$
- 65. Four boys and three girls stand in a queue for an interview, probability that they will in alternate position is

(a)
$$\frac{1}{34}$$
 (b) $\frac{1}{35}$

- (c) $\frac{1}{17}$ (d) $\frac{1}{68}$
- **66.** A mapping is selected at random from the set of all the mappings of the set *A* = {1, 2,...,*n*} into itself. The probability that the mapping selected is an injection is

(a)
$$\frac{1}{n^n}$$
 (b) $\frac{1}{n!}$
(c) $\frac{(n-1)!}{n^{n-1}}$ (d) $\frac{n!}{n^{n-1}}$

67. Among 15 players, 8 are batsmen and 7 are bowlers. Find the probability that a team is chosen of 6 batsmen and 5 bowlers

(a)
$$\frac{{}^{8}C_{6}\times^{7}C_{5}}{{}^{15}C_{11}}$$

(c) $\frac{15}{28}$ (d) None of these

(b) $\frac{{}^{8}C_{6} + {}^{7}C_{5}}{{}^{15}C_{11}}$

68. If *m* rupee coins and *n* ten paise coins are placed in a line, then the probability that the extreme coins are ten paise coins is

(a)
$${}^{m+n}C_m / n^m$$
 (b) ${n(n-1) \over (m+n)(m+n-1)}$
(c) ${}^{m+n}P_m / m^n$ (d) ${}^{m+n}P_n / n^m$

- 69. A bag contains 3 red, 4 white and 5 blue balls. All balls are different. Two balls are drawn at random. The probability that they are of different colour is
 - (a) $\frac{47}{66}$ (b) $\frac{10}{33}$ (c) $\frac{5}{22}$ (d) None of these
- 70. A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected randomwise, the probability that it is a black or red ball is

(a)
$$\frac{1}{3}$$
 (b) $\frac{1}{4}$ (c) $\frac{5}{12}$ (d) $\frac{2}{3}$

- 71. Six boys and six girls sit in a row randomly. The probability that the six girls sit together
 - (a) $\frac{1}{77}$ (b) $\frac{1}{132}$
 - (c) $\frac{1}{231}$ (d) None of these
- 72. Two friends *A* and *B* have equal number of daughters. There are three cinema tickets which are to be distributed among the daughters of *A* and *B*. The probability that all the tickets go to daughters of *A* is 1/20. The number of daughters each of them have is
 - (a) 4 (b) 5
 - (c) 6 (d) 3
- 73. If four vertices of a regular octagon are chosen at random, then the probability that the quadrilateral formed by them is a rectangle is
 - (a) $\frac{1}{2}$ (b) $\frac{2}{21}$
 - (c) $\frac{1}{32}$ (d) $\frac{1}{35}$
- 74. Two numbers are selected at random from 1, 2, 3100 and are multiplied, then the probability correct to two places of decimals that the product thus obtained is divisible by 3, is
 - (a) 0.55 (b) 0.44 (c) 0.22 (d) 0.33
- 75. Let *A* and *B* be two finite sets having *m* and *n* elements respectively such that $m \le n$. A mapping is selected at random from the set of all mappings from *A* to *B*. The probability that the mapping selected is an injection is

(a)
$$\frac{n!}{(n-m)!m^n}$$
 (b)
$$\frac{n!}{(n-m)!n^m}$$

(c)
$$\frac{m!}{(n-m)!n^m}$$
 (d)
$$\frac{m!}{(n-m)!m^n}$$

76. A drawer contains 5 brown socks and 4 blue socks well mixed. A man reaches the drawer and pulls out 2 socks at random. What is the probability that they match

- (a) $\frac{4}{9}$ (b) $\frac{5}{8}$ (c) $\frac{5}{9}$ (d) $\frac{7}{12}$
- 77. Fifteen persons among whom are *A* and *B*, sit down at random at a round table. The probability that there are 4 persons between *A* and *B*, is
 - (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{7}$ (d) $\frac{1}{7}$

- 78. Suppose $n \ge 3$ persons are sitting in a row. Two of them are selected at random. The probability that they are not together is
 - (a) $1 \frac{2}{n}$ (b) $\frac{2}{n-1}$ (c) $1 - \frac{1}{n}$ (d) None of these
- **79.** If odds against solving a question by three students are 2 : 1, 5:2 and 5:3 respectively, then probability that the question is solved only by one student is

(a)
$$\frac{31}{56}$$
 (b) $\frac{24}{56}$

(c) $\frac{25}{56}$ (d) None of these

80. Suppose that A, B, C are events such that $P(A) = P(B) = P(C) = \frac{1}{4}$, P(AB) = P(CB) = 0, $P(AC) = \frac{1}{8}$, then

- P(A+B) =
- (a) 0.125 (b) 0.25
- (c) 0.375 (d) 0.5
- 81. If the odds in favour of an event be 3 : 5, then the probability of non-occurrence of the event is
 - . . 3
 - (a) $\frac{3}{5}$ (b) $\frac{5}{3}$

(c)
$$\frac{3}{8}$$
 (d) $\frac{5}{8}$

82. If two events A and B are such that $P(A+B) = \frac{5}{6}$, $P(AB) = \frac{1}{3}$ and $P(\overline{A}) = \frac{1}{2}$, then the events A and

B are

- (a) Independent
- (b) Mutually exclusive
- (c) Mutually exclusive and independent
- (d) None of these
- 83. For an event, odds against is 6 : 5. The probability that event does not occur, is
 - (a) $\frac{5}{6}$ (b) $\frac{6}{11}$ (c) $\frac{5}{11}$ (d) $\frac{1}{6}$
- 84. In a horse race the odds in favour of three horses are 1:2, 1:3 and 1:4. The probability that one of the horse will win the race is
 - (a) $\frac{37}{60}$ (b) $\frac{47}{60}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

85. If A and B are two events such that $P(A \cup B) + P(A \cap B) = \frac{7}{8}$ and P(A) = 2P(B), then $P(A) = \frac{7}{8}$

