

PERIODICITY AND EXTREME VALUES

OBJECTIVES

1. Period of $f(x) = \sin\left(\frac{\pi x}{2}\right) + 2\cos\left(\frac{\pi x}{3}\right) - \tan\left(\frac{\pi x}{4}\right)$ is

- a) 4 b) 6 c) 12 d) 24

2. Period of $\cot\left(\frac{3-5x}{7}\right)$ is

- a) $-\frac{7\pi}{5}$ b) $\frac{5\pi}{7}$ c) $\frac{7\pi}{5}$ d) $\frac{14\pi}{5}$

3. Period of $\sec(x + 2x + 4x)$ is

- a) 2π b) 14π c) $2\pi/7$ d) $\pi/7$

4. Period of $\tan(x + 8x + 27x + \dots + n^3x)$ is

- a) $\frac{8\pi}{n^2(n+1)^2}$ b) $\frac{4\pi}{n^2(n+1)^2}$
c) $\frac{2\pi}{n^2(n+1)^2}$ d) $\frac{4\pi}{n(n+1)}$

5. Period of $\sin\left(\frac{\pi}{3} - x\right) \sin\left(\frac{\pi}{3} + x\right)$ is

- a) $\pi/2$ b) π c) $3\pi/2$ d) 2π

6. Period of $\frac{\cot\frac{x}{4} + \tan\frac{x}{4}}{1 + \tan\frac{x}{2} - \tan x}$ is

- a) $\pi/2$ b) π c) 4π d) 2π

7. Period of $3\sin^5x + \cos^3x$ is

- a) π b) 2π c) $\pi/2$ d) $3\pi/2$

8. The function whose period is 5π is

- a) Cosec (2x) b) sec (5x)
c) $\tan(5x/2)$ d) $\cot(x/5)$

9. The minimum and maximum values of $\cos \theta + 2\sqrt{2} \sin \theta$ is

- a) -3, 3 b) 3, -3 c) [-3, 3] d) [0, 3]

10. The maximum value of $3\cos \theta + 4\sin \theta$ is

- a) 3 b) 4 c) 5 d) $\sqrt{5}$

11. The range of $f(x) = -3\cos\sqrt{3+x+x^2}$ is

- a) [-1, 1] b) [-2, 2] c) [-3, 3] d) [-4, 4]

12. The greatest (or) least value of $2\sin^2\theta + 4\operatorname{cosec}^2\theta$ is

- a) $2\sqrt{2}$ b) $4\sqrt{2}$ c) 2 d) 4

13. Period of $\sin^4x + \cos^4x$ is

- a) $\pi/2$ b) π c) $3\pi/2$ d) 2π

14. Period of $\cos(x + 4x + 9x + \dots + n^2x)$ is

- a) $\frac{12\pi}{n(n+1)(2n+1)}$ b) $\frac{6\pi}{n(n+1)(2n+1)}$
c) $\frac{3\pi}{n(n+1)(2n+1)}$ d) $\frac{\pi}{12n(n+1)(2n+1)}$

15. The minimum value of $5 \cos x + 4\cos\left(\frac{\pi}{3} + x\right) + 8$ is

- a) 1 b) $\frac{1}{2}$ c) -1 d) 15

16. The range of $\cos x + 4\sqrt{2} \sin\left(x - \frac{\pi}{4}\right) + 6$ is

- a) [1, 11] b) [-1, 11] c) [2, 10] d) [10, 11]

17. Maximum value of $1 + 8\sin^2 x^2 \cos^2 x^2$ is

- a) 3 b) -1 c) -8 d) 9

18. $\frac{\sin^2 A + \sin A + 1}{\sin A} \geq K$ then $K =$

- a) 2 b) 1 c) 4 d) 3

19. The period of $\frac{\sin nx}{\cos(x/n)}$ is 4π , where $n \in \mathbb{Z}^+$ then $n =$

- a) 1 b) 2 c) 3 d) 4

20. Period of $\cos x \cos(120^\circ - x) \cos(120^\circ + x)$

- a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) π d) 2π

21. Period of $|\sin x| \cdot |\cos x|$ is

- a) $\pi/2$ b) π c) 4π d) 2π

22. Period of $|\cot x| + |\cos x| + |\tan x| + |\sin x|$ is

- a) π b) $\pi/2$ c) 2π d) $\pi/4$

23. Period of $\left(\frac{1}{3}\right)^{\sin x} + \left(\frac{1}{3}\right)^{\cos x}$ is

- a) π b) $\pi/2$ c) $\pi/4$ d) 2π

24. Period of $3x - [3x]$ is (where $[.]$ denotes greatest integer function $\leq x$)

- a) $1/2$ b) 1 c) 2 d) $1/3$

25. Period of $\sin(x - [x])$ is

- a) π b) 2π c) 1 d) $\pi/2$

26. Period of $\sin(\sin x) + \sin(\cos x)$ is

- a) π b) 2π c) $\pi/2$ d) $\pi/4$

27. Let $f(x) = \cos \sqrt{p} x$. where $p = [a]$ (integral part). If the period of $f(x)$ is π then $a \in$

- a) $[4, 5]$ b) $[4, 5)$ c) $(4, 5]$ d) $(4, 5)$

28. The period of $\sin \frac{nx}{n!} - \cos \frac{nx}{(n+1)!}$ is

- a) Not defined b) $2(n!)$
c) $n!$ d) $2((n+1)!)$

29. The maximum value of $\frac{3}{5 \sin x - 12 \cos x + 19}$ is

- a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

30. The range of $\log \log_{\sqrt{5}} [\sqrt{2}(\sin x - \cos x) + 3]$ is

- a) $[0, 2]$ b) $[1, 2]$ c) $[0, 3]$ d) $[1, 3]$

31. The minimum value of $2\cos x - 3\cos^2 x + 5$ is

- a) -1 b) 0 c) 1 d) 2

32. The range of $\cos^2 x + \sin^4 x$ is

- a) $[\frac{1}{2}, 1]$ b) $[1, \frac{3}{2}]$ c) $[\frac{3}{2}, 2]$ d) $[\frac{3}{4}, 1]$

