

PARTIAL FRACTIONS

OBJECTIVE PROBLEMS

1. If $\frac{x-4}{x^2-5x-2k} = \frac{2}{x-2} - \frac{2}{x+K}$, then **K** =
- a) -3 b) -2 c) 2 d) 3
2. If $\frac{ax-1}{(1-x+x^2)(2+x)} = \frac{x}{1-x+x^2} - \frac{1}{2+x}$ then **a** =
- a) 3 b) -3 c) 2 d) -2
3. If $\frac{3x+4}{x^2-3x+2} = \frac{A}{x-2} - \frac{B}{x-1}$, then **A, B**
- a) (7, 10) b) (10, 7) c) (10, -7) d) (-10, 7)
4. If $\frac{1-x+6x^2}{x-x^3} = \frac{A}{x} + \frac{B}{1-x} + \frac{C}{1+x}$, then **A** =
- a) 1 b) 2 c) 3 d) 4
5. If the remainders of the polynomial $f(x)$ when divided by $x - 1$, $x - 2$ are 2, 5 then the remainder of $f(x)$ when divided by $(x - 1)(x - 2)$ is
- a) 0 b) $1 - x$ c) $2x - 1$ d) $3x - 1$
6. The remainders of the polynomial $f(x)$ when divided by $x + 1$, $x + 2$, $x - 2$ are 6, 15, 3 the remainder of $f(x)$ when divided by $(x + 1)(x + 2)(x - 2)$ is
- a) $2x^2 - 3x + 1$ b) $3x^2 - 2x + 1$ c) $2x^2 - x - 3$ d) $3x^2 - 2x + 1$
7. If $\frac{2x+1}{(x-1)(x^2+2)} = \frac{A}{x-1} + \frac{Bx+c}{x^2+2}$ then **B** =
- a) 2 b) 1 c) -1 d) -2

8. If $\frac{3x}{(x-a)(x-b)} = \frac{2}{x-a} + \frac{1}{x-b}$ then **a : b =**

- a) 1 : 2 b) -2 : 1 c) 1 : 3 d) 3 : 1

9. Let **a, b, c** such that

$$\frac{1}{(1-x)(1-2x)(1-3x)} = \frac{a}{1-x} + \frac{b}{1-2x} + \frac{c}{1-3x},$$

$$\frac{a}{1} + \frac{b}{3} + \frac{c}{5} =$$

- a) $\frac{1}{15}$ b) $\frac{1}{6}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

10. If $\frac{x^4}{(x-a)(x-b)(x-c)} = P(x) + \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$ then **P(x) =**

- a) $x - a$ b) $x - a - b$
 c) $x - a - b - c$ d) $x + a + b + c$

11. $\frac{x^3}{(2x-1)(x+2)(x-3)} = A + \frac{B}{2x-1} + \frac{C}{x+2} + \frac{D}{x-3}$ then **A =**

- a) $\frac{1}{2}$ b) $-\frac{1}{50}$ c) $\frac{8}{25}$ d) $-\frac{27}{25}$

12. $\frac{3x^2+x+1}{(x-1)^4} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{(x-1)^4}$ then **A + B - C + D =**

- a) 0 b) 15 c) 1 d) 10

13. If $\frac{x^4+24x^2+28}{(x^2+1)^3} = \frac{A}{x^2+1} + \frac{B}{(x^2+1)^2} + \frac{C}{(x^2+1)^3}$ then **A + B + C =**

- a) 26 b) 27 c) 28 d) 29

14. The partial fractions of $\frac{1}{x^3(x+2)}$

a) $\frac{1}{8x} - \frac{1}{4x^2} + \frac{1}{2x^3} - \frac{1}{8(x+2)}$ b) $\frac{1}{8x} + \frac{1}{4x^2} + \frac{1}{2x^3} - \frac{1}{8(x+2)}$

c) $\frac{1}{8x} - \frac{1}{4x^2} - \frac{1}{2x^3} + \frac{1}{8(x+2)}$ d) $\frac{1}{8x} + \frac{1}{4x^2} + \frac{1}{2x^3} + \frac{1}{8(x+2)}$

15. If $\frac{(x+1)^2}{x^3+x} = \frac{A}{x} + \frac{Bx-C}{x^2+1}$, then $\sin^{-1}\left(\frac{A}{C}\right) =$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

16. If $\frac{x-4}{x^2-5x+6}$ can be expanded in the ascending powers of x, then the coefficient of x^3 is

a) $\frac{-73}{648}$ b) $\frac{73}{648}$ c) $\frac{71}{648}$ d) $\frac{-71}{648}$

17. The coefficient of x^n in $\frac{1}{(1-2x)(1-3x)}$ is

a) $3^{n+1} - 2^{n+1}$ b) $3^{n-1} + 2^{n+1}$
 c) $3^{n+1} - 2^{n-1}$ d) $3^{n-1} + 2^{n-1}$

18. The coefficient of x^n in $\frac{x+1}{(x-1)^2(x-2)}$ is

a) $1 - 2n - \frac{3}{2^{n+1}}$ b) $1 - 2n - \frac{3}{2^{n-1}}$
 c) $1 + 2n + \frac{3}{2^{n+1}}$ d) $1 + 2n - \frac{3}{2^{n-1}}$