(a) $\frac{7}{12}$ (b) $\frac{7}{24}$ (c) $\frac{5}{12}$ (d) $\frac{17}{24}$

86. If $P(A) = \frac{1}{4}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then $P(A \cap B) =$

- (a) $\frac{1}{8}$ (b) 0
- (c) $\frac{3}{4}$ (d) 1
- 87. If A and B are any two events, then the probability that exactly one of them occur is
 - (a) $P(A) + P(B) P(A \cap B)$ (b) $P(A) + P(B) 2P(A \cap B)$
 - (c) $P(A) + P(B) P(A \cup B)$ (d) $P(A) + P(B) 2P(A \cup B)$
- 88. If A and B are two events of a random experiment, P(A) = 0.25, P(B) = 0.5 and $P(A \cap B) = 0.15$,

then $P(A \cap \overline{B}) =$

- (a) 0.1 (b) 0.35
- (c) 0.15 (d) 0.6
- 89. The probability that at least one of A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then P(A') + P(B') =
 - (a) 0.9 (b) 1.15
 - (c) 1.1 (d) 1.2

90. If *A* and *B* are two independent events such that $P(A \cap B') = \frac{3}{25}$ and $P(A' \cap B) = \frac{8}{25}$, then $P(A) = \frac{3}{25}$

(a) $\frac{1}{5}$	465	(b) $\frac{3}{8}$
(c) $\frac{2}{5}$		(d) $\frac{4}{5}$

91. If the probability of X to fail in the examination is 0.3 and that for Y is 0.2, then the probability that either X or Y fail in the examination is

- (a) 0.5 (b) 0.44
- (c) 0.6 (d) None of these

- 92. The probability of happening at least one of the events *A* and *B* is 0.6. If the events *A* and *B* happens simultaneously with the probability 0.2, then $P(\overline{A}) + P(\overline{B}) =$
 - (a) 0.4 (b) 0.8
 - (c) 1.2 (d) 1.4
- 93. A, B, C are any three events. If P (S) denotes the probability of S happening then $P(A \cap (B \cup C)) =$
 - (a) $P(A) + P(B) + P(C) P(A \cap B) P(A \cap C)$ (b) P(A) + P(B) + P(C) P(B)P(C)
 - (c) $P(A \cap B) + P(A \cap C) P(A \cap B \cap C)$ (d) None of these
- 94. Let A and B be two events such that P(A) = 0.3 and $P(A \cup B) = 0.8$. If A and B are independent events, then P(B) =
 - (a) $\frac{5}{6}$ (b) $\frac{5}{7}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$
- **95.** Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where \overline{A} stands for

complement of event A. Then events A and B are

- (a) Independent but not equally likely
- (b) Mutually exclusive and independent
- (c) Equally likely and mutually exclusive
- (d) Equally likely but not independent

96. If *A* and *B* are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\overline{A}) = 2/3$, then $P(\overline{A} \cap B)$ is

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- (a) $\frac{5}{12}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{4}$
- 97. A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
<i>P</i> (0.1	0.2	0.1	0.1	0.2	0.0	0.0	0.0
<i>X</i>)	5	3	2	0	0	8	7	5

For the events $E = \{X \text{ is prime number}\}$ and $F = \{X < 4\}$, the probability of $P(E \cup F)$ is

- (a) 0.50 (b) 0.77
- (c) 0.35 (d) 0.87

- **98.** If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(\overline{A}) + P(\overline{B}) =$
 - (a) 0.3 (b) 0.5
 - (c) 0.7 (d) 0.9
- **99.** If *E* and *F* are independent events such that 0 < P(E) < 1 and 0 < P(F) < 1, then
 - (a) E and F^c (the complement of the event F) are independent
 - (b) E^c and F^c are independent

(c)
$$P\left(\frac{E}{F}\right) + P\left(\frac{E^c}{F^c}\right) = 1$$

- (d) All of the above
- **100.** If A and B are two independent events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{5}$, then
 - (a) $P\left(\frac{A}{B}\right) = \frac{1}{2}$ (b) $P\left(\frac{A}{A \cup B}\right) = \frac{5}{6}$
 - (c) $P\left(\frac{A \cap B}{A' \cup B'}\right) = 0$ (d) All of the above
- **101. If** $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ **and** $P(A \cap B) = \frac{1}{4}$, **then** $P\left(\frac{B}{A}\right) =$

(b)0

(d) $\frac{1}{2}$

- (a) 1
- (c) $\frac{1}{2}$

(c) $\frac{23}{40}$

102. If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{5}$, then $P\left(\frac{\overline{B}}{\overline{A}}\right) = \frac{1}{5}$

(a) $\frac{37}{40}$ (b) $\frac{37}{45}$

(d) None of these

103. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

- (a) $\frac{5}{17}$ (b) $\frac{12}{17}$
- (c) $\frac{17}{30}$ (d) $\frac{3}{5}$

104. If *A* and *B* are two events such that $P(A) \neq 0$ and $P(B) \neq 1$, then $P\left(\frac{\overline{A}}{\overline{B}}\right) =$

(a) $1 - P\left(\frac{A}{B}\right)$ (b) $1 - P\left(\frac{\overline{A}}{B}\right)$

(c)
$$\frac{1 - P(A \cup B)}{P(\overline{B})}$$
 (d) $\frac{P(\overline{A})}{P(\overline{B})}$

105. If two events A and B are such that $P(A^c) = 0.3$, P(B) = 0.4 and $P(AB^c) = 0.5$, then $P[B/(A \cup B^c)]$ is

equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) None of these

106. Let 0 < P(A) < 1, 0 < P(B) < 1 and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$. Then

(a) P(B/A) = P(B) - P(A) (b) $P(A^c \cup B^c) = P(A^c) + P(B^c)$

(c)
$$P(A \cup B)^{c} = P(A^{c})P(B^{c})$$
 (d) $P(A / B) = P(B / A)$

107. Let 0 < P(A) < 1, 0 < P(B) < 1 and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$. Then

- (a) P(B / A) = P(B) P(A)(b) $P(A^{c} \cup B^{c}) = P(A^{c}) + P(B^{c})$ (c) $P(A \cup B)^{c} = P(A^{c})P(B^{c})$ (d) P(A / B) = P(A)
- 108. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is
 - (a) $\frac{37}{40}$ (b) $\frac{1}{37}$ (c) $\frac{36}{37}$ (d) $\frac{1}{9}$

109. For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

(a) A and B are independent (b) $P\left(\frac{A'}{B}\right) = \frac{3}{4}$

(c) $P\left(\frac{B'}{A'}\right) = \frac{1}{2}$ (d) All of these

110. One ticket is selected at random from 100 tickets numbered 00, 01, 02, 98, 99. If X and Y denote the sum and the product of the digits on the tickets, then P(X = 9/Y = 0) equals

