PERMUTATIONS AND COMBINATIONS

OBJECTIVE PROBLEMS

1. There are 4 parcels and 5 post-offices. In how many different ways the registration of parcel can be made

(a) 20 (b) 4^5

(c) 5^4 (d) $5^4 - 4^5$

2. In how many ways n books can be arranged in a row so that two specified books are not together

(a) $n! - (n-2)!$ (b) $(n-1)!(n-2)$

(c) $n! - 2(n-1)$ (d) $(n-2)n!$

3. In how many ways can 10 true-false questions be replied

(a) 20 (b) 100

(c)512 (d)1024

4. There are 5 roads leading to a town from a village. The number of different ways in which a villager can go to the town and return back, is

(a) 25 (b) 20

(c) 10 (d) 5

5. Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to the number of heads is

(a) 20 (b) 9

(c) 120 (d) 40

6. Assuming that no two consecutive digits are same, the number of *n* digit numbers, is

(a) $n!$ (b) 9 !

(c) 9^n (d) n^9

7. The sum of all 4 digit numbers that can be formed by using the digits 2, 4, 6, 8 (repetition of digits not allowed) is

(a) 133320 (b) 533280

(c) 53328 (d) None of these

8. The number of numbers that can be formed with the help of the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places, is

(a) 24 (b) 18

(c) 12 (d) 30

9. The number of words which can be formed from the letters of the word MAXIMUM, if two consonants cannot occur together, is

(a) 4 ! (b) $3! \times 4!$

(c) 7 ! (d) None of these

10. The value of ^nP_r is equal to

(a) $^{n-1}P_r + r\,^{n-1}P_{r-1}$ (b) $n.\,^{n-1}P_r + ^{n-1}P_{r-1}$

(c) $n(^{n-1}P_r + ^{n-1}P_{r-1})$ (d) $^{n-1}P_{r-1} + ^{n-1}P_r$

11. Numbers greater than 1000 but not greater than 4000 which can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is allowed), are

(a) 350 (b) 375

(c) 450 (d) 576

12. In how many ways can 10 balls be divided between two boys, one receiving two and the other eight balls

(a) 45 (b) 75

(c) 90 (d) None of these

13. There are 4 parcels and 5 post-offices. In how many different ways the registration of parcel can be made

(a) 20 (b) 4^5

(c) 5^4 (d) $5^4 - 4^5$

14. How many words can be made from the letters of the word COMMITTEE

(a) $\frac{9!}{(2!)^2}$ (b) $\frac{9!}{(2!)^3}$

(c) $\frac{9!}{2!}$ (d) $9!$

15. In how many ways can 5 boys and 3 girls sit in a row so that no two girls are together

(a) $5! \times 3!$ (b) $^4P_3 \times 5!$

(c) $^6P_3 \times 5!$ (d) $^5P_3 \times 3!$

16. All the letters of the word 'EAMCET' are arranged in all possible ways. The number of such arrangements in which two vowels are not adjacent to each other is

(a) 360 (b) 114

(c) 72 (d) 54

17. How many numbers can be made with the digits 3, 4, 5, 6, 7, 8 lying between 3000 and 4000 which are divisible by 5 while repetition of any digit is not allowed in any number

(a) 60 (b) 12

(c) 120 (d) 24

18. How many numbers consisting of 5 digits can be formed in which the digits 3, 4 and 7 are used only once and the digit 5 is used twice

(a) 30 (b) 60

(c) 45 (d) 90

19. In how many ways 3 letters can be posted in 4 letter-boxes, if all the letters are not posted in the same letter-box

(a) 63 (b) 60

(c) 77 (d) 81

20. The total number of permutations of the letters of the word "BANANA" is

(a) 60 (b) 120

(c) 720 (d) 24

21. How many words can be formed with the letters of the word MATHEMATICS by rearranging them

(a) $\frac{11!}{2!2!}$ (b) $\frac{11!}{2!}$

(c) $\frac{11!}{2!2!2!}$ (d) $11!$

22. How many numbers less than 1000 can be made from the digits 1, 2, 3, 4, 5, 6 (repetition is not allowed)

(a) 156 (b) 160

(c) 150 (d) None of these

23. How many numbers greater than hundred and divisible by 5 can be made from the digits 3, 4, 5, 6, if no digit is repeated

(a) 6 (b) 12 (c) 24 (d) 30

24. If a denotes the number of permutations of $x+2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x-11$ things taken all at a time such that $a = 182\ bc$, then the value of x is

(a) 15 (b) 12

(c) 10 (d) 18

25. The product of any r consecutive natural numbers is always divisible by

(a) $r!$ (b) r^2

(c) r^n (d) None of these

26. The number of ways in which ten candidates $A_1, A_2, ...A_{10}$ can be ranked such that A_1 is always above A_{10} is

(a) $5!$ (b) $2(5!)$

(c) $10!$ (d) $\frac{1}{2}(10!)$

27. How many numbers lying between 999 and 10000 can be formed with the help of the digit 0,2,3,6,7,8 when the digits are not to be repeated

(a) 100 (b) 200

(c) 300 (d) 400

28. The number of 4 digit even numbers that can be formed using 0, 1, 2, 3, 4, 5, 6 without repetition is

(a) 120 (b) 300

(c) 420 (d) 20

29. The number of words that can be formed out of the letters of the word ARTICLE so that the vowels occupy even places is

(a) 36 (b) 574

(c) 144 (d) 754

30. If the letters of the word *SACHIN* arranged in all possible ways and these words are written out as in dictionary, then the word *SACHIN* appears at serial number

