MULTIPLE PRODUCTS

OBJECTIVES

1. **If** $\bar{a}=\bar{i}+2\bar{j}, \bar{b}=\bar{j}+2\bar{k}, \bar{c}=\bar{i}+2\bar{k}$, **then** $\bar{a}.(\bar{b}\times\bar{c})=$

   1) 8 2) 6 3) 4 5) 2

2. **If** $\bar{a}=3\bar{i}-\bar{j}+2\bar{k}$ **and** $\bar{b}=3\bar{i}+\bar{j}-\bar{k}$, **then** $\bar{a}.(\bar{a}\times\bar{b})=$

   1) 0 2) 1 3) 3 4) not defined

3. **The scalar** $\bar{a}.\{(\bar{b}\times\bar{c})\times(\bar{a}+\bar{b}+\bar{c})\}$ **is equal to**

   1) 0 2) $[\bar{a}\,\bar{b}\,\bar{c}]+[\bar{b}\,\bar{c}\,\bar{a}]$

   3) $[\bar{a}\,\bar{b}\,\bar{c}]$ 4) $2[\bar{a}\,\bar{b}\,\bar{c}]$

4. **If** $\bar{a}$ **is perpendicular to** $\bar{b}$ **and** $\bar{c}$, $|\bar{a}|=2, |\bar{b}|=3, |\bar{c}|=4$ **and the angle between** $\bar{b}$ **and** $\bar{c}$ **is** $2\frac{\pi}{3}$, **then** $|[\bar{a}\bar{b}\bar{c}]|$ =

   1) 12 2) $12\sqrt{3}$ 3) $\frac{12}{\sqrt{3}}$ 4) $12\sqrt{2}$

5. **The vector** $\bar{a}$ **lies in the plane of vectors** $\bar{b}$ **and** $\bar{c}$. **Which of the following is correct?**

   1) $\bar{a}.(\bar{b}\times\bar{c})=0$ 2) $\bar{a}.(\bar{b}\times\bar{c})=1$

   3) $\bar{a}.(\bar{b}\times\bar{c})=-1$ 4) $\bar{a}.(\bar{b}\times\bar{c})=3$

6. **For three vectors** $\bar{u},\bar{\upsilon},\bar{w}$ **which of the following expressions is not equal to any of the remaining three?**

   1) $\bar{u}.(\bar{\upsilon}\times\bar{w})$ 2) $(\bar{\upsilon}\times\bar{w}).\bar{u}$ 3) $\bar{\upsilon}.(\bar{u}.\bar{w})$ 4) $(\bar{u}\times\bar{\upsilon}).\bar{w}$

7. **Volume of the parallelepiped whose coterminous edges are** $2\bar{i}-3\bar{j}+4\bar{k}, \bar{i}+2\bar{j}-2\bar{k}$**,** $3\bar{i}-\bar{j}+\bar{k}$

1) 5 cu. Units 2) 6 cu. Units

3) 7 cu. Units 4) 8 cu. Units

8. **If** $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=2$**, then the volume of the parallelepiped whose coterminous edges are** $2\bar{a}+\bar{b}, 2\bar{b}+\bar{c}$ **and** $2\bar{c}+\bar{a}$ **is**

1) 9 cu. Units 2) 8 cu. Units

3) 18 cu. Units 4) 16 cu. Units

9. **If** $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=4$**, then the volume of the parallelepiped with** $\bar{a}+\bar{b}, \bar{b}+\bar{c}$ **and** $\bar{c}+\bar{a}$ **as coterminous edges is**

1) 6 2) 7 3) 8 4) 5

10. **If** $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=12$**, then** $\left[\bar{a}+\bar{b}\;\;\bar{b}+\bar{c}\;\;\bar{c}+\bar{a}\right]=$

1) 24 2) 36 3) 48 4) 26

11. **If** $\bar{a},\bar{b},\bar{c}$ **are linearly independent, then** $\dfrac{\left[2\bar{a}+\bar{b}\;\;2\bar{b}+\bar{c}\;\;2\bar{c}+\bar{a}\right]}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}=$

1) 9 2) 8 3) 7 4) 6

12. **If** $\bar{a},\bar{b},\bar{c}$ **are linearly independent and** $\dfrac{\left[(\bar{a}+2\bar{b})\times(2\bar{b}+\bar{c}).(5\bar{c}+\bar{a})\right]}{\bar{a}.\bar{b}\times\bar{c}}=k$**, then k is**

1) 10 2) 14 3) 18 4) 12

13. **If** $\bar{a},\bar{b},\bar{c}$ **are three non-coplanar vectors, then** $(\bar{a}+\bar{b}+\bar{c})\left[(\bar{a}+\bar{b})\times(\bar{a}+\bar{c})\right]$ **is equal to**

1) 0 2) $\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

3) $2\left[\bar{a}\,\bar{b}\,\bar{c}\right]$ 4) $-\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

14. **If** $\bar{a},\bar{b},\bar{c}$ **be three non-coplanar vectors and** $\left[\bar{a}-\bar{b}\;\;\bar{b}-\bar{c}\;\;\bar{c}-\bar{a}\right]=k\left[\bar{a}\;\bar{b}\;\bar{c}\right]$**, then k =**

1) 1 2) 0 3) -1 4) 2

15. **Let** $\bar{a}=\bar{i}-\bar{k},\bar{b}=x\bar{i}+\bar{j}+(1-x)\bar{k}$ **and** $\bar{c}=y\bar{i}+x\bar{j}+(1+x+y)\bar{k}$**. Then** $[\bar{a}\ \bar{b}\ \bar{c}]$ **depends on**

1) Only x 2) Only y

3) Neither x nor y 4) Both x and y

16. **The value of p such that the vectors** $\bar{i}+3\bar{j}-2\bar{k}$**,** $2\bar{i}-\bar{j}+4\bar{k}$ **and** $3\bar{i}+2\bar{j}+p\bar{k}$ **are coplanar, is**

1) 4 2) 2 3) 8 4) 10

17. **The vectors** $\lambda\bar{i}+\bar{j}+2\bar{k}$**,** $\bar{i}+\lambda\bar{j}-\bar{k}$ **and** $2\bar{i}-\bar{j}+\lambda\bar{k}$ **are coplanar if** $\lambda=$

1) $2,1\pm\sqrt{5}$ 2) $2,1\pm\sqrt{6}$

3) $-2,1\pm\sqrt{3}$ 4) $-3,1\pm\sqrt{2}$

18. **If** $(\bar{a}-\lambda\bar{b}).(\bar{b}-2\bar{c})\times(\bar{c}+3\bar{a})=0$ **then** λ **=**

1) $\frac{1}{6}$ 2) $-\frac{1}{6}$ 3) $\frac{1}{5}$ 4) $-\frac{1}{5}$

19. **If** $\bar{a},\bar{b},\bar{c}$ **are unit vectors perpendicular to each other, then** $[\bar{a}\,\bar{b}\,\bar{c}]^2=$

1) 1 2) 3 3) 2 4) 4

20. **If** $\bar{a}=2\bar{i}+3\bar{j},\bar{b}=\bar{i}+\bar{j}+\bar{k}$ **and** $\bar{c}=\lambda\bar{i}+4\bar{j}+2\bar{k}$ **are coterminous edges of a parallelepiped of volume 2 cu. Units, then a value of** λ **is**

1) 1 2) 2 3) 3 4) 4

21. $\frac{\bar{a}.(\bar{b}\times\bar{c})}{(\bar{c}\times\bar{a}).\bar{b}}+\frac{\bar{b}.(\bar{a}\times\bar{c})}{\bar{c}.(\bar{a}\times\bar{b})}$ **is**

1) 0 2) 1 3) -1 4) 2

22. **Let** $\bar{p}=\frac{\bar{a}\times\bar{c}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, $\bar{q}=\frac{\bar{c}\times\bar{a}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, $\bar{r}=\frac{\bar{a}\times\bar{b}}{[\bar{a}\,\bar{b}\,\bar{c}]}$ $\bar{a}\,\bar{b}\,\bar{c}$ **being any three non-coplanar vectors**

