LIMITS

OBJECTIVES

1. $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=$

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{1}{\sqrt{x}}$

(c) $2\sqrt{x}$ (d) $\sqrt{x}$

2. $\lim_{x\to 1}\frac{x-1}{2x^2-7x+5}=$

(a) 1/3 (b) 1/11

(c) –1/3 (d) None of these

3. $\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n}+\sqrt{n+1}}=$

(a) 1 (b) 1/2

(c) 0 (d) ∞

4. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}=$

(a) a/b (b) b/a

(c) 1 (d) None of these

5. $\lim_{x\to 0}\frac{x}{|x|+x^2}=$

(a) 1 (b) –1

(c) 0 (d) Does not exist

6. $\lim_{\alpha\to\pi/4}\frac{\sin\alpha-\cos\alpha}{\alpha-\frac{\pi}{4}}=$

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$

(c) 1 (d) None of these

7. $\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}=$

(a) –1 (b) 1

(c) 2 (d) –2

8. $\lim_{x\to 0+} \frac{xe^{1/x}}{1+e^{1/x}} =$

(a) 0 (b) 1

(c) ∞ (d) None of these

9. $\lim_{x\to 0} \frac{x(e^x-1)}{1-\cos x} =$

(a) 0 (b) ∞

(c) –2 (d) 2

10. $\lim_{x\to 0} x\log(\sin x) =$

(a) –1 (b) $\log_e 1$

(c) 1 (d) None of these

11. $\lim_{x\to a} \frac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a} =$

(a) $\frac{5}{3}(a+2)^{2/3}$ (b) $\frac{5}{3}(a+2)^{5/3}$

(c) $\frac{5}{3}a^{2/3}$ (d) $\frac{5}{3}a^{5/3}$

12. $\lim_{x\to 0}\left\{\frac{\sin x - x + \frac{x^3}{6}}{x^5}\right\} =$

(a) 1/120 (b) –1/120

(c) 1/20 (d) None of these

13. $\lim_{x\to 0} \frac{1-\cos 2x}{x} =$

(a) 0 (b) 1

(c) 2 (d) 4

14. $\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1} =$

(a) 0 (b) 1

(c) –1 (d) Does not exist

15. $\lim_{x\to 0} \frac{\sin x - x}{x^3} =$

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

16. $\lim_{x\to 0}\frac{x^2-\tan 2x}{\tan x}=$

(a) 2 (b) –2

(c) 0 (d) None of these

17. If $f(a)=2,\ f'(a)=1,\ g(a)=-1;\ g'(a)=2$, then $\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}=$

(a) 3 (b) 5

(c) 0 (d) –3

18. $\lim_{x\to\infty}\left[\frac{1^3+2^3+3^3+.......+n^3}{n^4}\right]=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) None of these

19. $\lim_{x\to 1}\frac{\log x}{x-1}=$

(a) 1 (b) –1

(c) 0 (d) ∞

20. $\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}=$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) None of these

21. $\lim_{x\to a}\frac{\cos x-\cos a}{\cos x-\cot a}=$

(a) $\frac{1}{2}\sin^3 a$ (b) $\frac{1}{2}\operatorname{cosec}^2 a$

(c) $\sin^3 a$ (d) $\operatorname{cosec}^3 a$

22. $\lim_{x\to\infty}\frac{\sqrt{x^2+a^2}-\sqrt{x^2+b^2}}{\sqrt{x^2+c^2}-\sqrt{x^2+d^2}}=$

(a) $\frac{a^2-b^2}{c^2-d^2}$ (b) $\frac{a^2+b^2}{c^2-d^2}$

(c) $\frac{a^2+b^2}{c^2+d^2}$ (d) None of these

23. $\lim_{x\to 0}\frac{x.2^x-x}{1-\cos x}=$

(a) 0 (b) $\log 4$

(c) $\log 2$ (d) None of these

24. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) $\frac{2}{3}$ (d) None of these

25. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$, where n is a positive integer, then $n =$