19. If $a_K = \frac{1}{K(K+1)}$ for $K = 1, 2, 3, \dots, n$, then $\left(\sum_{K=1}^n a_K\right)^2 =$

a) $\frac{n}{n+1}$ b) $\frac{n^2}{(n+1)^2}$ c) $\frac{n^4}{(n+1)^4}$ d) $\frac{n^6}{(n+1)^6}$

20. $\frac{x+1}{(2x-1)(3x+1)} = \frac{A}{2x-1} + \frac{B}{3x+1}$ then $16A + 9B =$

- a) 4 b) 5 c) 6 d) 8

21. If $\frac{(x+1)^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ then $\cos^{-1}\left(\frac{A}{C}\right) =$

- a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

22. If $\frac{3x-2}{(x+1)(2x^2+3)} = \frac{A}{x+1} - \frac{Bx+C}{2x^2+3}$, then $A + B - C =$

- a) 0 b) 2 c) 3 d) 5

23. If $\frac{x^2+5x+1}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)(x+2)} + \frac{C}{(x+1)(x+2)(x+3)}$ then $B =$

- a) 1 b) -5 c) 0 d) 10

24. $\frac{1}{x^4+1} =$

a) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2+\sqrt{2}x-1)} + \frac{\sqrt{2}-x}{2\sqrt{2}(x^2+\sqrt{2}x-1)}$

b) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2+\sqrt{2}x+1)} + \frac{\sqrt{2}-x}{2\sqrt{2}(x^2-\sqrt{2}x+1)}$

c) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2+\sqrt{2}x-1)} + \frac{\sqrt{2}-x}{2\sqrt{2}(x^2-\sqrt{2}x+1)}$

d) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2-\sqrt{2}x+1)} + \frac{\sqrt{2}-x}{2\sqrt{2}(x^2-\sqrt{2}x+1)}$

25. The coefficient of x^n in $\frac{(1+x)(1+2x)(1+3x)}{(1-x)(1-2x)(1-3x)}$ is

- a) $12 - 30 \cdot 2^n + 20 \cdot 3^n$ b) $12 + 30 \cdot 2^n + 20 \cdot 3^n$
 c) $12 + 30 \cdot 2^n - 20 \cdot 3^n$ d) $12 - 20 \cdot 2^n - 20 \cdot 3^n$

26. The coefficient of x^n in $\frac{x-4}{x^2-5x+6}$ is

- a) $\frac{1}{3^{n+1}} - \frac{1}{2^n}$ b) $\frac{1}{3^{n+1}} + \frac{1}{2^n}$
 c) $\frac{1}{5^{n+1}} + \frac{1}{2^n}$ d) $\frac{1}{5^{n+1}} + \frac{1}{3^n}$

27. The coefficient of x^n in $\frac{1}{(1-x)(1-2x)(1-3x)}$ is

- a) $\frac{3^{n+2} - 2^{n+3} + 1}{2}$ b) $\frac{3^{n+2} + 2^{n+3} - 1}{2}$
 c) $\frac{3^{n+2} - 2^{n+3} - 1}{2}$ d) $\frac{3^{n+2} + 2^{n+3} + 1}{2}$

28. If $\frac{1}{(1-2x)^2(1-3x)} = \frac{A}{1-2x} + \frac{B}{(1-2x)^2} + \frac{C}{1-3x}$ then match the following

- 1) A a) 9
 2) B b) -6
 3) C c) -2
 a) a, b, c b) b, c, a c) c, a, b d) c, b, a

29. $\frac{2x^4+3x^2+1}{(x^2+1)^4} = \frac{A}{(x^2+1)} + \frac{B}{(x^2+1)^2} + \frac{C}{(x^2+1)^3} + \frac{D}{(x^2+1)^4}$ then match the following

- 1) A a) 2
 2) B b) 1

3) C c) - 1

4) D d) 0

e) 1/2

a) d, a, c, d b) c, a, e, d c) b, a, e, d d) c, b, e, d

30. Observe the following lists

List - I

1) If $\frac{3x}{(x-a)(x-b)} = \frac{2}{x-a} + \frac{1}{x-b}$ then a : b is

2) If $\frac{x+4}{(x^2-4)(x+1)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+1}$ then A + B + C is

3) If $\frac{2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ then C =

List - II

a) slope of x-axis

b) $\sin \frac{3\pi}{2}$

c) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

d) Slope of the line $6x + 3y - 7 = 0$

a) 1 - d; 2 - a; 3 - c

b) 1 - d; 2 - b; 3 - c

c) 1 - d; 2 - a; 3 - b

d) 1 - b; 2 - c; 3 - a

KEY

1. a 2. a 3. b 4. a 5. d 6. a 7. c 8. b 9. a 10. d

11. a 12. c 13. c 14. a 15. c 16. a 17. a 18. a 19. b 20. c

21. c 22. b 23. c 24. b 25. a 26. a 27. a 28. b 29. a 30. a

PARTIAL FRACTIONS

HINTS AND SOLUTIONS

1- 4. Verification

5. $f(1)=2, f(2) =5, f(x)=Q(x).(x-1)(x-2)+AX+B$

6. $f(-1)=6, f(-2) =15, f(2)=3, f(x)=Q(x).(x+1)(x+2)(x-2)+Ax^2+Bx+C.$

7- 16. Verification

17. Standard problem

18. Standard problem

19. $a_k = \frac{1}{k} - \frac{1}{k+1}$

20-30. find the values of A,B,C by verification.