Then $\bar{p}(\bar{a}+\bar{b})+\bar{q}.(\bar{b}+\bar{c})+\bar{r}.(\bar{c}+\bar{a})$ **is equal to**

1) -3 2) 3 3) 0 4) -2

23. **If** $\left[2\bar{a}+4\bar{b}\ \bar{c}\ \bar{d}\right]=\lambda\left[\bar{a}\bar{c}\bar{d}\right]+\mu\left[\bar{b}\bar{c}\bar{d}\right]$**, then** $\lambda+\mu=$

1) 6 2) -6 3) 10 4) 8

24. **The position vectors of the points A,B,C,D are** $3\bar{i}-2\bar{j}-\bar{k}$, $2\bar{i}+3\bar{j}-4\bar{k}$, $-\bar{i}+\bar{j}+2\bar{k}$ **and** $4\bar{i}+5\bar{j}+\lambda\bar{k}$ **respectively. If A,B,C,D are coplanar, then** $\lambda=$

1) $-\frac{146}{17}$ 2) $\frac{146}{17}$ 3) $\frac{146}{15}$ 4) $-\frac{146}{15}$

25. **If the points (1,0,3), (-1,3,4), (1,2,1) and (a,2,5) are coplanar, then a =**

1) 1 2) -1 3) 2 4) -2

26. **Let** $\bar{a}$ **be a unit vector and** $\bar{b}$ **a non zero vector not parallel to** $\bar{a}$**. Then the angle between the vectors** $\bar{u}=\sqrt{3}\left(\bar{a}\times\bar{b}\right)$ **and** $\bar{\upsilon}=\left(\bar{a}\times\bar{b}\right)\times\bar{a}$ **is**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6}$

27. **The volume of the tetrahedron having vertices** $\bar{i}+\bar{j}-\bar{k}, 2\bar{i}+3\bar{j}+2\bar{k}, -\bar{i}+\bar{j}+3\bar{k}$ **and** $2\bar{k}$ **is**

1) $\frac{7}{3}$ cu. Units 2) $\frac{7}{6}$ cu. units

3) $\frac{7}{8}$ cu. Units 4) $\frac{5}{8}$ cu. units

28. **The volume of the tetrahedron with vertices (2,2,2),(4,3,3),(4,4,4) and (5,5,6) is**

1) $\frac{1}{2}$ cu. Units 2) $\frac{1}{4}$ cu. units 3) $\frac{1}{3}$ cu. units 4) $\frac{1}{5}$ cu. units

29. **Volume of the tetrahedron with vertices at (0,0,0), (1,0,0), (0,1,0) and (0,0,1) is**

1) $\frac{1}{6}$ cu. units 2) $\frac{1}{4}$ cu. units

3) $\frac{1}{3}$ cu. units 4) $\frac{1}{5}$ cu. units

30. The volume of the tetrahedron whose vertices are (1,-6,10), (-1,-3,7), (5,-1, λ) and (7,-4,7) is 11 cu. Units, then the value of λ is

1) 2 or 6   2) 3 or 4   3) 1 or 7   4) 5 or 6

31. If $\bar{i}, \bar{j}, \bar{k}$ are orthonormal unit vectors, then $\bar{i}(\bar{a}\times\bar{i}) + \bar{j}\times(\bar{a}\times\bar{j}) + \bar{k}(\bar{a}\times\bar{k})$ is

1) $\bar{a}$ 2) $2\bar{a}$ 3) $3\bar{a}$ 4) $4\bar{a}$

32. If $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$, and $\bar{c} = \bar{i} + 3\bar{j} - 2\bar{k}$ and $\bar{a}\times(\bar{b}\times\bar{c}) = p\bar{i} + q\bar{j} + r\bar{k}$, then

p+q+r =

1) -4   2) 4   3) 2   4) -2

33. If $\bar{a} = \bar{i} + \bar{j} + \bar{k}$, $\bar{b} = \bar{i} + \bar{j}$, $\bar{c} = \bar{i}$ and $(\bar{a}\times\bar{b})\times\bar{c} = \lambda\bar{a} + \mu\bar{b}$, then $\lambda + \mu =$

1) 1   2) 0   3) -1   4) 2

34. If $\bar{a} = 2\bar{i} - 3\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} - \bar{j} + \bar{k}$, then $\bar{a}\times(\bar{b}\times\bar{c})$ is perpendicular to

1) $\bar{a}$ 2) $\bar{b}$ 3) $\bar{c}$ 4) $\bar{a}\times\bar{b}$

35. $\bar{a}\times\{\bar{a}\times(\bar{a}\times\bar{b})\}$ is equal to

1) $(\bar{a}.\bar{a})(\bar{b}\times\bar{a})$ 2) $(\bar{a}.\bar{a})(\bar{a}\times\bar{b})$

3) $\bar{a}.\bar{a}$ 4) $\bar{a}\times\bar{b}$

36. $(\bar{a}\times\bar{b})\times(\bar{a}\times\bar{c}).\bar{d}$ is equal to

1) $[\bar{a}\,\bar{b}\,\bar{c}]$ 2) $[\bar{a}\,\bar{b}\,\bar{c}](\bar{c}.\bar{d})$

3) $[\bar{a}\,\bar{b}\,\bar{c}](\bar{c}.\bar{d})$ 4) $[\bar{b}\,\bar{a}\,\bar{c}](\bar{b}.\bar{d})$

37. Which of the following statement is not true?

1) $\bar{a}(\bar{b}+\bar{c}) = \bar{a}.\bar{b} + \bar{a}.\bar{c}$ 2) $(\bar{a}\times\bar{b})\times\bar{c} = \bar{a}\times(\bar{b}\times\bar{c})$

3) $\bar{a}.(\bar{b}\times\bar{c}) = (\bar{a}\times\bar{b}).\bar{c}$ 4) $\bar{a}\times(\bar{b}+\bar{c}) = (\bar{a}\times\bar{b}) + (\bar{a}\times\bar{c})$

38. $\{\bar{a}.(\bar{b}\times\bar{i})\}\bar{i} + \{\bar{a}.(\bar{b}\times\bar{j})\}\bar{j} + \{\bar{a}.(\bar{b}\times\bar{k})\}\bar{k} =$

1) $2(\bar{a}\times\bar{b})$ 2) $3(\bar{a}\times\bar{b})$ 3) $\bar{a}\times\bar{b}$ 4) $(\bar{a}\times\bar{b})$

39. The position vectors of three non- collinear points A,B,C are $\bar{a},\bar{b},\bar{c}$ respectively. The distance of the origin from the plane through A,B,C is

1) $\dfrac{1}{\left|(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})\right|}$ 2) $\dfrac{2[\bar{a}\,\bar{b}\,\bar{c}]}{\left|(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})\right|}$

3) $\dfrac{[\bar{a}\,\bar{b}\,\bar{c}]}{\left|(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})\right|}$ 4) $\dfrac{3[\bar{a}\,\bar{b}\,\bar{c}]}{\left|(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})\right|}$

40. The shortest distance between the lines $\bar{r}=(3\bar{i}+8\bar{j}+3\bar{k})+s(3\bar{i}-\bar{j}+\bar{k})$ **and** $\bar{r}=(-3\bar{i}-7\bar{j}+6\bar{k})+t(-3\bar{i}+2\bar{j}+4\bar{k})$ **is**

1) $\sqrt{30}$ 2) $2\sqrt{30}$ 3) $3\sqrt{30}$ 4) $4\sqrt{30}$

41. If the four points $\bar{a},\bar{b},\bar{c},\bar{d}$ are coplanar, then $[\bar{b}\,\bar{c}\,\bar{d}]+[\bar{c}\,\bar{a}\,\bar{d}]+[\bar{a}\,\bar{b}\,\bar{d}]=$