(a) 603 (b) 602

(c) 601 (d) 600

31. The number of arrangements of the letters of the word BANANA in which two N's do not appear adjacently is

(a) 40 (b) 60

(c) 80 (d) 100

32. If a man and his wife enter in a bus, in which five seats are vacant, then the number of different ways in which they can be seated is

(a) 2 (b) 5

(c) 20 (d) 40

33. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are

(a) 216 (b) 375

(c) 400 (d) 720

34. Let the eleven letters A, B .....,K denote an arbitrary permutation of the integers (1, 2,.....11), then $(A-1)(B-2)(C-3).....(K-11)$

(a) Necessarily zero (b) Always odd

(c) Always even (d) None of these

35. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(a) $6! \times 5!$ (b) 30

(c) $5! \times 4!$ (d) $7! \times 5!$

36. If eleven members of a committee sit at a round table so that the President and Secretary always sit together, then the number of arrangements is

(a) $10! \times 2$ (b) $10!$

(c) $9! \times 2$ (d) None of these

37. The number of ways in which 5 beads of different colours form a necklace is

(a) 12 (b) 24

(c) 120 (d) 60

38. In how many ways 7 men and 7 women can be seated around a round table such that no two women can sit together

(a) $(7!)^2$ (b) $7! \times 6!$

(c) $(6!)^2$ (d) $7!$

39. In how many ways a garland can be made from exactly 10 flowers

(a) $10\,!$ (b) $9\,!$

(c) $2(9\,!)$ (d) $\frac{9\,!}{2}$

40. In how many ways can 5 boys and 5 girls sit in a circle so that no two boys sit together

(a) $5! \times 5!$ (b) $4! \times 5!$

(c) $\frac{5\,! \times 5\,!}{2}$ (d) None of these

41. The number of ways that 8 beads of different colours be string as a necklace is

(a) 2520 (b) 2880

(c) 5040 (d) 4320

42. In how many ways can 12 gentlemen sit around a round table so that three specified gentlemen are always together

(a) 9 ! (b) 10 !

(c) 3 !10 ! (d) 3 !9 !

43. The number of ways in which 5 male and 2 female members of a committee can be seated around a round table so that the two female are not seated together is

(a) 480 (b) 600

(c) 720 (d) 840

44. $^nC_r + ^nC_{r-1}$ **is equal to**

(a) $^{n+1}C_r$ (b) $^nC_{r+1}$

(c) $^{n+1}C_{r+1}$ (d) $^{n-1}C_{r-1}$

45. A man has 7 friends. In how many ways he can invite one or more of them for a tea party

(a) 128 (b) 256

(c) 127 (d) 130

46. If $^nC_{r-1} = 36, ^nC_r = 84$ **and** $^nC_{r+1} = 126$ **, then the value of** r **is**

(a) 1 (b) 2

(c) 3 (d) None of these

47. Everybody in a room shakes hand with everybody else. The total number of hand shakes is 66. The total number of persons in the room is

(a) 11 (b) 12 (c) 13 (d) 14

48. If n is even and the value of $^{n}C_{r}$ is maximum, then $r =$

(a) $\frac{n}{2}$ (b) $\frac{n+1}{2}$

(c) $\frac{n-1}{2}$ (d) None of these

49. If $^{2n}C_3 : {}^{n}C_2 = 44 : 3$, then for which of the following values of r, the value of $^{n}C_{r}$ will be 15

(a) $r = 3$ (b) $r = 4$

(c) $r = 6$ (d) $r = 5$

50. If $^{15}C_{3r} = {}^{15}C_{r+3}$, then the value of r is

(a) 3 (b) 4

(c) 5 (d) 8

51. In an election there are 8 candidates, out of which 5 are to be choosen. If a voter may vote for any number of candidates but not greater than the number to be choosen, then in how many ways can a voter vote

(a) 216 (b) 114

(c) 218 (d) None of these

52. In a city no two persons have identical set of teeth and there is no person without a tooth. Also no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, then the maximum population of the city is

(a) 2^{32} (b) $(32)^2 - 1$

(c) $2^{32} - 1$ (d) 2^{32-1}

53. How many words can be formed by taking 3 consonants and 2 vowels out of 5 consonants and 4 vowels

(a) $^{5}C_3 \times {}^{4}C_2$ (b) $\frac{^{5}C_3 \times {}^{4}C_2}{5}$

(c) $^{5}C_3 \times {}^{4}C_3$ (d) $(^{5}C_3 \times {}^{4}C_2)(5)!$

54. There are 15 persons in a party and each person shake hand with another, then total number of handshakes is

(a) $^{15}P_2$ (b) $^{15}C_2$ (c) $15!$ (d) $2(15!)$

55. There are 15 persons in a party and each person shake hand with another, then total number of handshakes is

(a) ${}^{15}P_2$ (b) ${}^{15}C_2$

(c) $15!$ (d) $2(15!)$

56. If ${}^{n}C_r = 84, {}^{n}C_{r-1} = 36$ **and** ${}^{n}C_{r+1} = 126$ **, then** n **equals**

(a) 8 (b) 9

(c) 10 (d) 5

57. In an election the number of candidates is 1 greater than the persons to be elected. If a voter can vote in 254 ways, then the number of candidates is

(a) 7 (b) 10

(c) 8 (d) 6

58. If ${}^{n+1}C_3 = 2\,{}^{n}C_2$, **then** n **=**

(a) 3 (b) 4

(c) 5 (d) 6

59. In an election there are 5 candidates and three vacancies. A voter can vote maximum to three candidates, then in how many ways can he vote