- (a) $\frac{1}{19}$ (b) $\frac{2}{19}$
- (c) $\frac{3}{19}$ (d) None of these

111. If \overline{E} and \overline{F} are the complementary events of events *E* and *F* respectively and if 0 < P(F) < 1, then

- (a) $P(E/F) + P(\overline{E}/F) = 1$ b) $P(E/F) + P(E/\overline{F}) = 1$
- (c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ (d) None of these
- 112. A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then the probability for the ball chosen be white is

(a)
$$\frac{2}{15}$$
 (b) $\frac{7}{15}$

- (c) $\frac{8}{15}$ (d) $\frac{14}{15}$
- 113. Cards are drawn one by one at random from a well shuffled full pack of 52 cards until two aces are obtained for the first time. If N is the number of cards required to be drawn, then $P_c\{N = n\}$, where $2 \le n \le 50$, is

(a)
$$\frac{(n-1)(52-n)(51-n)}{50\times49\times17\times13}$$
 (b)
$$\frac{2(n-1)(52-n)(51-n)}{50\times49\times17\times13}$$
 (c)
$$\frac{3(n-1)(52-n)(51-n)}{50\times49\times17\times13}$$
 (d)
$$\frac{4(n-1)(52-n)(51-n)}{50\times49\times17\times13}$$

114. If (1+3p)/3, (1-p)/4 and (1-2p)/2 are the probabilities of three mutually exclusive events,

then the set of all values of *p* is

(a)
$$\frac{1}{3} \le p \le \frac{1}{2}$$
 (b) $\frac{1}{3} (c) $\frac{1}{2} \le p \le \frac{2}{3}$ (d) $\frac{1}{2}$$

115. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, is a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

- (a) $\frac{1}{3}$ (b) $\frac{1}{6}$
- (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

- 116. An anti-aircraft gun take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. The probability that the gun hits the plane is
 - (a) 0.25 (b) 0.21
 - (c) 0.16 (d) 0.6976
- 117. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4,.....,12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8, is
 - (a) 0.24 (b) 0.244 (c) 0.024 (d) None of these

118. If *E* and *F* are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then

- (a) Occurrence of $E \Rightarrow$ Occurrence of F
- (b) Occurrence of $F \Rightarrow$ Occurrence of E
- (c) Non-occurrence of $E \Rightarrow$ Non-occurrence of F
- (d) None of the above implications holds
- 119. If *n* positive integers are taken at random and multiplied together, the probability that the last digit of the product is 2, 4, 6 or 8, is

(a)
$$\frac{4^n + 2^n}{5^n}$$
 (1)

(c)
$$\frac{4^n - 2^n}{5^n}$$
 (d) None of these

120. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is

(a) 16/81 (b) 1/81 (c) 80/81 (d) 65/81

121. If A and B are two events, then the probability of the event that at most one of A, B occurs, is

```
(a) P(A' \cap B) + P(A \cap B') + P(A' \cap B')
```

- (b) $1 P(A \cap B)$
- (C) $P(A') + P(B') + P(A \cup B) 1$
- (d) All of the these

122. Two numbers are selected at random from the numbers 1, 2,n. The probability that the difference between the first and second is not less than *m* (where 0<*m*<*n*), is

(a)
$$\frac{(n-m)(n-m+1)}{(n-1)}$$
 (b) $\frac{(n-m)(n-m+1)}{2n}$
(c) $\frac{(n-m)(n-m-1)}{2n(n-1)}$ (d) $\frac{(n-m)(n-m+1)}{2n(n-1)}$

123. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

(a)
$$\frac{1}{2}$$
 (b) $\frac{7}{15}$

- (c) $\frac{2}{15}$ (d) $\frac{1}{3}$
- 124. The probability of happening an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of happening neither A nor B is
 - (a) 0.6 (b) 0.2
 - (c) 0.21 (d) None of these
- **125.** If *A* and *B* are two events such that $P(A \cup B) = P(A \cap B)$, then the true relation is
 - (a) P(A) + P(B) = 0

(b)
$$P(A) + P(B) = P(A)P\left(\frac{B}{A}\right)$$

$$(\mathbf{C}) P(A) + P(B) = 2 P(A) P\left(\frac{B}{A}\right)$$

PROBABILITY

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- 1. (c) Since there are one *A*, two *I* and one *o*, hence the required probability $=\frac{1+2+1}{11}=\frac{4}{11}$.
- 2. (b) Required probability $=\left(\frac{1}{2}\right)\left(\frac{1}{6}\right)=\frac{1}{12}$.
- 3. (d) The chance of head $=\frac{1}{2}$ and not of head $=\frac{1}{2}$

.. Probability of A's winning

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^4 \cdot \frac{1}{2} + \dots$$
$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots = \frac{2}{3}.$$

4. (a) Required probability is 1-P (no ace of heart)

$$= 1 - \frac{51}{52} \cdot \frac{51}{52} = \frac{(52+51)}{52 \cdot 52} = \frac{103}{2704}$$

5. (a) Favourable ways {29,92,38,83,47,74,56,65}

Hence required probability $=\frac{8}{100}=\frac{2}{25}$.

- 6. (a) Required probability $=1-\left(\frac{1}{2}\right)^4 = \frac{15}{16}$.
- 7. (c) Required probability is

P(getting 8) + P(9) + P(10) + P(11) + P(12)

$$=\frac{5}{36}+\frac{4}{36}+\frac{3}{36}+\frac{2}{36}+\frac{1}{36}=\frac{15}{36}=\frac{5}{12}.$$

8. (c) Required probability = P(Diamond). P(king)

$$=\frac{13}{52}\cdot\frac{4}{52}=\frac{1}{52}.$$

9. (b) Required probability is 1-P (All letters in right envelope) $=1-\frac{1}{n!}$

10. (d) The probability of students not solving the problem are $1 - \frac{1}{3} = \frac{2}{3}$, $1 - \frac{1}{4} = \frac{3}{4}$ and $1 - \frac{1}{5} = \frac{4}{5}$

Therefore the probability that the problem is not solved by any one of them $=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{2}{5}$

11. (b) Here
$$P(A) = 0.4$$
 and $P(\overline{A}) = 0.6$

Probability that A does not happen at all $=(0.6)^3$

Thus required Probability $= 1 - (0.6)^3 = 0.784$.

12. (b) Court cards are king, queen and jack

Hence required probability $=\frac{12}{52}=\frac{3}{13}$.