1) 0 2) $[\bar{a}\,\bar{b}\,\bar{c}]$

2) $2[\bar{a}\,\bar{b}\,\bar{c}]$ 4) $3[\bar{a}\,\bar{b}\,\bar{c}]$

42. Let $\bar{a}=a_1\bar{i}+a_2\bar{j}+a_3\bar{k}$, , $\bar{b}=b_1\bar{i}+b_2\bar{j}+b_3\bar{k}$, **and** $\bar{c}=c_1\bar{i}+c_2\bar{j}+c_3\bar{k}$ **be three non-zero and non coplanar vectors such that $\bar{c}$ is a unit vector perpendicular to both $\bar{a}$ and $\bar{b}$. If the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{6}$, then** $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=$

1) 0 2) $\pm\frac{1}{2}\sqrt{(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)}$

3) 1 4) -1

43. If $\bar{a}$ $\bar{b}$ are non-zero and non-collinear vectors, then $[\bar{a}\,\bar{b}\,\bar{i}]\bar{i}+[\bar{a}\,\bar{b}\,\bar{j}]\bar{j}+[\bar{a}\,\bar{b}\,\bar{k}]\bar{k}$ **is**

1) $\bar{a}+\bar{b}$ 2) $\bar{a}\times\bar{b}$ 3) $\bar{a}-\bar{b}$ 4) $\bar{b}\times\bar{a}$

44. If three unit vectors $\bar{a}$, $\bar{b}$, $\bar{c}$ are such that $\bar{b}\times(\bar{b}\times\bar{c})=\frac{1}{2}\bar{b}$, then the vector $\bar{a}$ makes with $\bar{b}$ and $\bar{c}$ respectively the angles

1) 40°, 80° 2) 45°, 45° 3) 30°, 60° 4) 90°, 60°

45 Let $\bar{a},\bar{b},\bar{c}$ be three non-coplanar vectors each having a unit magnitude. If $\bar{a}\times(\bar{b}\times\bar{c})=\frac{1}{\sqrt{2}}(\bar{b}+\bar{c})$, then the angle between $\bar{a}$ and $\bar{b}$

1) 60° 2) 30° 3) 135° 4) 120°

46. Let $\bar{a},\bar{b},\bar{c}$ be three vectors having magnitude 1, 1 and 2 respectively. If $\bar{a}\times(\bar{a}\times\bar{c})+\bar{b}=\bar{O}$, then the acute angle between $\bar{a}$ and $\bar{c}$ is

1) 30° 2) 60° 3) 45° 4) 75°

47. Let $\bar{p},\bar{q},\bar{r}$ be the three mutually perpendicular vectors of the same magnitude. If a vector $\bar{x}$ satisfies the equation $\bar{p}\times\{(\bar{x}-\bar{r})\times\bar{p}\}+\bar{q}\times\{(\bar{x}-\bar{r})\times\bar{q}\}+\bar{r}\times\{(\bar{x}-\bar{p})\times\bar{r}\}=\bar{O}$, then $\bar{x}$ is given by

1) $\frac{1}{2}(\bar{p}+\bar{q}-2\bar{r})$ 2) $\frac{1}{2}(\bar{p}+\bar{q}+2\bar{r})$

3) $\frac{1}{3}(\bar{p}+\bar{q}+\bar{r})$ 4) $\frac{1}{3}(2\bar{p}+\bar{q}-\bar{r})$

48. If $a(\bar{\alpha}\times\bar{\beta})+b(\bar{\beta}\times\bar{\gamma})+c(\bar{\gamma}\times\bar{\alpha})=\bar{O}$ and at least one of a a,b and c is non-zero, then vectors $\bar{\alpha},\bar{\beta}\bar{\gamma}$

1) Parallel 2) Mutually Perpendicular

3) Coplanar 4) None of these

49. If $(\bar{b}\times\bar{c})\times(\bar{c}\times\bar{a})=3\bar{c}$, then $[\bar{b}\times\bar{c}\,\bar{c}\times\bar{a}\,\bar{a}\times\bar{b}]=$

1) 2 2) 7 3) 9 4) 11

50. **Vector** $\bar{c}$ **is perpendicular to** $\bar{a}$ $\bar{a} = 2\bar{i} + 3\bar{j} - \bar{k}$ **and** $\bar{b} = \bar{i} - 2\bar{j} + 3\bar{k}$. **Also** $\bar{c}.\left(2\bar{i} - \bar{j} + \bar{k}\right) = 6$. Then the value of $\left[\bar{a}\,\bar{b}\,\bar{c}\right]$ is

1) 36 2) -36 3) -63 4) 63

51. **If** $\bar{b}$ **and** $\bar{c}$ **are any two non-collinear unit vectors and** $\bar{a}$ **is any vector, then**

$$\left(\bar{a}.\bar{b}\right)\bar{b} + \left(\bar{a}.\bar{c}\right)\bar{c} + \frac{\bar{a}.\left(\bar{b}\times\bar{c}\right)}{\left|\bar{b}\times\bar{c}\right|^2}\left(\bar{b}\times\bar{c}\right) =$$

1) $2\bar{a}$ 2) $3\bar{a}$ 3) $\bar{a}$ 4) $4\bar{a}$

52. **The position vectors of vertices of** ΔABC **are** $\bar{a}, \bar{b}, \bar{c}$. **Given that** $\bar{a}.\bar{a} = \bar{b}.\bar{b} = \bar{c}.\bar{c} = 9$ **and** $\left[\bar{a}\,\bar{b}\,\bar{c}\right] = o$. **Then the position vector of the orthocentre of** ΔABC **is**

1) $\bar{a} - \bar{b} + \bar{c}$ 2) $\bar{a} + \bar{b} - \bar{c}$ 3) $\bar{a} + \bar{b} + \bar{c}$ 4) $\bar{b} + \bar{c} - \bar{a}$

53. $\left[\bar{a}\times\left(3\bar{b} + 2\bar{c}\right), \bar{b}\times\left(\bar{c} - 2\bar{c}\right), 2\bar{c}\times\left(\bar{a} - 3\bar{b}\right)\right]$

1) $-1\left[\bar{a}\times\bar{b}, \bar{b}\times\bar{c}, \bar{c}\times\bar{a}\right]$ 2) $18\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2$

3) $-18\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2$ 4) $6\left[\bar{a}\times\bar{b}, \bar{b}\times\bar{c}, \bar{c}\times\bar{a}\right]$

MULTIPLE PRODUCTS

HINTS AND SOLUTIONS

Scalar and Vector Triple Products

1. (2)

$$\bar{a}\cdot(\bar{b}\times\bar{c})[\bar{a}\ \bar{b}\ \bar{c}|=\begin{vmatrix}1 & 2 & 0\\ 0 & 1 & 2\\ 1 & 0 & 2\end{vmatrix}$$

$$=1(2-0)-2(0-2)=2+4=6.$$

2. (4)

$\bar{a}\cdot\bar{b}$ is a scalar. Hence $\bar{a}\times(\bar{a}\cdot\bar{b})$ i.e. cross product between a vector and a scalar is not defined.