(a) 125 (b) 60

(c) 10 (d) 25

60. Six '+' and four '–' signs are to be placed in a straight line so that no two '–' signs come together, then the total number of ways are

(a) 15 (b) 18

(c) 35 (d) 42

61. The number of ways of dividing 52 cards amongst four players equally, are

(a) $\frac{52!}{(13!)^4}$ (b) $\frac{52!}{(13!)^2\ 4!}$

(c) $\frac{52!}{(12!)^4\ (4!)}$ (d) None of these

62. Out of 10 white, 9 black and 7 red balls, the number of ways in which selection of one or more balls can be made, is

(a) 881 (b) 891

(c) 879 (d) 892

63. A total number of words which can be formed out of the letters a, b, c, d, e, f taken 3 together such that each word contains at least one vowel, is

(a) 72 (b) 48

(c) 96 (d) None of these

64. Out of 6 books, in how many ways can a set of one or more books be chosen

(a) 64 (b) 63

(c) 62 (d) 65

65. All possible two factors products are formed from numbers 1, 2, 3, 4, ..., 200. The number of factors out of the total obtained which are multiples of 5 is

(a) 5040 (b) 7180

(c) 8150 (d) None of these

66. The numbers of permutations of n things taken r at a time, when p things are always included, is

(a) ${}^{n}C_r\ p!$ (b) ${}^{n-p}C_r\ r!$

(c) ${}^{n-p}C_{r-p}\ r!$ (d) None of these

67. In a touring cricket team there are 16 players in all including 5 bowlers and 2 wicket-keepers. How many teams of 11 players from these, can be chosen, so as to include three bowlers and one wicket-keeper

(a) 650 (b) 720

(c) 750 (d) 800

68. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(a) ${}^{56}C_3$ (b) ${}^{56}C_4$

(c) ${}^{55}C_4$ (d) ${}^{55}C_3$

69. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five question. The number of choices available to him is

(a) 140 (b) 196

(c) 280 (d) 346

70. **If** ${}^{n}C_{r}$ **denotes the number of combinations of** ***n*** **things taken** ***r*** **at a time, then the expression** ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2 \times {}^{n}C_{r}$ **equals**

(a) ${}^{n+2}C_{r}$ (b) ${}^{n+2}C_{r+1}$ (c) ${}^{n+1}C_{r}$ (d) ${}^{n+1}C_{r+1}$

71. **A father with 8 children takes them 3 at a time to the Zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is**

(a) 336 (b) 112

(c) 56 (d) None of these

72. ${}^{n-1}C_{r} = (k^2 - 3).{}^{n}C_{r+1}$ **if** $k \in$

(a) $[-\sqrt{3}, \sqrt{3}]$ (b) $(-\infty, -2)$

(c) $(2, \infty)$ (d) $(\sqrt{3}, 2)$

73. **The number of ways in which thirty five apples can be distributed among 3 boys so that each can have any number of apples, is**

(a) 1332 (b) 666

(c) 333 (d) None of these

74. **A person is permitted to select at least one and at most** ***n*** **coins from a collection of** $(2n+1)$ **distinct coins. If the total number of ways in which he can select coins is 255, then** ***n*** **equals**

a) 4 (b) 8

(c) 16 (d) 32

75. **The number of ways in which four letters of the word 'MATHEMATICS' can be arranged is given by**

(a) 136 (b) 192

(c) 1680 (d) 2454

76. **In how many ways can 5 red and 4 white balls be drawn from a bag containing 10 red and 8 white balls**

(a) ${}^{8}C_{5} \times {}^{10}C_{4}$ (b) ${}^{10}C_{5} \times {}^{8}C_{4}$

(c) ${}^{18}C_{9}$ (d) None of these

77. A student is allowed to select at most n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select one book is 63, then the value of n is

(a) 2 (b) 3

(c) 4 (d) None of these

78. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21,$ then n equals

(a) 5 (b) 7

(c) 6 (d) 4

79. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total number of m points are taken on I_1, n points on I_2, k points on I_3. The maximum number of triangles formed with vertices at these points are

(a) ${}^{m+n+k}C_3$ (b) ${}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$

(c) ${}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3$ (d) None of these

80. If a polygon has 44 diagonals, then the number of its sides are

(a) 7 (b) 11

(c) 8 d) None of these

81. The number of triangles that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line, is

(a) 185 (b) 175

(c) 115 (d) 105

82. There are m points on a straight line AB and n points on another line AC, none of them being the point A. Triangles are formed from these points as vertices when (i) A is excluded (ii) A is included. Then the ratio of the number of triangles in the two cases is

(a) $\frac{m+n-2}{m+n}$ (b) $\frac{m+n-2}{2}$

(c) $\frac{m+n-2}{m+n+2}$ (d) None of these

83. The greatest possible number of points of intersection of 8 straight lines and 4 circles is

(a) 32 (b) 64

(c) 76 (d) 104

84. There are n straight lines in a plane, no two of which are parallel and no three pass through the same point. Their points of intersection are joined. Then the number of fresh lines thus obtained is

(a) $\frac{n(n-1)(n-2)}{8}$ (b) $\frac{n(n-1)(n-2)(n-3)}{6}$

(c) $\frac{n(n-1)(n-2)(n-3)}{8}$ (d) None of these

85. A parallelogram is cut by two sets of m lines parallel to its sides. The number of parallelograms thus formed is

(a) $({}^mC_2)^2$ (b) $({}^{m+1}C_2)^2$

(c) $({}^{m+2}C_2)^2$ (d) None of these

86. Out of 18 points in a plane, no three are in the same straight line except five points which are collinear. The number of (i) straight lines, (ii) triangles which can be formed by joining them is