- 13. (d) Total no. of ways placing 3 letters in three envelops = 3!, out of these ways only one way is correct.
- **14.** (c) Obviously numbers will be $\begin{pmatrix} I & II \\ 5, & 1 \\ 4, & 2 \\ 2, & 4 \\ 1, & 5 \end{pmatrix}$. Hence required probability = $\frac{4}{6 \cdot 5} = \frac{2}{15}$.
- 15. (b) Favourable cases for one are three *i.e.* 2, 4 and 6 and for other are two *i.e.* 3, 6.

Hence required probability $=\left[\left(\frac{3\times 2}{36}\right)2 - \frac{1}{36}\right] = \frac{11}{36}$

- **16.** (d) Required probability $=\frac{64}{64}$.
- **17.** (a) $P(A' \cap B') = \frac{4}{5} \cdot \frac{3}{4} = \frac{3}{5}$.
- **18.** (a) Let E_1 be the event that man will be selected and E_2 the event that woman will be selected. Then

$$P(E_1) = \frac{1}{4}$$
 SO $P(\overline{E}_1) = 1 - \frac{1}{4} = \frac{3}{4}$ and $P(E_2) = \frac{1}{3}$

So
$$P(\overline{E}_2) = \frac{2}{3}$$

Clearly E_1 and E_2 are independent events.

So,
$$P(\overline{E}_1 \cap \overline{E}_2) = P(\overline{E}_1) \times P(\overline{E}_2) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$
.

19. (d) We have $P(\overline{A}) = 0.05 \implies P(A) = 0.95$

Hence the probability that the event will take place in 4 consecutive occasions

$$= \{P(A)\}^4 = (0.95)^4 = 0.81450625$$
.

20. (c) *P* (Tail in 3rd). *P* (Tail in 4th) = $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

21. (b) Required probability is $1 - P(\text{no girl}) = 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$.

22. (b) Required probability $=\frac{52-16}{52}=\frac{36}{52}=\frac{9}{13}$.

23. (a) Required probability

 $= \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5}.$

- 24. (a) It is a fundamental concept.
- **25.** (c) concept
- **26.** (b) A determinant of order 2 is of the form $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$
- 27. (b) Concept.
- **28.** (b) Here $p_1 = \frac{1}{3}$, $p_2 = \frac{2}{7}$ and $p_3 = \frac{3}{8}$ $\Rightarrow q_1 = \frac{2}{3}$, $q_2 = \frac{5}{7}$ and $q_3 = \frac{5}{8}$

29. (b) The probability of husband is not selected = $1 - \frac{1}{7} = \frac{6}{7}$

The probability that wife is not selected $=1-\frac{1}{5}=\frac{4}{5}$

The probability that only husband selected $=\frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

The probability that only wife selected $=\frac{1}{5} \times \frac{6}{7} = \frac{6}{35}$

Hence required probability = $\frac{6}{35} + \frac{4}{35} = \frac{10}{35} = \frac{2}{7}$

30. (a) The required probability

$$= \frac{2}{9} \times \frac{1}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times 1 \times 1 \times 1 = \frac{1}{1260}.$$

31. (c) P(5 or 6 or 7) in one draw $=\frac{3}{7}$

... Probability that in each of 3 draws, the chits bear 5 or 6 or $7 = \left(\frac{3}{7}\right)^3$.

- **32.** (c) The sample space is [LWW, WLW]
 - $\therefore P(LWW) + P(WLW)$

= Probability that in 5 match series, it is India's second win

$$= P(L)P(W)P(W) + P(W)P(L)P(W) = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}.$$

33. (b) Let A denote the event that the stranger succeeds at the k^{th} trial. Then

$$P(A') = \frac{999}{1000} \times \frac{998}{999} \times \dots \times \frac{1000 - k + 1}{1000 - k + 2} \times \frac{1000 - k}{1000 - k + 1}$$
$$\Rightarrow P(A') = \frac{1000 - k}{1000} \Longrightarrow P(A) = 1 - \frac{1000 - k}{1000} = \frac{k}{1000}.$$

34. (d) *A* denote the event getting I;

B denote the event getting II;

c denote the event getting III;

and *D* denote the event getting fail.

Obviously, these four event are mutually exclusive and exhaustive, therefore

 $P(A) + P(B) + P(C) + P(D) = 1 \Longrightarrow P(D) = 1 - 0.95 = 0.05$.

35. (d) concept

- **36.** (c) $P(\text{non-occurrence of } A_i) = 1 \frac{1}{i+1} = \frac{i}{i+1}$
 - \therefore *P* (Non-occurrence of any of events)

$$=\left(\frac{1}{2}\right)\cdot\left(\frac{2}{3}\right)\cdots\cdots\left\{\frac{n}{n+1}\right\}=\frac{1}{n+1}.$$

37. (d) The probability that one test is held $= 2 \times \frac{1}{5} \times \frac{4}{5} = \frac{8}{25}$

Probability that one test is held on both days

$$=\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$$

Thus the probability that the student misses at least one test $=\frac{8}{25}+\frac{1}{25}=\frac{9}{25}$.

- **38.** (b) We have total number of balls = 10
 - \therefore Number of red balls = 3

And number of black balls = 7

And number of balls in the bag = 3 + 7 = 10

... The probability for taking out one red ball out of 10 balls = $\frac{3}{10}$ and the probability for taking out one red ball out of remaining 9 balls = $\frac{2}{9}$

.. Probability for both balls to be red

i.e.,
$$p = \frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$$
.

- **39.** (c) Required combinations are (2, 2, 1), (1, 2, 2), (2, 1, 2), (1, 3, 1,), (3, 1, 1) and (1, 1, 3)
 - \therefore Required probability $=\frac{6}{4^3}=\frac{6}{64}=\frac{3}{32}$.
- **40.** (a) Total number of ways $=2^n$

If head comes odd times, then favourable ways = 2^{n-1} .

- \therefore Required probability $=\frac{2^{n-1}}{2^n}=\frac{1}{2}$.
- **41.** (a) Required probability $=\frac{3}{6}=\frac{1}{2}$.
- **42.** (d) Probability of correct bit appearing is (1 p)

:. Probability of correct number $=(1-p)^{16}$

and hence probability of incorrect number $= 1 - (1 - p)^{16}$.