3. (1)

G.E. = $\bar{a}\cdot\{(\bar{b}\times\bar{c})\times(\bar{a}+\bar{b}+\bar{c})\}$

$$=\bar{a}\cdot\{(\bar{b}\times\bar{a})+(\bar{b}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})+(\bar{c}\times\bar{b})+(\bar{c}\times\bar{c})\}$$
$$=\bar{a}\cdot\{(\bar{b}\times\bar{a})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})+(\bar{c}\times\bar{b})\}$$
$$=\bar{a}\cdot(\bar{b}\times\bar{a})+\bar{a}\cdot(\bar{b}\times\bar{c})+\bar{a}\cdot(\bar{c}\times\bar{a})+\bar{a}\cdot(\bar{c}\times\bar{b})$$
$$=[\bar{a}\ \bar{b}\ \bar{a}]+[\bar{a}\ \bar{b}\ \bar{c}]+[\bar{a}\ \bar{c}\ \bar{a}]+[\bar{a}\ \bar{c}\ \bar{b}]$$
$$=0+[\bar{a}\ \bar{b}\ \bar{c}]+0-[\bar{a}\ \bar{b}\ \bar{c}]=0.$$

4. (2)

$\bar{a}\perp\bar{b},\ \bar{a}\perp\bar{c}\Rightarrow\bar{a}\,//(\bar{b}\times\bar{c})$

Now $|\bar{b}\times\bar{c}|=|\bar{b}|\cdot|\bar{c}|\sin 120°$

$$=3(4)\left(\frac{\sqrt{3}}{2}\right)=6\sqrt{3}.$$

Then $\left|[\bar{a}\ \bar{b}\ \bar{c}]\right|=\left|\bar{a}\cdot(\bar{b}\times\bar{c})\right|$

$=|\bar{a}|\cdot|\bar{b}\times\bar{c}|\cos(\bar{a},\bar{b}\times\bar{c})$

$=2(6\sqrt{3})\cos 0° = 12\sqrt{3}.$

5. (1)

Given $\bar{a},\bar{b},\bar{c}$ are coplanar. But $\bar{b}\times\bar{c}$ is a vector perpendicular to the plane containing $\bar{b}$ and $\bar{c}$.

Hence $\bar{b}\times\bar{c}$ is also perpendicular to $\bar{a}$

$\therefore \bar{a}\cdot(\bar{b}\times\bar{c}) = 0$.

6. (3)

$\bar{v}\cdot(\bar{u}\cdot\bar{w})$ is not equal to the remaining three.

7. (3)

Volume = $\left\|\begin{matrix} 2 & -3 & 4 \\ 1 & 2 & 2 \\ 3 & -1 & -1 \end{matrix}\right\|$

$=|2(2+2)+3(1-6)+4(-1-6)|=7$

8. (3)

Volume = $\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix}[\bar{a}\ \bar{b}\ \bar{c}]$

$=[2(4-0)-0+1(1-0)](2)=18$.

9. (3)We have

$[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}] = 2[\bar{a}\ \bar{b}\ \bar{c}] = 2(4) = 8$.

10. (1)

$[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}] = 2[\bar{a}\ \bar{b}\ \bar{c}] = 2(12) = 24$

11. (1)

$[2\bar{a}+\bar{b}\ \ 2\bar{b}+\bar{c}\ \ 2\bar{c}+\bar{a}]$

$$=\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix}[\bar{a}\ \bar{b}\ \bar{c}]$$

$$=9[\bar{a}\ \bar{b}\ \bar{c}]$$

$$\text{G.E.}=\frac{9[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{a}\ \bar{b}\ \bar{c}]}=9.$$

12. (4)

$$\frac{\begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 5 \end{vmatrix}[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{a}\ \bar{b}\ \bar{c}]}=k \Rightarrow 12=k$$

13. (4)

$$\text{G.E.}=(\bar{a}+\bar{b}+\bar{c})\cdot[(\bar{a}+\bar{b})\times(\bar{a}+\bar{c})]$$

$$(\bar{a}+\bar{b}+\bar{c})\cdot[(\bar{a}\times\bar{b})+(\bar{a}\times\bar{c})+(\bar{b}\times\bar{a})+(\bar{b}\times\bar{c})]$$

$$=\bar{a}(\bar{a}\times\bar{c})+\bar{a}(\bar{b}\times\bar{a})+\bar{a}(\bar{b}\times\bar{c})+\bar{b}(\bar{a}+\bar{c})+\bar{b}(\bar{b}+\bar{a})+\bar{b}(\bar{b}\times\bar{c})+\bar{c}(\bar{a}\times\bar{c})+\bar{c}(\bar{b}\times\bar{a})+\bar{c}(\bar{b}\times\bar{c})$$

$$(\because \bar{a}\times\bar{a}=\bar{0})$$

$$=0+0+[\bar{a}\ \bar{b}\ \bar{c}]-[\bar{b}\ \bar{a}\ \bar{c}]+0+0+0+[\bar{c}\ \bar{b}\ \bar{a}]+0$$

$$=[\bar{a}\ \bar{b}\ \bar{c}]-[\bar{b}\ \bar{c}\ \bar{a}]-[\bar{c}\ \bar{a}\ \bar{b}]$$

$$=[\bar{b}\ \bar{c}\ \bar{a}]-[\bar{b}\ \bar{c}\ \bar{a}]-[\bar{a}\ \bar{b}\ \bar{c}]=-[\bar{a}\ \bar{b}\ \bar{c}]$$

14. (2)

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix}[\bar{a}\ \bar{b}\ \bar{c}]=k[\bar{a}\ \bar{b}\ \bar{c}]$$

$$\Rightarrow 0=k[\bar{a}\ \bar{b}\ \bar{c}]$$

$$\Rightarrow k=0\ \{\because [\bar{a}\ \bar{b}\ \bar{c}]\neq 0\}$$

15. (3)

$$[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix}$$

Applying: $C_3 \to C_3 + C_1$

$= 1(1 + x) - 1.x = 1 + x - x = 1$

(Expanding along R_1)

This depends neither on x nor on y.

16. (2)

Given vectors are coplanar

$$\Rightarrow \begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 3 & 2 & p \end{vmatrix} = 0$$

$\Rightarrow 1(-p-8) - 2(3p+4) + 3(12-2) = 0$

$\Rightarrow 7p + 14 = 0 \Rightarrow p = 2.$

17. (3)

Given vectors are coplanar $\Leftrightarrow$

$$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda(\lambda^2 - 1) - 1(\lambda + 2) + 2(-1 - 2\lambda) = 0$

$\Rightarrow \lambda^3 - 6\lambda - 4 = 0$

By inspection one value of λ is –2.

$\therefore (\lambda + 2)(\lambda^2 - 2\lambda - 2) = 0$.

Now $\lambda^2 - 2\lambda - 2 = 0 \Rightarrow \lambda = \dfrac{2 \pm \sqrt{4+8}}{2}$

$$= \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

$\therefore \lambda = -2, 1 \pm \sqrt{3}$

18. (2)

$[\bar{a} - \lambda\bar{b} \;\; \bar{b} - 2\bar{c} \;\; \bar{c} + 3\bar{a}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & -\lambda & 0 \\ 0 & 1 & -2 \\ 3 & 0 & 1 \end{vmatrix} [\bar{a} \; \bar{b} \; \bar{c}] = 0$$

$\Rightarrow 1(1-0) + \lambda(0+6) = 0$

Since $[\bar{a} \; \bar{b} \; \bar{c}] \neq 0 \Rightarrow \lambda = -1/6.$

19. (1)

Given

$|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$ and $\bar{a} \perp \bar{b}, \bar{b} \perp \bar{c}, \bar{c} \perp \bar{a}$.

Now $\bar{b} \times \bar{c} = |\bar{b}| \cdot |\bar{c}| \sin 90° \bar{n}$

$= 1 \cdot 1 \cdot 1 \cdot \bar{n} \Rightarrow \bar{n}$ is $\perp \bar{b}$ and $\bar{c}$

But $\bar{a}$ is $\perp$ to $\bar{b}$ and $\bar{c} \Rightarrow \bar{a} \parallel \bar{n}$

$\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot \bar{n} = |\bar{a}| \cdot |\bar{n}| \cos 0°$

or $|\bar{a}||\bar{n}| \cos \pi = 1$ or -1.