(i) (a) 140 (b) 142 (c) 144 (d) 146

(ii) (a) 816 (b) 806 (c) 800 (d) 750

87. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is

(a) 6 (b) 18

(c) 12 (d) 9

88. Out of 10 points in a plane 6 are in a straight line. The number of triangles formed by joining these points are

(a) 100 (b) 150

(c) 120 (d) None of these

89. Out of 10 points in a plane 6 are in a straight line. The number of triangles formed by joining these points are

(a) 100 (b) 150

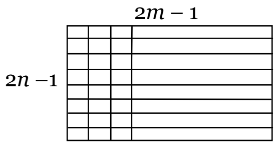
(c) 120 (d) None of these

90. Given six line segments of lengths 2, 3, 4, 5, 6, 7 units, the number of triangles that can be formed by these lines is

(a) ${}^6C_3-7$ (b) ${}^6C_3-6$

(c) ${}^6C_3-5$ (d) ${}^6C_3-4$

91. **There is a rectangular sheet of dimension** $(2m-1)\times(2n-1)$**, (where** $m>0, n>0$)**. It has been divided into square of unit area by drawing lines perpendicular to the sides. Find number of rectangles having sides of odd unit length**



(a) $(m+n+1)^2$ (b) $mn(m+1)(n+1)$

(c) 4^{m+n-2} (d) m^2n^2

92. **The sum** $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, $\left(\text{where}\binom{p}{q}=0 \text{ if } p<q\right)$**, is maximum when** ***m*** **is**

(a) 5 (b) 15

(c) 10 (d) 20

93. **If** ${}^nP_r=840, {}^nC_r=35,$ **then** n **is equal to**

(a) 1 (b) 3

(c) 5 (d) 7

94. **The number of way to sit 3 men and 2 women in a bus such that total number of sitted men and women on each side is 3**

(a) 5! (b) ${}^6C_5\times 5!$

(c) $6!\times{}^6P_5$ (d) $5!+{}^6C_5$

95. **An** ***n*****-digit number is a positive number with exactly** n **digits. Nine hundred distinct** ***n*****-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of** n **for which this is possible is**

(a) 6 (b) 7

(c) 8 (d) 9

96. **Number of ways of selection of 8 letters from 24 letters of which 8 are** a**, 8 are** b **and the rest unlike, is given by**

(a) 2^7 (b) $8\,.\,2^8$

(c) $10\,.\,2^7$ (d) None of these

97. A set contains $(2n+1)$ elements. The number of sub-sets of the set which contain at most n elements is

(a) 2^n (b) 2^{n+1}

(c) 2^{n-1} (d) 2^{2n}

98. The number of numbers of 4 digits which are not divisible by 5 are

(a) 7200 (b) 3600

(c) 14400 (d) 1800

99. The number of ordered triplets of positive integers which are solutions of the equation $x+y+z=100$ is

(a) 6005 (b) 4851

(c) 5081 (d) None of these

100. The number of divisors of 9600 including 1 and 9600 are

(a) 60 (b) 58

(c) 48 (d) 46

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1. (c) Required number of ways are 5^4.

2. (b) Total number of arrangements of n books $= n!$.

If two specified books always together then number of ways $=(n-1)!\times 2$

Hence required number of ways $= n! - (n-1)!\times 2$

$= n(n-1)! - (n-1)\times 2 = (n-1)!(n-2)$.

3. (d) Required number of ways are $2^{10}=1024$, because every question may be answered in 2 ways.

4. (a) The man can go in 5 ways and he can return in 5 ways. Hence, total number of ways are $5\times 5 = 25$.

5. (a) Required number of ways $= \frac{6!}{3!3!} = \frac{720}{6\times 6} = 20$.

6. (a) concept

7. (a) Sum of the digits in the unit place is $6(2+4+6+8)=120$ units. Similarly, sum of digits in ten place is 120 tens and in hundredth place is 120 hundreds etc. Sum of all the 24 numbers is $120(1+10+10^2+10^3)=120\times 1111=133320$.

8. (b) The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places in $\frac{4!}{2!2!}=6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places in $\frac{3!}{2!}=3$ ways. Hence the required number of ways $=6\times 3=18$.

9. (a) Concept

10. (a) ${}^{n-1}C_r+{}^{n-1}C_{r-1}={}^nC_r$

$\Rightarrow \frac{{}^{n-1}P_r}{r!}+\frac{{}^{n-1}P_{r-1}}{(r-1)!}=\frac{{}^nP_r}{r!}\Rightarrow {}^{n-1}P_r+r.{}^{n-1}P_{r-1}={}^nP_r$.

11. (b) Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the first place with 0 in each of remaining places.

After fixing 1^{st} place, the second place can be filled by any of the 5 numbers. Similarly third place can be filled up in 5 ways and 4^{th} place can be filled up in 5 ways. Thus there will be $5\times 5\times 5=125$ ways in which 1 will be in first place but this include 1000 also hence there will be 124 numbers having 1 in the first place. Similarly 125 for each 2 or 3. One number will be in which 4 in the first place and *i.e.* 4000. Hence the required numbers are $124+125+125+1=375$ ways.

12. (c) A gets 2, B gets 8; $\frac{10!}{2!8!}=45$

A gets 8, B gets 2; $\frac{10!}{8!2!}=45$

$\therefore 45+45=90$.

13. (c) Required number of ways are 5^4.

14. (b) CONCEPT

15. (c) Since the 5 boys can sit in $5!$ ways. In this case there are 6 places are vacant in which the girls can sit in ${}^6P_3$ ways. Therefore required number of ways are ${}^6P_3\times 5!$.