43. (b) Let 100 students studying in which 60 % girls and 40 % boys.

$$Boys = 40, Girls = 60$$

25% of boys offer Maths = $\frac{25}{100} \times 40 = 10$ Boys

10% of girls offer Maths $=\frac{10}{100} \times 60 = 6$ Girls

It means, 16 students offer Maths.

- \therefore Required probability $=\frac{6}{16}=\frac{3}{8}$.
- **44.** (a) concept.
- **45.** (b) Required probability $=\frac{1}{2}\left(\frac{3}{5}+\frac{2}{6}\right)=\frac{9+5}{30}=\frac{7}{15}$.
- **46.** (a) Probability problem is not solved by $A = 1 \frac{1}{2} = \frac{1}{2}$

Probability problem is not solved by $B = 1 - \frac{1}{3} = \frac{2}{3}$

Probability problem is not solved by $C = 1 - \frac{1}{4} = \frac{3}{4}$

Probability of solving the problem = 1 - P (not solved by any body)

:
$$P = 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$
.

47. (c) Here $P(A) = \frac{3}{4}, P(B) = \frac{4}{5}$

:. Required probability = $P(A).P(\overline{B}) + P(\overline{A}).P(B) = \frac{7}{20}$.

48. (d) For a particular house being selected, Probability = $\frac{1}{3}$

Probability (all the persons apply for the same house)

 $= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) 3 = \frac{1}{9} \bullet$

49. (b) Required probability

$$= \left(\frac{5}{6}\right) \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) + \frac{5}{6} = \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{1 - \left(\frac{5}{6}\right)^2} = \frac{5}{36 - 25} = \frac{5}{11}.$$

50. (c) Total number of digits in any number at the unit place is 10.

$$\therefore n(S) = 10$$

To get the last digit in product is 1, 3, 5, or 7, it is necessary the last digit in each number must be 1, 3, 5 or 7.

$$n(A) = 4, \quad \therefore P(A) = \frac{4}{10} = \frac{2}{5}$$

- : Required probability $=\left(\frac{2}{5}\right)^4 = \frac{16}{625}$.
- **51.** (c) Total number of ways $=\frac{10!}{2!}$

Favourable number of ways for '*I*' come together is 9!

Thus probability that 'I' come together

$$=\frac{9!\times 2!}{10!}=\frac{2}{10}=\frac{1}{5}.$$

Hence required probability $=1-\frac{1}{5}=\frac{4}{5}$

52. (a) Mohan can gets one prize, 2 prizes or 3 prizes and his chance of failure means he get no prize.

Number of total ways $={}^{12}C_3 = 220$

Favourable number of ways to be failure $={}^9C_3 = 84$

Hence required probability $= 1 - \frac{84}{220} = \frac{34}{55}$.

53. (b) Total ways of arrangements $=\frac{8!}{2! \cdot 4!}$

No of arrangements under given conditions = $5\left(\frac{4!}{2!}\right)$

Hence required probability $=\frac{5.4!2!4!}{2!8!}=\frac{1}{14}$.

54. (c) Required probability is 1-P

$$=1 - \frac{{}^{13}C_2}{{}^{25}C_2} = 1 - \frac{13.12}{25.24} = \frac{37}{50}.$$

55. (c) Required probability $=\frac{{}^{26}C_3 \cdot {}^{26}C_3}{{}^{52}C_6}$.

- **56.** (b) Required probability $=\frac{{}^{3}C_{1}\times{}^{3}C_{1}}{{}^{6}C_{2}}=\frac{3\times3}{15}=\frac{3}{5}$.
- **57.** (b) Required probability $=\frac{{}^{7}C_{2}}{{}^{11}C_{2}}=\frac{7 \cdot 6}{11 \cdot 10}=\frac{21}{55}$.
- **58.** (c) Required probability $=\frac{{}^{4}C_{2} \times {}^{6}C_{1}}{{}^{18}C_{3}} = \frac{3}{68}$.
- **59.** (a) Let E_i denote the event that the *i*th object goes to the *i*th place, we have $P(E_i) = \frac{(n-1)!}{n!} = \frac{1}{n}, \forall i$

and
$$P(E_i \cap E_j \cap E_k) = \frac{(n-3)!}{n!}$$
 for $i < j < k$

Since we can choose 3 places out of n in ${}^{n}C_{3}$ ways.

The probability of the required event is ${}^{n}C_{3} \cdot \frac{(n-3)!}{n!} = \frac{1}{6}$.

60. (a) In 50 tickets 14 are of prize and 36 are blank. Number of ways both the tickets are blank $={}^{36}C_2$

Thus the probability of not winning the prize $=\frac{{}^{36}C_2}{{}^{50}C_2}=\frac{18}{35}$.

Hence probability of winning the prize $=1-\frac{18}{35}=\frac{17}{35}$.

61. (c) Total number of triangles which can be formed is equal to ${}^{6}C_{3} = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

Number of equilateral triangles = 2

:. Required probability $=\frac{2}{20}=\frac{1}{10}$.

62. (a) Total number of ways = n!. Favourable cases = 2(n-1)!

Hence required probability $=\frac{2(n-1)!}{n!}=\frac{2}{n}$.

- **63.** (a) Required probability $=\frac{{}^{3}C_{1}}{{}^{7}C_{1}}\times\frac{{}^{2}C_{1}}{{}^{6}C_{1}}=\frac{1}{7}$.
- 64. (c) Number of ways of selecting two good mangoes = ${}^{6}C_{2} = 15$. Number of ways that at least one of the two selected mangoes is to be good = ${}^{6}C_{1} \times {}^{9}C_{1} = 54$

- \therefore Required probability $=\frac{15}{64}=\frac{5}{18}$.
- 65. (b) Four boys can be arranged in 4! ways and three girls can be arranged in 3! ways.
 - \therefore The favourable cases = 4!×3!

Hence the required probability $\frac{=4!\times3!}{7!} = \frac{6}{7\times6\times5} = \frac{1}{35}$.

66. (c) The total number of functions from A to itself is n^n and the total number of bijections from A to itself is n!. {Since A is a finite set, therefore every injective map from A to itself is bijective also}.

:. The required probability $=\frac{n!}{n^n}=\frac{(n-1)!}{n^{n-1}}$.

67. (a) Total number of ways $= {}^{15}C_{11}$

Favourable cases $= {}^{8}C_{6} \times {}^{7}C_{5}$

Required probability = $\frac{{}^{8}C_{6} \times {}^{7}C_{5}}{{}^{15}C_{11}}$.