$\Rightarrow [\bar{a} \; \bar{b} \; \bar{c}] = \pm 1$

$\Rightarrow [\bar{a} \; \bar{b} \; \bar{c}]^2 = 1.$

20. (4)

$$\text{Volume} = \left\| \begin{matrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ \lambda & 4 & 2 \end{matrix} \right\| = 2 \text{(given)}$$

$\Rightarrow |2(2-4) - 3(2-\lambda) + 0| = 2$

$\Rightarrow |3\lambda - 10| = 2$

$3\lambda - 10 = 2 \Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4.$

21. (1)

$$\text{G.E.} = \frac{[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{c}\ \bar{a}\ \bar{b}]} + \frac{[\bar{b}\ \bar{a}\ \bar{c}]}{[\bar{c}\ \bar{a}\ \bar{b}]}$$

$$= \frac{[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{c}\ \bar{a}\ \bar{b}]} - \frac{[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{c}\ \bar{a}\ \bar{b}]} = 0$$

22. (2)

$$\text{G.E.} = \frac{(\bar{b}\times\bar{c})\cdot(\bar{a}+\bar{b})}{[\bar{a}\ \bar{b}\ \bar{c}]} +$$

$$\frac{(\bar{c}\times\bar{a})\cdot(\bar{b}+\bar{c})}{[\bar{a}\ \bar{b}\ \bar{c}]} + \frac{(\bar{a}\times\bar{b})\cdot(\bar{c}+\bar{a})}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

$$= \frac{\left[\begin{array}{l}\bar{a}\cdot(\bar{a}\times\bar{c})+\bar{b}\cdot(\bar{b}\times\bar{c})+\bar{b}\cdot(\bar{c}\times\bar{a})+ \\ \qquad \bar{c}\cdot(\bar{c}\times\bar{a})+\bar{c}(\bar{a}\times\bar{b})+\bar{a}(\bar{a}\times\bar{b})\end{array}\right]}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

$$= \frac{[\bar{a}\ \bar{b}\ \bar{c}]+0+[\bar{b}\ \bar{c}\ \bar{a}]+0+[\bar{c}\ \bar{a}\ \bar{b}]+0}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

$$= \frac{3[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{a}\ \bar{b}\ \bar{c}]} = 3.$$

23. (1)

$$(2\bar{a}+4\bar{b})\cdot(\bar{c}\times\bar{d}) = 2(\bar{a}\cdot\bar{c}\times\bar{d})+4(\bar{b}\cdot\bar{c}\times\bar{d})$$

$$= 2[\bar{a}\ \bar{c}\ \bar{d}]+4[\bar{b}\ \bar{c}\ \bar{d}]+\lambda[\bar{a}\ \bar{b}\ \bar{c}]+\mu[\bar{b}\ \bar{c}\ \bar{d}]$$

$$\Rightarrow \lambda = 2, \mu = 4$$

$$\Rightarrow \lambda+\mu = 6.$$

24. (1)

Given A(3, –2, –1), B(2, 3, –4),

C = (–1, 1, 2), D = (4, 5, λ)

$\overline{AB} = (-1, 5, -3), \overline{AC} = (-4, 3, 3)$, $\overline{AD} = (1, 7, \lambda+1)$

Given A, B, C, D are coplanar

$$\Rightarrow [\overline{AB}\ \overline{AC}\ \overline{AD}] = 0 \Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda+1 \end{vmatrix} = 0$$

$\Rightarrow 17\lambda = -146 \Rightarrow \lambda = -146/17.$

25. (2)

Let A = (1, 0, 3), B = (–1, 3, 4),

C = (1, 2, 1), D = (a, 2, 5)

Then $\overline{AB} = (-2, 3, 1), \overline{AC} = (0, 2, -2)$

$\overline{AD} = (a-1, 2, 2)$

Given points are coplanar

$\Rightarrow [\overline{AB}\ \overline{AC}\ \overline{AD}] = 0$

$\Rightarrow 8a = -8 \Rightarrow a = -1.$

26. (3)

If θ is the angle between the given vectors, then $\cos\theta = \dfrac{\sqrt{3}(\overline{a}\times\overline{b})\cdot\{(\overline{a}\times\overline{b})\times\overline{a}\}}{\sqrt{3}\,|\overline{a}\times\overline{b}|\cdot|(\overline{a}\times\overline{b})\times\overline{a}|}$

$$= \frac{\sqrt{3}(\overline{a}\times\overline{b})\cdot(\overline{a}\times\overline{b})\overline{a}}{\sqrt{3}\,|\overline{a}\times\overline{b}|\cdot|(\overline{a}\times\overline{b})\times\overline{a}} = 0$$

$\Rightarrow \theta = 90° = \pi/2.$

27. (1)

Let A = (1, 1, –1), B = (2, 3, 2),

C = (–1, 1, 3), D = (0, 0, 2)

$\overline{AB} = (2-1, 3-1, 2+1) = (1, 2, 3)$

$\overline{AC} = (-2, 0, 4), \overline{AD} = (-1, -1, 3)$

Volume $= \dfrac{1}{6}[\overline{AB}\ \overline{AC}\ \overline{AD}]$

$$=\frac{1}{6}\begin{vmatrix}1 & 2 & 3\\ -2 & 0 & 4\\ -1 & -1 & 3\end{vmatrix}=\frac{1}{6}(14)=\frac{7}{3}\text{cu.units}$$

28. (3)

Let A = (2, 2, 2), B = (4, 3, 3), C = (4, 4, 4) and D = (5, 5, 6), be the vertices. Then volume of the tetrahedron

$$=\frac{1}{6}[\overline{AB}\ \overline{AC}\ \overline{AD}]$$

$$=\frac{1}{6}\left|[(2,1,1),(2,2,2),(3,3,4)]\right|$$

$$=\frac{1}{6}\left\|\begin{matrix}2 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 4\end{matrix}\right\|=\frac{1}{6}(2)=\frac{1}{3}\text{cu.units.}$$

29. (1)

Volume $=\frac{1}{6}[\overline{OA}\ \overline{OB}\ \overline{OC}]$ where O, A, B, C are the given points.

$$\Rightarrow V=\frac{1}{6}\left\|\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right\|=\frac{1}{6}(1)=\frac{1}{6}\text{cu.units.}$$

30. (3)

Let A = (1, –6, 10), B = (–1, –3, 7),

C = (5, –1, λ), D = (7, –4, 7)

Then $\overline{AB}=(-2,3,-3),\overline{AC}=(4,5,\lambda-10)$ and $\overline{AD}=(6,2,-3)$

$$\text{Volume} = \frac{1}{6}\begin{vmatrix}-2 & 4 & 6\\ 3 & 5 & 2\\ -3 & \lambda-10 & -3\end{vmatrix}=11$$

$$\Rightarrow 22\lambda-88=\pm 66\Rightarrow \lambda=7 \text{ or } 1$$

31. (2)

$$G.E. = (\overline{i}\cdot\overline{i})a - (\overline{i}\cdot\overline{a}) + (\overline{j}\cdot\overline{j})\overline{a}$$
$$-(\overline{j}\cdot\overline{a})\overline{j} + (\overline{k}\cdot\overline{k})\overline{a} - (\overline{k}\cdot\overline{a})\overline{k}$$
$$= 3\overline{a} - \left[(\overline{i}\cdot\overline{a})\overline{i} + (\overline{j}\cdot\overline{a})\overline{j} + (\overline{k}\cdot\overline{a})\overline{k}\right]$$
$$= 3\overline{a} - \overline{a} = 2\overline{a}$$

32. (1)

$$(\overline{a}\cdot\overline{c})\overline{b} - (\overline{a}\cdot\overline{b})\overline{c} = p\overline{i} + q\overline{i} + r\overline{k}$$

$$\Rightarrow (1-6+6)(2\overline{i} + \overline{j} - \overline{k}) -$$
$$(2-2+3)(\overline{i} + 3\overline{j} - 2\overline{k}) = p\overline{i} + q\overline{j} + r\overline{k}$$

$$\Rightarrow -\overline{i} - 8\overline{j} + 5\overline{k} = p\overline{i} + q\overline{j} + r\overline{k}$$
$$\Rightarrow p + q + r = -1 - 8 + 5 = -4$$