16. (c) First, we arrange 3 consonants in $3!$ ways and then at four places (two places between them and two places on two sides) 3 vowels can be placed in ${}^4P_3\times\frac{1}{2!}$ ways.

Hence the required number $=3!\times {}^4P_3\times\frac{1}{2!}=72$.

17. (b) 3 must be at thousand place and since the number should be divisible by 5, so 5 must be at unit place. Now we have to filled two place (ten and hundred) *i.e.*, ${}^4P_2 = 12$.

18. (b) Required number of ways are $\frac{5!}{2!} = 60$.

19. (b) Three letters can be posted in 4 letter boxes in $4^3 = 64$ ways but it consists the 4 ways that all letters may be posted in same box. Hence required ways = 60.

20. (a) Total no. of permutations $= \frac{6!}{3!2!} = 60$.

21. (c) Since there are 2 M's, 2 A's and 2T's.

$\therefore$ Required number of ways are $\frac{11!}{2!2!2!}$.

22. (a) Number of 1 digit numbers $= {}^6P_1$

Number of 2 digit numbers $= {}^6P_2$

Number of 3 digit numbers $= {}^6P_3$

The required number of numbers $= 6 + 30 + 120 = 156$.

23. (b) Standard problem

24. (b) We have $a = {}^{x+2}P_{x+2} = (x+2)!, b = {}^xP_{11} = \frac{x!}{(x-11)!}$

And $c = {}^{x-11}P_{x-11} = (x-11)!$

Now $a = 182\ bc \Rightarrow (x+2)! = 182 . \frac{x!}{(x-11)!}(x-11)!$

$\Rightarrow (x+2)! = 182x! \Rightarrow (x+2)(x+1) = 182 \Rightarrow x = 12$.

25. (a) concept.

26. (d) Without any restriction the 10 persons can be ranked among themselves in $10!$ ways; but the number of ways in which A_1 is above A_{10} and the number of ways in which A_{10} is above A_1 make up $10!$. Also the number of ways in which A_1 is above A_{10} is exactly same as the number of ways in which A_{10} is above A_1.

Therefore the required number of ways $= \frac{1}{2}(10!)$.

27. (c) The numbers between 999 and 10000 are of four digit numbers.

The four digit numbers formed by digits 0, 2,3,6,7,8 are ${}^6P_4 = 360$.

But here those numbers are also involved which begin from 0. So we take those numbers as three digit numbers.

Taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2,3,6,7,8 are ${}^5P_3 = 60$

So the required numbers = 360-60 = 300.

28. (c) The units place can be filled in 4 ways as any one of 0, 2, 4 or 6 can be placed there. The remaining three places can be filled in with remaining 6 digits in ${}^6P_3$ = 120 way. So, total number of ways = 4 × 120 = 480. But, this includes those numbers in which 0 is fixed in extreme left place. Numbers of such numbers $= 3 \times {}^5P_2 = 3 \times 5 \times 4 = 60$

0	×	×	×
Fix	${}^5P_2$ ways		3 ways (only 2, 4 or 6)

∴ Required number of ways = 480 – 60 = 420.

29. (c) Out of 7 places, 4 places are odd and 3 even. Therefore 3 vowels can be arranged in 3 even places in ${}^3P_3$ ways and remaining 4 consonants can be arranged in 4 odd places in ${}^4P_4$ ways.

Hence required no. of ways = ${}^3P_3$ × ${}^4P_4$ = 144.

30. (c) Words starting with A, C, H, I, N are each equals to 5!

∴ Total words = 5×5! =600

The first word starting with *S* is *SACHIN*.

∴*SACHIN* appears in dictionary at serial number 601.

31. (a) Required number of arrangements

= (Total number of arrangements)

– (Number of arrangements in which N'*s* are together)

$= \frac{6!}{2! \times 3!} - \frac{5!}{3!} = 60 - 20 = 40$.

32. (c) There are five seats in a bus are vacant. A man can sit on any one of 5 seats in 5 ways. After the man is seated, his wife can be seated in any of 4 remaining seats in 4 ways. Hence total number of ways of seating them $= 5 \times 4 = 20$.

33. (d) 0, 1, 2, 3, 5, 7 : Six digits

The last place can be filled in by 1, 3, 5, 7. *i.e.*,4 ways as the number is to be odd. We have to fill in the remaining 3 places of the 4 digit number *i.e.* I, II, III place. Since repetition is allowed each place can be filled in 6 ways. Hence the 3 place can be filled in 6 × 6 × 6 = 216 ways.

But in case of 0 = 216 – 36 = 180 ways.

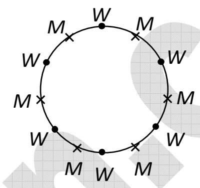
Hence by fundamental theorem, the total number will be = 180 × 4 = 720.

34. (c) Given set of numbers is {1, 2,....11} in which 5 are even six are odd, which demands that in the given product it is not possible to arrange to subtract only even number from odd numbers. There must be at least one factor involving subtraction of an odd number form another odd number. So at least one of the factors is even. Hence product is always even.

35. (a) No. of ways in which 6 men can be arranged at a round table = (6 – 1)!

Now women can be arranged in 6! ways.

Total Number of ways = 6! × 5!



36. (c) Required number of ways $9!\times 2$.

37. (a) The number of ways in which 5 beads of different colours can be arranged in a circle to form a necklace are $(5-1)!=4!$.

But the clockwise and anticlockwise arrangement are not

Hence the total number of ways of arranging the beads $=\frac{1}{2}(4!)=12$.