- **68.** (b) standard problem
- **69.** (a) We have the following three pattern :
 - (i) Red, white $P(A) = \frac{3 \times 4}{{}^{12}C_2}$
 - (ii) Red, blue $P(B) = \frac{3 \times 5}{1^2 C_2}$
 - (iii) Blue, white $P(C) = \frac{4 \times 5}{\frac{12}{C}}$

Since all these cases are exclusive, so the required probability $=\frac{(12+15+20)}{{}^{12}C_2}=\frac{(47\times2)}{(12\times11)}=\frac{47}{66}$.

- **70.** (d) $P(\text{Black or Red}) = \frac{{}^{5}C_{1} + {}^{3}C_{1}}{{}^{12}C_{1}} = \frac{2}{3}$.
- 71. (b) 6 boys and 6 girls can be arranged in a row in 12 ! ways. If all the 6 girls are together, then the number of arrangement are 7 !×6 !.

Hence required probability = $\frac{7!.6!}{12!}$

 $^{=\}frac{6\times5\times4\times3\times2}{12\times11\times10\times9\times8}=\frac{1}{132}.$

- 72. (d) Let each of the friend have x daughters. Then the probability that all the tickets go to the daughters of A is $\frac{{}^{x}C_{3}}{{}^{2x}C_{3}}$. Therefore $\frac{{}^{x}C_{3}}{{}^{2x}C_{3}} = \frac{1}{20} \Rightarrow x = 3$.
- 73. (d) Here only 2 rectangles are formed ADEH, GFCB.
 - \therefore Number of favourable cases = 2

and total number of cases $={}^{8}C_{4}$

.: Required probability

$$= \frac{\frac{2}{{}^{8}C_{4}}}{=} = \frac{1}{35} .$$

$$= \frac{{}^{2}C_{1} \times {}^{3}C_{1} \times {}^{4}C_{1}}{{}^{9}C_{3}} = \frac{2 \times 3 \times 4}{\left(\frac{9 \times 8 \times 7}{3 \times 2}\right)} = \frac{2}{7} .$$



74. (a) Total number of cases obtained by taking multiplication of only two numbers out of $100 = {}^{100}C_2$. Out of hundred (1, 2,...., 100) given numbers, there are the numbers 3, 6, 9, 12,..., 99, which are 33 in number such that when any one of these is multiplied with any one of remaining 67 numbers or any two of these 33 are multiplied, then the resulting products is divisible by 3. Then the number of numbers which are the products of two of the given number are divisible by $3={}^{33}C_1\times{}^{67}C_1+{}^{33}C_2$. Hence the required probability

$$=\frac{{}^{33}C_1\times{}^{67}C_1+{}^{33}C_2}{{}^{100}C_2}=\frac{2739}{4950}=0.55.$$

- 75. (b) As we know the total number of mappings is n^m and number of injective mappings is $\frac{n!}{(n-m)!n^m}$.
- **76.** (a) Out of 9 socks, 2 can be drawn in ${}^{9}C_{2}$ ways.

Two socks drawn from the drawer will match if either both are brown of both are blue. Therefore favourable number of cases is ${}^{5}C_{2} + {}^{4}C_{2}$.

Hence the required probability $=\frac{{}^{5}C_{2}+{}^{4}C_{2}}{{}^{9}C_{2}}=\frac{4}{9}$.

77. (d) Let A occupy any seat

at the round table. Then there are 14 seats available for *B*.

If there are to be four persons between A and B.

Then *B* has only two ways to sit, as show in the fig.

Hence required probability $=\frac{2}{14}=\frac{1}{7}$.

- 78. (a) standard problem
- **79.** (c) The probability of solving the question by these three students are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively.

 $P(A) = \frac{1}{3}; P(B) = \frac{2}{7}; P(C) = \frac{3}{8}$

Then probability of question solved by only one student

 $= P(A \ \overline{B} \ \overline{C} \ \text{ or } \overline{A} \ B \ \overline{C} \ \text{ or } \overline{A} \ \overline{B} \ C)$

- **80.** (d) $P(A+B) = P(A) + P(B) P(AB) = \frac{1}{4} + \frac{1}{4} 0 = \frac{1}{2}$.
- **81.** (d) Required probability $=1-\frac{3}{8}=\frac{5}{8}$.
- 82. (a) We have P(A+B) = P(A) + P(B) P(AB)

$$\Rightarrow \frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3} \Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$$

- Thus, $P(A) \cdot P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} = P(AB)$
- 83. (b) Required probability $=\frac{6}{6+5}=\frac{6}{11}$
- **84.** (b) Probabilities of winning the race by three horses are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$.

Hence required probability $=\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{47}{60}$.

85. (a) Since we have

$$P(A \cup B) + P(A \cap B) = P(A) + P(B) = P(A) + \frac{P(A)}{2}$$

$$\Rightarrow \frac{7}{8} = \frac{3P(A)}{2} \Rightarrow P(A) = \frac{7}{12}.$$

86. (a) $P(A \cap B) = \frac{2}{8} + \frac{5}{8} - \frac{6}{8} = \frac{1}{8}$.

87. (b) Required probability = A occurs and B does not occur or B occurs and A does not occur

$$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$$
$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - 2P(A \cap B).$$

88. (a) Since $P(A \cap \overline{B}) + P(A \cap B) = P(A)$

$$\Rightarrow P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.25 - 0.15 = 0.1$$

B)

89. (c) $1 - P(A' \cap B') = 0.6$, $P(A \cap B) = 0.3$, then

 $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

$$\implies 1 - P(A \cap B) = P(A') + P(B') - 0.4$$

 $\Longrightarrow P(A') + P(B') = 0.7 + 0.4 = 1.1.$

90. (a) Since events are independent.

So,
$$P(A \cap B') = P(A) \times P(B') = \frac{3}{25}$$

 $\Rightarrow P(A) \times \{1 - 2P(B)\} = \frac{3}{25}$ (i)

Similarly, $P(B) \times \{1 - P(A)\} = \frac{8}{25}$ (ii)

On solving (i) and (ii), we get $P(A) = \frac{1}{5}$ and $\frac{3}{5}$.

91. (b) Here P(X) = 0.3; P(Y) = 0.2

Now $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

Since these are independent events, so

$$P(X \cap Y) = P(X). P(Y)$$

Thus required probability = 0.3 + 0.2 - 0.06 = 0.44.