33. (2)

$$(\overline{c}\cdot\overline{a})\overline{b} - (\overline{c}\cdot\overline{b})\overline{a} = \lambda\overline{a} + \mu\overline{b}$$
$$\Rightarrow \lambda = -(\overline{c}\cdot\overline{b}) = -1$$
and $\mu = \overline{c}\cdot\overline{a} = 1 \Rightarrow \lambda + \mu = -1 + 1 = 0$

34. (1)

$$\overline{a}\times(\overline{b}\times\overline{c}) = (\overline{a}\cdot\overline{c})\overline{b} - (\overline{a}\cdot\overline{b})\overline{c}$$
$$= (2-3+4)(\overline{i} + \overline{j} - \overline{k}) - (2+3-4)(\overline{i} - \overline{j} + \overline{k})$$
$$= 2\overline{i} + 4\overline{j} - 4\overline{k}$$
Now $\overline{a}\times(\overline{b}\times\overline{c}) \perp \overline{a}$

since $[\overline{a}\times(\overline{b}\times\overline{c})]\cdot\overline{a} = 4 + 12 - 16 = 0$

35. (1)

$$\overline{a}\times\{\overline{a}\times(\overline{a}\times\overline{b})\} = \overline{a}\{(\overline{a}\cdot\overline{b})\overline{a} - (\overline{a}\cdot\overline{a})\overline{b}\}$$
$$(\overline{a}\times\overline{a})(\overline{a}\cdot\overline{b}) - (\overline{a}\cdot\overline{a})(\overline{a}\times\overline{b})$$
$$= \overline{0} + (\overline{a}\cdot\overline{a})(\overline{b}\times\overline{a}) = (\overline{a}\cdot\overline{a})(\overline{b}\times\overline{a})$$

36. (3)

$(\bar{a}\times\bar{b})\times(\bar{a}\times\bar{c})\cdot\bar{d}$

$=\left[\{(\bar{a}\times\bar{b})\bar{c}\}\bar{a}-\{(\bar{a}\times\bar{b})\bar{a}\}\bar{c}\right]\bar{d}$

$=\{[\bar{a}\ \bar{b}\ \bar{c}]\bar{a}-[\bar{a}\ \bar{b}\ \bar{a}]\bar{c}\}\bar{d}=[\bar{a}\ \bar{b}\ \bar{c}](\bar{a}\cdot\bar{d}).$

37. (2)

$(\bar{a}\times\bar{b})\times\bar{c}\neq\bar{a}\times(\bar{b}\times\bar{c})$.

Hence (2) is not true.

38. (3)

$\text{G.E.}=\{\bar{a}\cdot(\bar{b}\times\bar{i})\}i+\{\bar{a}\cdot(\bar{b}\times\bar{j})\}\bar{j}+\{\bar{a}\cdot(\bar{b}\times\bar{k})\}\bar{k}$

Interchanging $\cdot$ and $\times$, we get

$=\left\{(\bar{a}\times\bar{b})\cdot\bar{i}\right\}\bar{i}+\left\{(\bar{a}\times\bar{b})\cdot\bar{j}\right\}\bar{j}+\left\{(\bar{a}\times\bar{b})\cdot\bar{k}\right\}\bar{k}$

Let $\bar{a}\times\bar{b}=x\bar{i}+y\bar{j}+z\bar{k}$

$\Rightarrow(\bar{a}\times\bar{b})\cdot\bar{i}=x,(\bar{a}\times\bar{b})\cdot\bar{j}=y,(\bar{a}\times\bar{b})\cdot\bar{k}=z$

Hence

G.E. = $[\bar{a}\ \bar{b}\ \bar{i}]\bar{i}+[\bar{a}\ \bar{b}\ \bar{j}]\bar{j}+[\bar{a}\ \bar{b}\ \bar{k}]\bar{k}$

$=x\bar{i}+y\bar{j}+z\bar{k}=\bar{a}\times\bar{b}.$

39. (3)

The equation of the plane passing through the points $A(\bar{a}),B(\bar{b}),C(\bar{c})$ is

$\bar{r}\cdot\{(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})\}=[\bar{a}\ \bar{b}\ \bar{c}]$

$\therefore$ Distance from the origin to the plane

$$=\frac{[\bar{a}\ \bar{b}\ \bar{c}]}{|(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})}.$$

40. (3)

Here $\bar{a} = 3\bar{i} + 8\bar{j} + 3\bar{k}, \bar{b} = 3\bar{i} - \bar{j} + \bar{k},$

$\bar{c} = -3\bar{i} - 7\bar{j} + 6\bar{k}, \bar{d} = -3\bar{i} + 2\bar{j} + 4\bar{k}$

$$\bar{b} \times \bar{d} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$= \bar{i}(-4-2) - \bar{j}(12+3) + \bar{k}(6-3)$

$= -6\bar{i} - 15\bar{j} + 3\bar{k}$

41. (2)

Points $A(\bar{a}), B(\bar{b}), C(\bar{c}), D(\bar{d})$ are coplanar.

$\Rightarrow \overline{AB}, \overline{AC}, \overline{AD}$ are coplanar

$\Rightarrow [\overline{AB}, \overline{AC}, \overline{AD}] = 0$

$\Rightarrow [\bar{b} - \bar{a} \;\; \bar{c} - \bar{a} \;\; \bar{d} - \bar{a}] = 0$

$\Rightarrow (\bar{b} - \bar{a}) \times (\bar{c} - \bar{a}) \cdot (\bar{d} - \bar{a}) = 0$

$\Rightarrow \{(\bar{b} \times \bar{c}) - (\bar{b} \times \bar{a}) - (\bar{a} \times \bar{c})\} \cdot (\bar{d} - \bar{a}) = 0$

$\Rightarrow \{(\bar{b} \times \bar{c}) - (\bar{a} \times \bar{b}) + (\bar{c} \times \bar{a})\} \cdot (\bar{d} - \bar{a}) = 0$

$\Rightarrow (\bar{b} \times \bar{c}) \cdot \bar{d} - (\bar{b} \times \bar{c}) \cdot \bar{a} + (\bar{a} \times \bar{b}) \cdot \bar{d} - (\bar{a} \times \bar{b}) \cdot \bar{a} + (\bar{c} \times \bar{a}) \cdot \bar{d} - (\bar{c} \times \bar{a}) \cdot \bar{a} = 0$

$\Rightarrow [\bar{b} \;\; \bar{c} \;\; \bar{d}] + [\bar{a} \;\; \bar{b} \;\; \bar{d}] + [\bar{c} \;\; \bar{a} \;\; \bar{d}] = [\bar{a} \;\; \bar{b} \;\; \bar{c}]$

since $[\bar{a} \;\; \bar{b} \;\; \bar{c}] = 0$ and $[\bar{c} \;\; \bar{a} \;\; \bar{a}] = 0$ and

$(\bar{b} \times \bar{c}) \cdot \bar{a} = \bar{a} \cdot (\bar{b} \times \bar{c})$.