38. (b) Fix up 1 man and the remaining 6 men can be seated in 6! ways. Now no two women are to sit together and as such the 7 women are to be arranged in seven empty seats between two consecutive men and number of arrangement will be 7!. Hence by fundamental theorem the total number of ways = 7! × 6!.

39. (d) A garland can be made from 10 flowers in $\frac{1}{2}(9!)$ ways.

40. (b) Since total number of ways in which boys can occupy any place is $(5-1)!=4!$ and the 5 girls can be sit accordingly in $5!$ ways.

Hence required number of ways are $4!\times 5!$.

41. (a) 8 different beads can be arranged in circular form in (8 – 1)! = 7! ways. Since there is no distinction betweenthe clockwise and anticlockwise arrangement. So the required number of arrangements = $\frac{7!}{2}=2520$.

42. (d) It is obvious by fundamental property of circular permutations.

43. (a) Fix up a male and the remaining 4 male can be seated in 4! ways. Now no two female are to sit together and as such the 2 female are to be arranged in five empty seats between two consecutive male and number of arrangement will be ${}^5P_2$. Hence by fundamental theorem the total number of ways is

$= 4! \times {}^5P_2 = 24 \times 20 = 480$ ways.

44. (a) It is a fundamental property

45. (c) Required number of ways = $2^7 - 1 = 127$.

{Since the case that no friend be invited *i.e.*, ${}^7C_0$ is excluded}.

46. (c) Here $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{36}{84}$ and $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{84}{126}$.

$\Rightarrow 3n - 10r = -3$ and $4n - 10r = 6$

On solving, we get $n = 9, r = 3$.

47. (b) ${}^nC_2 = 66 \Rightarrow n(n-1) = 132 \Rightarrow n = 12$.

48. (a) It is obvious.

49. (b) $\frac{(2n)!}{(2n-3)!.3!} \times \frac{2! \times (n-2)!}{n!} = \frac{44}{3}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{3n(n-1)} = \frac{44}{3}$

$\Rightarrow 4(2n-1) = 44 \Rightarrow 2n = 12 \Rightarrow n = 6$

Now ${}^6C_r = 15 \Rightarrow {}^6C_r = {}^6C_2$ or ${}^6C_4 \Rightarrow r = 2, 4$.

50. (a) ${}^{15}C_{3r} = {}^{15}C_{r+3} \Rightarrow {}^{15}C_{15-3r} = {}^{15}C_{r+3}$

$\Rightarrow 15 - 3r = r + 3 \Rightarrow r = 3$.

51. (c) Required number of ways $= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5$

$= 8 + 28 + 56 + 70 + 56 = 218$

52. (c) We have 32 places for teeth. For each place we have two choices either there is a tooth or there is no tooth. Therefore the number of ways to fill up these places is 2^{32}. As there is no person without a tooth, the maximum population is $2^{32} - 1$.

53. (d) The letters can be select in ${}^5C_3 \times {}^4C_2$ ways.

Therefore the number of arrangements are $({}^5C_3 \times {}^4C_2)\, 5!$.

54. (b) Total number of handshakes = ${}^{15}C_2$.

55. (b) ${}^nC_2 = 153 \Rightarrow \frac{n(n-1)}{2} = 153 \Rightarrow n = 18$.

56. (b) $\frac{n-r+1}{r}=\frac{84}{36}=\frac{7}{3}$ and $\frac{n-r}{r+1}=\frac{126}{84}=\frac{3}{2}$

$\therefore \frac{7}{3}r-1=n-r=\frac{3}{2}(r+1)$

Or $14r-6=9r+9$ or $r=3$. So, $n=9$.

57. (c) Let there are n candidates then

${}^nC_1+{}^nC_2+......+{}^nC_{n-1}=254 \Rightarrow 2^n-2=254$

$\Rightarrow 2^n=2^8 \Rightarrow n=8$.

58. (c) ${}^{n+1}C_3=2.{}^nC_2$

$\Rightarrow \frac{(n+1)!}{3!.(n-2)!}=2.\frac{n!}{2!.(n-2)!} \Rightarrow \frac{n+1}{3.2!}=\frac{2}{2!}$

$\Rightarrow n+1=6 \Rightarrow n=5$.

59. (d) A voter can vote in ${}^5C_1+{}^5C_2+{}^5C_3=25$ ways.

60. (c) The arrangement can be make as $.+.+.+.+.+.+.$ *i.e.*, the $(-)$ signs can be put in 7 vacant (pointed) place.

Hence required number of ways $={}^7C_4=35$.

61. (a) Required number of ways

$={}^{52}C_{13}\times{}^{39}C_{13}\times{}^{26}C_{13}\times{}^{13}C_{13}$

$=\frac{52\,!}{39\,!\times13\,!}\times\frac{39\,!}{26\,!\times13\,!}\times\frac{26\,!}{13\,!\times13\,!}\times\frac{13\,!}{13\,!}=\frac{52\,!}{(13\,!)^4}$.

62. (c) The required number of ways are

$(10+1)(9+1)(7+1)-1=11\times10\times8-1=879$.

63. (c) The required number of words is

$({}^2C_1\times{}^4C_2+{}^2C_2\times{}^4C_1)3\,!=96$.

64. (b) Required number of ways

$={}^6C_1+{}^6C_2+{}^6C_3+{}^6C_4+{}^6C_5+{}^6C_6=2^6-1=63$.

65. (b) The total number of two factor products $={}^{200}C_2$. The number of numbers from 1 to 200 which are not multiples of 5 is 160. Therefore total number of two factor products which are not multiple of 5 is ${}^{160}C_2$.