92. (c) We are given that
$$P(A \cup B) = 0.6$$
 and $P(A \cap B) = 0.2$.

We know that if A and B are any two events, then

 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.6 = 1 - P(\overline{A}) + 1 - P(\overline{B}) - 0.2$$

 $\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - 0.8 = 1.2.$

93. (c) $P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$

 $= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$

94. (b) $0.8 = 0.3 + x - 0.3x \Rightarrow x = 5/7$.

95. (a)
$$P(\overline{A \cup B}) = \frac{1}{6}$$
; $P(A \cap B) = \frac{1}{4}$,
 $P(\overline{A}) = \frac{1}{4} \Rightarrow P(A) = \frac{3}{4}$,
 $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B)$
96. (a) $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$
 $P(\overline{A}) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$

 $\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$\frac{1}{4} = \frac{1}{3} + P(B) - \frac{3}{4} \Longrightarrow P(B) = \frac{2}{3}$$
$$P(\overline{A} \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{8 - 3}{12} = \frac{5}{12}.$$

97. (b) $E = \{x \text{ is a prime number}\}$

P(E) = P(2) + P(3) + P(5) + P(7) = 0.62,

 $F = \{x < 4\}, P(F) = P(1) + P(2) + P(3) = 0.50$

and $P(E \cap F) = P(2) + P(3) = 0.35$

 $\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$= 0.62 + 0.50 - 0.35 = 0.77.$$

98. d)
$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.8 = 0.2$$

$$P(A'\cup B') = 1 - P(A \cap B) = 1 - 0.3 = 0.7$$

 $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

 $\implies 0.7 = P(A') + P(B') - 0.2 \implies P(A') + P(B') = 0.9 \ .$

99. (d)
$$P(E \cap F) = P(E). P(F)$$

Now, $P(E \cap F^c) = P(E) - P(E \cap F) = P(E)[1 - P(F)] = P(E).P(F^c)$

and $P(E^{c} \cap F^{c}) = 1 - P(E \cup F) = 1 - [P(E) + P(F) - P(E \cap F)]$

 $= [1 - P(E)][1 - P(F)] = P(E^{c})P(F^{c})$

Also
$$P(E/F) = P(E)$$
 and $P(E^c/F^c) = P(E^c)$

 $\Rightarrow P(E/F) + P(E^c/F^c) = 1.$

100. (d) P(A | B) = P(A) as independent event $=\frac{1}{2}$.

$$P\{A/(A \cup B)\} = \frac{P[A \cap (A \cup B)]}{P(A \cup B)}$$

{Since $A \cap (A \cup B) = A \cap [A - B - A \cap B]$
 $= A - A \cap B - A \cap B = a\}$
 $\Rightarrow P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)} = \frac{\frac{1}{2}}{\frac{1}{2} - \frac{1}{5} - \frac{1}{10}} = \frac{\frac{1}{2}}{\frac{6}{10}} = \frac{5}{6}$
and similarly $P\left(\frac{A \cap B}{A' \cup B'}\right)$.
101. (c) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2}$.

102. (a)
$$P\left(\frac{\overline{B}}{\overline{A}}\right) = \frac{1 - P(A \cup B)}{P(\overline{A})} = \frac{1 - \frac{23}{60}}{1 - \frac{1}{3}} = \frac{37}{60} \times \frac{3}{2} = \frac{37}{40}.$$

103. (b) We define the following events :

 A_1 : Selecting a pair of consecutive letter from the word LONDON.

 A_2 : Selecting a pair of consecutive letters from the word CLIFTON.

E : Selecting a pair of letters 'ON'.

Then $P(A_1 \cap E) = \frac{2}{5}$; as there are 5 pairs of consecutive letters out of which 2 are ON.

 $P(A_2 \cap E) = \frac{1}{6}$; as there are 6 pairs of consecutive letters of which one is ON.

... The required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}} = \frac{12}{17}$$

104. (c)
$$P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})}.$$

105. (c) $P[B/(A \cup B^c)] = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$

$$= \frac{P(A \cap B)}{P(A) + P(B^{c}) - P(A \cap B^{c})}$$
$$= \frac{P(A) - P(A \cap B^{c})}{P(A) + P(B^{c}) - P(A \cap B^{c})} = \frac{0.7 - 0.5}{0.8} = \frac{1}{4}.$$

106. (c) Since $P(A \cap B) = P(A)P(B)$

It means *A* and *B* are independent events so A^c and B^c will also be independent. Hence $P(A \cup B)^c = P(A^c \cap B^c) = P(A^c)P(B^c)$ (Demorgan's law)

As A is independent of B, hence

 $P(A / B) = P(A), \qquad \{:: P(A \cap B) = P(B)P(A / B)\}.$

107. (d) Let E_1 be the event that the ball is drawn from bag A, E_2 the event that it is drawn from bag *B* and *E* that the ball is red. We have to find $P(E_2 / E)$.

Since both the bags are equally likely to be selected, we have $P(E_1) = P(E_2) = \frac{1}{2}$

Also $P(E/E_1) = 3/5$ and $P(E/E_2) = 5/9$.

Hence by Bay's theorem, we have

$$P(E_2 / E) = \frac{P(E_2)P(E / E_2)}{P(E_1)P(E / E_1) + P(E_2)P(E / E_2)}$$

$$=\frac{\frac{1}{2}\cdot\frac{5}{9}}{\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{5}{9}}=\frac{25}{52}$$

108. (b) apply baye's theorem

109. (d)
$$P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{1}{2} \Rightarrow P(B \cap A) = \frac{1}{8}$$

 $P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \Rightarrow P(B) = \frac{1}{2}$
 $P(A \cap B) = \frac{1}{8} = P(A) \cdot P(B)$

 \therefore Events A and B are independent.