42. (2)

$\bar{c} \parallel (\bar{a} \times \bar{b})$ and $|\bar{c}| = 1$. Also $(\bar{a}, \bar{b}) = 30°$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = [\bar{a}\ \bar{b}\ \bar{c}] = \bar{c}\cdot(\bar{a}\times\bar{b})$$

$=|\bar{c}|.|\bar{a}\times\bar{b}|\cos 0°$ or $\cos 180°$

$=\pm 1\cdot|\bar{a}|\cdot|\bar{b}|\sin 30°$

$=\pm\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}\,\frac{1}{2}$

$=\pm\frac{1}{2}\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}$

43. (2)

Let $\bar{a}\times\bar{b} = x\bar{i}+y\bar{j}+z\bar{k}$

$\therefore (\bar{a}\times\bar{b})\cdot\bar{i} = x, (\bar{a}\times\bar{b})\cdot\bar{j} = y, (\bar{a}\times\bar{b})\cdot\bar{k} = z$

$\therefore [\bar{a}\ \bar{b}\ \bar{i}]\bar{i}+[\bar{a}\ \bar{b}\ \bar{j}]\bar{j}+[\bar{a}\ \bar{b}\ \bar{k}]\bar{k}$

$= x\bar{i}+y\bar{j}+z\bar{k} = \bar{a}\times\bar{b}$

44. (4)

Given $\bar{a}\times(\bar{b}\times\bar{c}) = \frac{\bar{b}}{2}$

$\Rightarrow (\bar{a}\cdot\bar{c}) - (\bar{a}\cdot\bar{b})\bar{c} = \frac{\bar{b}}{2}$ ... (1)

Taking dot product with $\bar{b}$,

$(\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{b}) - (\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{b}) = \frac{1}{2}(\bar{b}\cdot\bar{b})$

$\Rightarrow \bar{a}\cdot\bar{c} - (\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{b}) = \frac{1}{2}$ ... (2)

In (1) take dot product with $\bar{c}$:

$(\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{c}) - (\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{c}) = \frac{\bar{c}\cdot\bar{b}}{2}$

$$\Rightarrow (\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{c})-(\bar{a}\cdot\bar{b})=\frac{\bar{c}\cdot\bar{b}}{2}$$

$$\Rightarrow \left[\frac{1}{2}+(\bar{a}\cdot\bar{b})(\bar{b}\cdot\bar{c})\right](\bar{b}\cdot\bar{c})-\bar{a}\cdot\bar{b}=\frac{\bar{c}\cdot\bar{b}}{2}$$

Using (2)

$$\Rightarrow \frac{\bar{b}\cdot\bar{c}}{2}-(\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{c})^2-\bar{a}\cdot\bar{b}=\frac{\bar{b}\cdot\bar{c}}{2}$$

$$\Rightarrow (\bar{b}\cdot\bar{c})^2=1$$

$$\Rightarrow \bar{b}\cdot\bar{c}=1 \text{ or } \bar{a}\cdot\bar{b}=0$$

$$\therefore \bar{a}\cdot\bar{c}=\frac{1}{2}\Rightarrow 1\cdot 1\cos(\bar{a},\bar{c})=\frac{1}{2}$$

$$\Rightarrow (\bar{a}\cdot\bar{c})=60^\circ \text{ and } \bar{a}\cdot\bar{b}=0$$

$$\Rightarrow (\bar{a},\bar{b})=90^\circ$$

45. (3)

Given $\bar{a}\times(\bar{b}\times\bar{c})=\frac{1}{\sqrt{2}}(\bar{b}+\bar{c})$

$$\Rightarrow (\bar{a}\cdot\bar{c})\bar{b}-(\bar{a}\cdot\bar{b})\bar{c}=\frac{1}{\sqrt{2}}(\bar{b}+\bar{c})$$

Taking cross product with $\bar{b}$:

$$(\bar{a}\cdot\bar{c})(\bar{b}\times\bar{b})-(\bar{a}\cdot\bar{b})(\bar{b}\times\bar{c})$$
$$=\frac{1}{\sqrt{2}}(\bar{b}\times\bar{b}+\bar{b}\times\bar{c})$$

$$\Rightarrow -(\bar{a}\cdot\bar{b})(\bar{b}\times\bar{c})=\frac{1}{\sqrt{2}}(\bar{b}\times\bar{c})$$

$$\Rightarrow (\bar{b}\times\bar{c})\left(\bar{a}\cdot\bar{b}+\frac{1}{\sqrt{2}}\right)=0$$

$$\Rightarrow \bar{a}\cdot\bar{b}=-\frac{1}{\sqrt{2}}(\because \bar{c}\times\bar{b}\neq 0)$$

$$\Rightarrow 1\cdot 1\cos\theta=-\frac{1}{\sqrt{2}}\Rightarrow \theta=135^\circ$$

46. (1)

Given $|\bar{a}|=1, |\bar{b}|=1$ and $|\bar{c}|=2$

Also given $\bar{a}\times(\bar{a}\times\bar{c})+\bar{b}=\bar{0}$

$\Rightarrow (\bar{a}\cdot\bar{c})\bar{a}-(\bar{a}\cdot\bar{a})\bar{c}+\bar{b}=\bar{0}$

$\Rightarrow (\bar{a}\cdot\bar{c})\bar{a}-\bar{c}=-\bar{b} \qquad \ldots(1)$

$\Rightarrow \bar{a}\times(\bar{a}\times\bar{c})=-\bar{b}$

$\Rightarrow |\bar{a}\times(\bar{a}\times\bar{c})|=|-\bar{b}|=1$

But $(\bar{a}, \bar{a}\times\bar{c})=\pi/2\ |\bar{a}|\cdot|\bar{a}\times\bar{c}|\sin\frac{\pi}{2}=1$

$\Rightarrow |\bar{a}|\cdot|\bar{c}|\sin\theta=1$

$\Rightarrow \sin\theta=\frac{1}{2}\Rightarrow\theta=30°$

$\Rightarrow (\bar{a}\cdot\bar{c})\bar{a}-\bar{c}=-\bar{b}$ [From (1)]

$\Rightarrow \{(\bar{a}\cdot\bar{c})\bar{a}-\bar{c}\}\cdot\{(\bar{a}\cdot\bar{c})\bar{a}-\bar{c}\}=\bar{b}\cdot\bar{b}$

$\Rightarrow (\bar{a}\cdot\bar{c})^2(\bar{a}\cdot\bar{c})+\bar{c}\cdot\bar{c}-2(\bar{a}\cdot\bar{c})(\bar{a}\cdot\bar{c})=1$

$\Rightarrow (\bar{a}\cdot\bar{c})^2+4-2(\bar{a}\cdot\bar{c})^2=1$

$\Rightarrow (\bar{a}\cdot\bar{c})^2=3\Rightarrow\bar{a}\cdot\bar{c}=\sqrt{3}$

$\Rightarrow |\bar{a}|\cdot|\bar{c}|\cos(\bar{a},\bar{c})=\sqrt{3}$

$\Rightarrow \cos(\bar{a},\bar{c})=\frac{\sqrt{3}}{1.2}=\cos\frac{\pi}{6}$

$\Rightarrow (\bar{a},\bar{c})=30°$

47. (2)

Given $|\bar{p}|=|\bar{q}|=|\bar{r}|=\lambda$ (say) and

$\bar{p}\cdot\bar{q}=0, \bar{p}\cdot\bar{r}=0, \bar{q}\cdot\bar{r}=0$

$\bar{p}\times\{(\bar{x}-\bar{q})\times\bar{p}\}+\bar{q}\times\{(\bar{x}-\bar{r})\times\bar{q}\}+\bar{r}\times\{(\bar{x}-\bar{p})\times\bar{r}\}=\bar{0}$

$\Rightarrow(\bar{p}\cdot\bar{p})(\bar{x}-\bar{q})-\{\bar{p}\cdot(\bar{x}-\bar{q})\}\cdot\bar{p}+(\bar{q}\cdot\bar{q})(\bar{x}-\bar{r})-\{\bar{q}\cdot(\bar{x}-\bar{r})\}+(\bar{r}\cdot\bar{r})(\bar{x}-\bar{p})-\{\bar{r}\cdot(\bar{x}-\bar{p})\}\cdot\bar{r}=\bar{0}$

$\Rightarrow\lambda^2(\bar{x}-\bar{q}+\bar{x}-\bar{r}+\bar{x}-\bar{p})-(\bar{p}\cdot\bar{x})\bar{p}+(\bar{p}\cdot\bar{q})\bar{p}-(\bar{q}\cdot\bar{x})\bar{q}+(\bar{q}\cdot\bar{r})\bar{q}-(\bar{r}\cdot\bar{x})\bar{r}+(\bar{r}\cdot\bar{p})\bar{r}=\bar{0}$

$\Rightarrow\lambda^2\{3\bar{x}-(\bar{p}+\bar{q}+\bar{r})\}-[(\bar{p}\cdot\bar{x})\bar{p}+(\bar{q}\cdot\bar{x})\bar{q}+(\bar{r}\cdot\bar{x})\bar{r}]=\bar{0}$

Clearly this is satisfied by $\bar{x}=\frac{1}{2}(\bar{p}+\bar{q}+\bar{r})$

48. (3)

Given $a(\bar{\alpha}\times\bar{\beta})+b(\bar{\beta}\times\bar{\gamma})+c(\bar{\gamma}\times\bar{\alpha})=\bar{O}$

Taking dot product with $\bar{\alpha},\bar{\beta},\bar{\gamma}$ respectively

$a[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}]=0, b[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}]=0, c[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}]=0$

Also given that at least one of $\bar{a},\bar{b},\bar{c}$ is non-zero.