Hence the required number of factors

$={}^{200}C_2-{}^{160}C_2=7180$.

66. (c) Since number of selections are ${}^{n-p}C_{r-p}$. Therefore the arrangement of r things can be done in r !ways. Hence the total permutations are ${}^{n-p}C_{r-p}\, r!$

67. (b) Required number of ways

$= {}^5C_3 \times {}^2C_1 \times {}^9C_7 = 10 \times 2 \times 36 = 720$.

68. (b) ${}^{50}C_4 + \left({}^{50}C_3 + {}^{51}C_3 + {}^{52}C_3 + ... {}^{55}C_3\right)$. Taking first two terms together and adding them and following the same pattern, we get ${}^{56}C_4$, [As ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$].

69. (b) As for given question two cases are possible.

70. (b) Expression = ${}^nC_{r+1} + {}^nC_{r-1} + {}^nC_r + {}^nC_r$

$= {}^nC_{r+1} + {}^{n+1}C_r + {}^nC_r = {}^{n+1}C_{r+1} + {}^{n+1}C_r = {}^{n+2}C_{r+1}$.

71. (c) The number of times he will go to the garden is same as the number of selecting 3 children from 8.

Therefore the required number $= {}^8C_3 = 56$.

72. (d) We have $\frac{(n-1)!}{(n-r-1)!\,r!} = \frac{(k^2-3)\,n!}{(n-r-1)!\,(r+1)!}$, $0 \le r \le n-1$

$\Rightarrow k^2 = \frac{r+1}{n} + 3, \frac{1}{n} \le \frac{r+1}{n} \le 1 \Rightarrow k^2 \in \left[\frac{1}{n}+3, 4\right], n \ge 2$

$k \in \left[-2, -\sqrt{\frac{1}{n}+3}\right] \cup \left[\sqrt{\frac{1}{n}+3}, 2\right]; n \ge 2$.

73. (b) The required number $= {}^{3+35-1}C_{3-1} = {}^{37}C_2 = 666$

74. (a) Since the person is allowed to select at most n coins out of $(2n + 1)$ coins, therefore in order to select one, two, three, ….,n coins. Thus, if T is the total number of ways of selecting one coin, then

$T = {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = 255$ …..(i)

Again the sum of binomial coefficients

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n+1} = (1+1)^{2n+1} = 2^{2n+1}$

$\Rightarrow {}^{2n+1}C_0 + 2\left({}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n\right) + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow 1 + 2(T) + 1 = 2^{2n+1} \Rightarrow 1 + T = \frac{2^{2n+1}}{2} = 2^{2n}$

$\Rightarrow 1 + 255 = 2^{2n} \Rightarrow 2^{2n} = 2^8 \Rightarrow n = 4$.

75. (d) Word 'MATHEMATICS' has 2*M*, 2*T*, 2*A*, *H*, *E*, *I*, *C*, *S*. Therefore 4 letters can be chosen in the following ways.

Case I: 2 alike of one kind and 2 alike of second kind *i.e.*, ${}^3C_2 \Rightarrow$ No. of words $= {}^3C_2 \dfrac{4!}{2!\,2!} = 18$

Case II: 2 alike of one kind and 2 different

i.e., ${}^3C_1 \times {}^7C_2 \Rightarrow$ No. of words $= {}^3C_1 \times {}^7C_2 \times \dfrac{4!}{2!} = 756$

Case III : All are different

i.e., ${}^8C_4 \Rightarrow$ No. of words $= {}^8C_4 \times 4! = 1680$.

76. (b) Required number $= {}^{10}C_5 \times {}^8C_4$.

77. (b) Since the student is allowed to select at most *n* books out of $(2n+1)$ books, therefore in order to select one book he has the choice to select one, two, three, ...,*n* books.

Thus, if *T* is the total number of ways of selecting one book then

$T = {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = 63$ .....(i)

Again the sum of binomial coefficients

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n+1} = (1+1)^{2n+1} = 2^{2n+1}$$

Or ${}^{2n+1}C_0 + 2({}^{2n+1}C_1 + {}^{2n+1}C_2 + .. + {}^{2n+1}C_n) + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow 1 + 2(T) + 1 = 2^{2n+1} \Rightarrow 1 + T = \dfrac{2^{2n+1}}{2} = 2^{2n}$

$\Rightarrow 1 + 63 = 2^{2n} \Rightarrow 2^6 = 2^{2n} \Rightarrow n = 3$.

78. (b) Clearly, ${}^nC_3 = T_n$.

So, ${}^{n+1}C_3 - {}^nC_3 = 21 \Rightarrow ({}^nC_3 + {}^nC_2) - {}^nC_3 = 21$

$\therefore {}^nC_2 = 21$ or $n(n-1) = 42 = 7.6 \;\; \therefore n = 7$.

79. (b) Total number of points are $m+n+k$, the $\Delta's$ formed by these points $= {}^{m+n+k}C_3$

Joining 3 points on the same line gives no triangle, such $\Delta's$ are ${}^mC_3 + {}^nC_3 + {}^kC_3$

Required number $= {}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$.

80. (b) Since ${}^nC_2 - n = 44 \Rightarrow n = 11$.

81. (a) Required number of ways $= {}^{12}C_3 - {}^7C_3$

$= 220 - 35 = 185$.

82. (a) **Case I:** When A is excluded.

Number of triangles = selection of 2 points from AB and one point from $AC +$ selection of one point from AB and two points from AC

$$= {}^mC_2\,{}^nC_1 + {}^mC_1\,{}^nC_2 = \frac{1}{2}(m+n-2)mn \qquad \text{.....(i)}$$

Case II: When A is included.