Now,
$$P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(A')P(B)}{P(B)} = \frac{3}{4}$$

and
$$P\left(\frac{B'}{A'}\right) = \frac{P(B' \cap A')}{P(A')} = \frac{P(B')P(A')}{P(A')} = \frac{1}{2}$$

110. (b) Event (Y = 0) is $\{00, 01, 09, 10, 20, \dots, 90\}$

Also $(X = 9) \cap (Y = 0) = 09, 90$, we have

$$P(Y=0) = \frac{19}{100}$$
 and $P(X=9) \cap (Y=0) = \frac{2}{100}$

Hence required probability

$$= P\{(X=9)/(Y=0)\} = \frac{\{P(X=9) \cap (Y=0)\}}{P(Y=0)} = \frac{2}{19}$$

111. (a)
$$P(E/F) + P(\overline{E}/F) = \frac{P(E \cap F) + P(\overline{E} \cap F)}{P(F)}$$

 $=\frac{P\{(E\cap F)\cup (\overline{E}\cap F)\}}{P(F)}$

[$: E \cap F$ and $\overline{E} \cap F$ are disjoint]

$$=\frac{P\{(E\cup\overline{E})\cap F\}}{P(F)}=\frac{P(F)}{P(F)}=1$$

Similarly we can show that (b) and (c) are not true while (d) is true.

$$P\left(\frac{E}{\overline{F}}\right) + P\left(\frac{\overline{E}}{\overline{F}}\right) = \frac{P(E \cap \overline{F})}{P(F)} + \frac{P(\overline{E} \cap \overline{F})}{P(F)} = \frac{P(\overline{F})}{P(\overline{F})} = 1$$

112. (c)Let *A* be the event of selecting bag *X*, *B* be the event of selecting bag *Y* and *E* be the event of drawing a white ball, then P(A)=1/2, P(B)=1/2, P(E/A)=2/5 P(E/B)=4/6=2/3.

 $P(E) = P(A)P(E / A) + P(B)P(E / B) = \frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{2}{3} = \frac{8}{15}.$

113. (a) Here the least number of draws to obtain 2 aces are 2 and the maximum number is 50 thus

n can take value from 2 to 50.

Since we have to make *n* draws for getting two aces, in (n-1) draws, we get any one of the 4 aces and in the *n*th draw we get one ace. Hence the required probability

$$= \frac{{}^{4}C_{1} \times {}^{48}C_{n-2}}{{}^{52}C_{n-1}} \times \frac{3}{52 - (n-1)}$$
$$= \frac{4 \times (48)!}{(n-2)!(48 - n+2)!} \times \frac{(n-1)!(52 - n+1)!}{(52)!} \times \frac{3}{52 - n+1}$$
$$= \frac{(n-1)(52 - n)(51 - n)}{50 \times 49 \times 17 \times 13} \quad \text{(on simplification).}$$

114. (a) Standard problem.

115. (b) This is a problem of without replacement.

 $P = \frac{\text{one def. from 2 def.}}{\text{any one from 4}} \times \frac{1 \text{ def. from remaining 1 def.}}{\text{any one from remaining 3}}$

Hence required probability $=\frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

116. (d) Let $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.2$ and $p_4 = 0.1$

P (the gun hits the plane) = P (the plane is hit in once)

=1-P (the plane is hit in none of the shots)

 $= 1 - (1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4) = 0.6976 .$

117. (b) Required probability = probability that either the number is 7 or the number is 8.

i.e., Required Probability = $P_7 + P_8$

Now
$$P_7 = \frac{1}{2} \cdot \frac{1}{11} + \frac{1}{2} \cdot \frac{6}{36} = \frac{1}{2} \left(\frac{1}{11} + \frac{1}{6} \right)$$

 $P_8 = \frac{1}{2} \cdot \frac{1}{11} + \frac{1}{2} \cdot \frac{5}{36} = \frac{1}{2} \left(\frac{1}{11} + \frac{5}{36} \right)$
 $\therefore P = \frac{1}{2} \left(\frac{2}{11} + \frac{11}{36} \right) = 0.244.$

118. (d) $P(E) \le P(F) \Rightarrow n(E) \le n(F)$

$$P(E \cap F) > 0 \Longrightarrow E \cap F \neq \phi$$

These do not mean that *E* is a sub-set of *F* or *F* is a sub-set of *E.i.e.*, $E \subseteq F$ or $F \subseteq E$ or $\overline{E} \subseteq \overline{F}$.

119. (c) The last digit of the product will be 1,2,3,4,6,7,8 or 9 if and only if each of the *n* positive integers ends in any of these digits. Now the probability of an integer ending in 1,2,3,4,6,7,8 or 9 is $\frac{8}{10}$. Therefore the probability that the last digit of the product of *n* integers in 1,2,3,4,6,7,8 or 9 is $\left(\frac{4}{5}\right)^n$. The probability for an integer to end in 1,3,7 or 9 is $\frac{4}{10} = \frac{2}{5}$.

Therefore the probability for the product of *n* positive integers to end in 1,3,7 or 9 is $\left(\frac{2}{5}\right)^n$.

Hence the required probability $=\left(\frac{4}{5}\right)^n - \left(\frac{2}{5}\right)^n = \frac{4^n - 2^n}{5^n}$.

120. (a) *P*(minimum face value not less than 2 and maximum face value is not greater than 5)

 $= P(2 \text{ or } 3 \text{ or } 4 \text{ or } 5) = \frac{4}{6} = \frac{2}{3}$

Hence required probability $={}^{4}C_{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{0}=\frac{16}{81}$.

121. (d) Concept

122. (d) Let the first number be x and second is y.

Let *A* denotes the event that the difference between the first and second number is at least *m*. Let E_x denote the event that the first number chosen is *x*, we must have $x - y \ge m$ or $y \le x - m$. Therefore x > m and y < n - m. Thus $P(E_x) = 0$ for $0 < x \le m$ and $P(E_x) = \frac{1}{n}$ for $m < x \le n$.

Also $P(A/E_x) = \frac{(x-m)}{(n-1)}$

Therefore, $P(A) = \sum_{x=1}^{n} P(E_x) P(A / E_x)$

$$= \sum_{x=m+1}^{n} P(E_x) P(A/E_x) = \sum_{x=m+1}^{n} \frac{1}{n} \cdot \frac{x-m}{n-1}$$
$$= \frac{1}{n(n-1)} [1+2+3+\dots+(n-m)]$$
$$= \frac{(n-m)(n-m+1)}{2n(n-1)}.$$

123. (b) standard problem

124. (b) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Since *A* and *B* are mutually exclusive, so

 $P(A \cup B) = P(A) + P(B)$

Hence required probability = 1 - (0.5 + 0.3) = 0.2.

125. (c)
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

 $\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cap B)$
 $\Rightarrow 2P(A \cap B) = P(A) + P(B)$
 $\Rightarrow 2P(A) \cdot \frac{P(A \cap B)}{P(A)} = P(A) + P(B)$
 $\Rightarrow 2P(A) \cdot P\left(\frac{B}{A}\right) = P(A) + P(B)$.