33. The minimum value of $\sin x (\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x})$ is

- a) 1 b) 2 c) 3 d) 4

34. If $\sqrt{3} \sin x + \cos x$ is maximum, then $x =$

- a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{2\pi}{5}$ d) $\frac{\pi}{2}$

35. Range of $\sin^3 x + \cos^3 x$ is

- a) $[\frac{1}{\sqrt{2}}, 1]$ b) $[1, 2]$ c) $[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$ d) $[-1, 1]$

36. The period of $f(x) = \cos 3x + \sin \sqrt{3} x$ is

- a) Non defined b) $\frac{2\pi}{3}$
c) $\frac{2\pi}{\sqrt{3}}$ d) $2\sqrt{3}\pi$

37. The period of $f(x) = |\sin x/2| + |\cos x|$ is

- a) 2π b) 3π c) 4π d) π

38. Period of $|\sin x| + |\cos x|$ is

- a) $\pi/2$ b) c) $\frac{3\pi}{2}$ d) 2π

39. The minimum and maximum values of $1 - 8 \sin^2 x \cos^2 x$ are

- a) $-1/4, 1/4$ b) $-4, 4$ c) $-1, 1$ d) $-1/8, 1/8$

40. The range of $\cos^2(\frac{2\pi}{3} - x) + \cos^2(\frac{2\pi}{3} + x)$ is

- a) $[1/2, 1]$ b) $[1, 3/2]$ c) $[3/2, 2]$ d) $[1/2, 3/2]$

41. The range of $\sin^2 x + 4 \sin x + 5$ lies in

- a) $[2, 10]$ b) $[-2, 10]$ c) $[10, 5]$ d) $[5, 10]$

42. The value of $(\sin x + \cos x)^2 + \cos^2(\frac{\pi}{4} + x)$ lies is

- a) $[0, 1]$ b) $[0, 2]$ c) $[1, 2]$ d) $[0, 3]$

43. The value of $\sec x + 2 \cot 2x \sin x$ lies in

- a) $[-1, 1]$ b) $[-2, 2]$ c) $[-3, 3]$ d) $[-4, 4]$

44. The extreme values of $\cos^6 x + \sin^6 x$ are

- a) $1/4, 1$ b) $7/8, 9/8$ c) $3/8, -3/8$ d) $1/4, 1/4$

45. Period of $\tan \theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right)$ is

- a) $\pi/2$ b) π c) 2π d) $\pi/3$

46. The period of $\sin\left(\frac{2\pi x}{a}\right) + 3\cos\left(\frac{2\pi x}{b}\right)$ when $a = 12$, $b = 9$ is

- a) 18 b) 36 c) 108 d) 54

47. The period of $f(x) = \sin\left[\left(\frac{3}{2} + \frac{5}{4} + \frac{7}{8} + \dots + \infty\right)x\right]$ is

- a) $\pi/6$ b) $\pi/3$ c) $\pi/5$ d) $2\pi/5$

48. The minimum value of $a^{\cos^2 x} + a^{\sin^2 x} \quad \forall (a > 0)$ is

- a) $2a$ b) $\sqrt{2}a$ c) $2\sqrt{a}$ d) \sqrt{a}

49. $2^{\sin x} + 2^{\cos x} \geq K$, then $K =$

- a) 2 b) $2^{\frac{1}{2}}$ c) $2^{1-\frac{1}{\sqrt{2}}}$ d) $2^{\frac{1}{\sqrt{2}}}$

50. In a triangle ABC $\cos A + \cos B + \cos C \leq P$ then $P =$

- a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{3\sqrt{3}}{2}$ d) $\frac{3}{2}$

51. In a triangle ABC $\tan^2 \frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2} \geq K$ then $K =$

- a) 1 b) 2 c) 3 d) 4

52. If the sides of a triangle inscribed in a given circle subtend angles α, β, γ at the centre then the minimum value of the A.M. of $\cos(\alpha + \pi/2)$, $\cos(\beta + \pi/2)$ and $\cos(\gamma + \pi/2)$ is

- a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) $-\frac{\sqrt{3}}{2}$

53. The maximum value of $\cos \alpha_1, \cos \alpha_2, \dots, \cos \alpha_n$ under the restriction $\theta \leq \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \leq \pi/2$ and $\cot \alpha_1, \cot \alpha_2, \dots, \cot \alpha_n = 1$ is

- a) $\frac{1}{2^n}$ b) 1 c) $1/2^{n/2}$ d) $-\frac{1}{2^n}$

54. $A \leq \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta} \leq B$ then the values of (A, B) =

a) $(a + b, \sqrt{2(a^2 + b^2)})$

b) $(a, \sqrt{a^2 + b^2})$

c) $(\sqrt{a^2 + b^2}, b)$

d) (a, b)

ANSWERS

1. c 2. c 3. c 4. b 5. b 6. c 7. b 8. d 9. a 10. c
11. c 12. b 13. a 14. a 15. a 16. a 17. a 18. d 19. b 20. a
21. a 22. b 23. d 24. d 25. c 26. b 27. b 28. d 29. b 30. a
31. b 32. d 33. b 34. b 35. d 36. a 37. a 38. a 39. c 40. a
41. a 42. c 43. b 44. a 45. d 46. b 47. d 48. c 49. c 50. d
51. a 52. d 53. c 54. a