The triangles with one vertex at $A =$ selection of one point from AB and one point from $AC = mn$.

$\therefore$ Number of triangles

$$= mn + \frac{1}{2}mn(m+n-2) = \frac{1}{2}mn(m+n) \qquad \text{.....(ii)}$$

$\therefore$ Required ratio $= \dfrac{(m+n-2)}{(m+n)}$.

83. (d) The required number of points

$$= {}^8C_2 \times 1 + {}^4C_2 \times 2 + \left({}^8C_1 \times {}^4C_1\right) \times 2$$

$$= 28 + 12 + 32 \times 2 = 104$$

84. (c) Since no two lines are parallel and no three are concurrent, therefore n straight lines intersect at ${}^nC_2 = N$ (say) points. Since two points are required to determine a straight line, therefore the total number of lines obtained by joining N points ${}^NC_2$. But in this each old line has been counted ${}^{n-1}C_2$ times, since on each old line there will be $n-1$ points of intersection made by the remaining $(n-1)$ lines.

Hence the required number of fresh lines is

$${}^NC_2 - n\,.\,{}^{n-1}C_2 = \frac{N(N-1)}{2} - \frac{n(n-1)(n-2)}{2}$$

$$= \frac{{}^nC_2({}^nC_2 - 1)}{2} - \frac{n(n-1)(n-2)}{2} = \frac{n(n-1)(n-2)(n-3)}{8}.$$

85. (c) Each set is having $m+2$ parallel lines and each parallelogram is formed by choosing two straight lines from the first set and two straight lines from the second set. Two straight lines from the first set can be chosen in ${}^{m+2}C_2$ ways and two straight lines from the second set can be chosen in ${}^{m+2}C_2$ ways.

Hence the total number of parallelograms formed $= {}^{m+2}C_2\,.\,{}^{m+2}C_2 = \left({}^{m+2}C_2\right)^2$.

86. (c,b) 18 points, 5 collinear :

(i) Number of lines $= {}^{18}C_2 - {}^{5}C_2 + 1 = 153 - 10 + 1 = 144$

(ii) Number of $\Delta's = {}^{18}C_3 - {}^{5}C_3 = 816 - 10 = 806$.

87. (b) Required number of ways $= {}^{4}C_2 \times {}^{3}C_2 = 18$.

88. (a) Number of triangles $= {}^{10}C_3 - {}^{6}C_3 = 120 - 20 = 100$.

89. (d) Required number $= {}^{20}C_2 - {}^{4}C_2 + 1$

$$= \frac{20 \times 19}{2} - \frac{4 \times 3}{2} + 1 = 190 - 6 + 1 = 185.$$

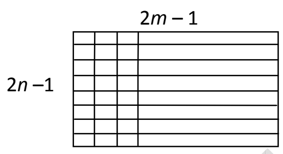
90. (b) No. of triangles = ${}^{6}C_3 - 6$.

91. (d) Along horizontal side one unit can be taken in $(2m–1)$ ways and 3 unit side can be taken in $2m - 3$ ways.

$\therefore$ The number of ways of selecting a side horizontally is

$(2m - 1 + 2m - 3 + 2m - 5 + ... + 3 + 1)$

Similarly the number of ways along vertical side is $(2n - 1 + 2n - 3 + ... + 5 + 3 + 1)$.



$\therefore$Total number of rectangles

92. (b)

93. (d) $\frac{{}^{n}P_r}{{}^{n}C_r} = 24 \Rightarrow r\,! = 24 \Rightarrow r = 4$

$\therefore {}^{n}C_4 = 35 \Rightarrow n = 7$.

94. (b)

3 men and 2 women equal to 5. A group of 5 members make 5! permutation with each other.

$\therefore$ The number of ways to sit 5 members = 5!

6 Places are filled by 5 members by ${}^{6}C_5$ ways

$\therefore$ The total number of ways to sit 5 members on 6 seats of a bus = ${}^{6}C_5 \times 5!$.

95. (b) Since at any place, any of the digits 2, 5 and 7 can be used, total number of such positive n-digit numbers are 3^n. Since we have to form 900 distinct numbers, hence $3^n \geq 900 \Rightarrow n = 7$.

96. (c) The number of selections = coefficient of x^8 in $(1 + x + x^2 + ... + x^8)(1 + x + x^2 + ... + x^8).(1 + x)^8$

97. (d) The number of sub-sets of the set which contain at most n elements is

$^{2n+1}C_0 + ^{2n+1}C_1 + ... + ^{2n+1}C_n = S$ (Say)

Then $2S = 2(^{2n+1}C_0 + ^{2n+1}C_1 + ... + ^{2n+1}C_n)$

$= (^{2n+1}C_0 + ^{2n+1}C_{2n+1}) + (^{2n+1}C_1 + ^{2n+1}C_{2n}) + ... \quad .... + (^{2n+1}C_n + ^{2n+1}C_{n+1})$

$= ^{2n+1}C_0 + ^{2n+1}C_1 + ... + ^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow S = 2^{2n}$.

98. (a) The total number of 4 digits are $9999 - 999 = 9000$.

The numbers of 4 digits number divisible by 5 are $90 \times 20 = 1800$. Hence required number of ways are $9000 - 1800 = 7200$.

99. (b) The number of triplets of positive integers which are solutions of $x + y + z = 100$.

$=$ Coefficient of x^{100} in $(x + x^2 + x^3 + ...)^3$

100. (c) Since $9600 = 2^7 \times 3 \times 5^2$

Hence, number of divisors $= (7+1)(1+1)(2+1) = 48$.