Hence $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ are coplanar.

49. (3)

Given $(\bar{b}\times\bar{c})\times(\bar{c}\times\bar{a})=3\bar{c}$

$\Rightarrow\left\{(\bar{b}\times\bar{c})\cdot\bar{a}\right\}\bar{c}-\left\{(\bar{b}\times\bar{c})\cdot\bar{c}\right\}\bar{a}=3\bar{c}$

$\Rightarrow[\bar{b}\ \bar{c}\ \bar{a}]\bar{c}-[\bar{b}\ \bar{c}\ \bar{c}]\bar{a}=3\bar{c}$

$\Rightarrow[\bar{b}\ \bar{c}\ \bar{a}]\bar{c}=3\bar{c}$

$\left\{\because[\bar{b}\ \bar{c}\ \bar{c}]=0\right\}\Rightarrow[\bar{b}\ \bar{c}\ \bar{a}]=3$

$[\bar{b}\times\bar{c},\bar{c}\times\bar{a},\bar{a}\times\bar{b}]=[\bar{a}\ \bar{b}\ \bar{c}]^2=[\bar{b}\ \bar{c}\ \bar{a}]^2=9$

50. (4)

Given $\bar{c}$ is $\perp$ to $\bar{a}$ and $\bar{b}$

$\Rightarrow\bar{c}$ is parallel to $\bar{a}\times\bar{b}$

$$\bar{a}\times\bar{b}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & 3 & -1\\ 1 & -2 & 3\end{vmatrix}$$

$=\bar{i}(9-2)-\bar{j}(6+1)+\bar{k}(-4-3)$

$=7\bar{i}-7\bar{j}-7\bar{k}=7(\bar{i}-\bar{j}-\bar{k})$

$\therefore\ \bar{c}$ is parallel to $\bar{i}-\bar{j}-\bar{k}$.

Let $\bar{c}=\lambda(\bar{i}-\bar{j}-\bar{k})$

Also given $\bar{c}\cdot(2\bar{i}-\bar{j}+\bar{k})=6$

$\Rightarrow\lambda(\bar{i}-\bar{j}-\bar{k})\cdot(2\bar{i}-\bar{j}+\bar{k})=6$

$\Rightarrow\lambda(2+1-1)=6\Rightarrow 2\lambda=6$

$\therefore\lambda=3$

Hence $\bar{c}=3(\bar{i}-\bar{j}-\bar{k})$

Now $[\bar{a}\ \bar{b}\ \bar{c}]=(\bar{a}\times\bar{b})\cdot\bar{c}$

$=7(\bar{i}-\bar{j}-\bar{k})\cdot 3(\bar{i}-\bar{j}-\bar{k})$

$=21(1+1+1)=63$

51. (3)

We know that $\bar{b},\bar{c}$ and $\bar{b}\times\bar{c}$ are mutually $\perp$ vectors.

$\therefore$ Any vector $\bar{a}$ can be expressed in terms of $\bar{b},\bar{c}$, $\bar{b}\times\bar{c}$

$\Rightarrow\bar{a}=x\bar{b}+y\bar{c}+z(\bar{b}\times\bar{c})$ ...(1)

Taking dot product on (1) with $\bar{b}\times\bar{c}$, we get

$\bar{a}\cdot(\bar{b}\times\bar{c})=x\{\bar{b}\cdot(\bar{b}\times\bar{c})\}+y\{\bar{c}\cdot(\bar{b}\times\bar{c})\}+z(\bar{b}\times\bar{c})^2$

$=x(0)+y(0)+z(\bar{b}\times\bar{c})^2=z(\bar{b}\times\bar{c})^2$

$$\Rightarrow z=\frac{\bar{a}\cdot(\bar{b}\times\bar{c})}{|\bar{b}\times\bar{c}|^2}$$

Given $|\bar{b}|=1,|\bar{c}|=1$

Again taking dot product on (1) with $\bar{b}$ and $\bar{c}$

$\Rightarrow \bar{a}\cdot\bar{b} = x(\bar{b}\cdot\bar{b}) + y(\bar{c}\cdot\bar{b}) + z(\bar{b}\times\bar{c})\cdot\bar{c}$

$= x(1) + y(0) + z[\bar{b}\ \bar{c}\ \bar{c}] = x + y(0) + z(0) = x$

Also $\bar{a}\cdot\bar{c} = x(\bar{b}\cdot\bar{c}) + y(\bar{c}\cdot\bar{c}) + z(\bar{b}\times\bar{c})\cdot\bar{c}$

$= x(0) + y(1) + z[\bar{b}\ \bar{c}\ \bar{c}] = 0 + y + 0 = y$

$\therefore \bar{a} = (\bar{a}\cdot\bar{b})\bar{b} + (\bar{a}\cdot\bar{c})\bar{b} + \dfrac{\bar{a}\cdot(\bar{b}\times\bar{c})}{|\bar{b}\times\bar{c}|^2}.$

52. (3)O, A, B, C are coplanar since $[\bar{a}\ \bar{b}\ \bar{c}] = 0$

OA = OB = OC $\Rightarrow |\bar{a}| = |\bar{b}| = |\bar{c}| = 3$

$(\because |\bar{a}|^2 = |\bar{b}|^2 = |\bar{c}|^2 = 9$ given)

Hence origin O is the circumcentre.

P.V. op G i.e., centroid = $\dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

We know that orthocenter H divides GO in the ratio 3 : 2 externally

$$\therefore \text{P.V.of } H = \frac{\dfrac{3(\bar{a}\times\bar{b}\times\bar{c})}{3} - 2(\bar{O})}{3-2} = \bar{a}+\bar{b}+\bar{c}$$

53. (3)

G.E. = $[3(\bar{a}\times\bar{b}) + 2(\bar{a}\times\bar{c}), \bar{b}\times\bar{c} - 2(\bar{b}\times\bar{a}), 2(\bar{c}\times\bar{a}) - 6(\bar{c}\times\bar{b})]$

Let $\bar{a}\times\bar{b} = \bar{p}, \bar{b}\times\bar{c} = \bar{q}, \bar{c}\times\bar{a} = \bar{r}$

$\therefore$ G.E. $= [3\bar{p} - 2\bar{r}, \bar{q} + 2\bar{p}, 2\bar{r} + 6\bar{q}]$

$$= \begin{vmatrix} 3 & 0 & -2 \\ 2 & 1 & 0 \\ 0 & 6 & 2 \end{vmatrix} [\bar{p}\ \bar{q}\ \bar{r}]$$

$= [3(2-0) - 0 - 2(12-0)][\bar{p}\ \bar{q}\ \bar{r}]$

$= -18[\bar{p}\ \bar{q}\ \bar{r}] = -18[\bar{a}\times\bar{b}, \bar{b}\times\bar{c}, \bar{c}\times\bar{a}]$

$= -18[\bar{a}\ \bar{b}\ \bar{c}]^2$