(a) 3 (b) 5

(c) 2 (d) None of these

26. $\lim_{x \to \pi/2} \frac{1 + \cos 2x}{(\pi - 2x)^2} =$

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{2}$

27. $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} =$

(a) n (b) 1

(c) –1 (d) None of these

28. $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x} =$

(a) 1 (b) e

(c) $1/e$ (d) None of these

29. $\lim_{x \to 0} \frac{(1+x)^{1/2} - (1-x)^{1/2}}{x} =$

(a) 0 (b) 1/2

(c) 1 (d) –1

30. $\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta^2} =$

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

31. $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx} =$

(a) m/n (b) n/m

(c) $\frac{m^2}{n^2}$ (d) $\frac{n^2}{m^2}$

32. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sin^{-1}x}=$

(a) 2 (b) 1

(c) –1 (d) None of these

33. $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}=$

(a) $a\cos a+a^2\sin a$ (b) $a\sin a+a^2\cos a$

(c) $2a\sin a+a^2\cos a$ (d) $2a\cos a+a^2\sin a$

34. $\lim_{x\to 2}\frac{|x-2|}{x-2}=$

(a) 1 (b) –1

(c) Does not exist (d) None of these

35. $\lim_{\theta\to 0}\frac{\sin 3\theta-\sin\theta}{\sin\theta}=$

(a) 1 (b) 2

(c) 1/3 (d) 3/2

36. $\lim_{x\to\infty}[x(a^{1/x}-1)]$, $(a>1)=$

(a) $\log x$ (b) 1

(c) 0 (d) $-\log\frac{1}{a}$

37. If $f(x)=\frac{\sin(e^{x-2}-1)}{\log(x-1)}$, then $\lim_{x\to 2}f(x)$ is given by

(a) –2 (b) –1

(c) 0 (d) 1

38. $\lim_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{x}=$

(a) $\alpha+\beta$ (b) $\frac{1}{\alpha}+\beta$

(c) $\alpha^2-\beta^2$ (d) $\alpha-\beta$

39. If $f(x)=\begin{cases}x, & \text{when } x>1\\ x^2, & \text{when } x<1\end{cases}$**, then** $\lim_{x\to 1}f(x)=$

(a) x^2 (b) x

(c) -1 (d) 1

40. The value of $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x}$ **;** $(a, b, c > 0)$ **is**

(a) $(abc)^3$ (b) abc

(c) $(abc)^{1/3}$ (d) None of these

41. If a, b, c, d **are positive, then** $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx} =$

(a) $e^{d/b}$ (b) $e^{c/a}$

(c) $e^{(c+d)/(a+b)}$ (d) e

42. $\lim_{x\to 0}\left[\frac{\sin(x+a)+\sin(a-x)-2\sin a}{x\sin x}\right] =$

(a) $\sin a$ (b) $\cos a$

(c) $-\sin a$ (d) $\frac{1}{2}\cos a$

43. The value of $\lim_{x\to 0^+} x^m(\log x)^n, m, n \in N$ **is**

(a) 0 (b) $\frac{m}{n}$

(c) mn (d) None of these

44. $\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3-x}}{x} =$

(a) –1 (b) 0

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

45. $\lim_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$ is equal to

(a) 0 (b) $\frac{1}{2}$

(c) $\log 2$ (d) e^4

46. $\lim_{x\to\frac{\pi}{2}}\{(1-\sin x)\tan x\}$ is

(a) $\frac{\pi}{2}$ (b) 1

(c) 0 (d) ∞

47. $\lim_{x\to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\operatorname{cosec} x}$ is equal to

(a) e (b) $\frac{1}{e}$

(c) 1 (d) None of these

48. $\underset{x\to -1}{}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$ is given by

(a) $\frac{1}{\sqrt{\pi}}$ (b) $\frac{1}{\sqrt{2\pi}}$

(c) 1 (d) 0

49. $\lim_{x\to 0}\frac{\log(a+x)-\log a}{x}+k\lim_{x\to e}\frac{\log x-1}{x-e}=1$, then

(a) $k=e\left(1-\frac{1}{a}\right)$

(b) $k=e(1+a)$

(c) $k=e(2-a)$

(d) The equality is not possible

50. $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b}=$

(a) 1 (b) e^{b-a}

(c) e^{a-b} (d) e^{b}

51. The value of the limit of $\frac{x^3-8}{x^2-4}$ as x tends to 2 is

(a) 3 (b) $\frac{3}{2}$

(c) 1 (d) 0

52. If $f(x)=\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ and $g(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)},\ 0<a<\frac{1}{2}$ is

(a) $\frac{3}{2(1+a^2)}$ (b) $\frac{3}{2(1+x^2)}$

(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

53. $\lim_{n\to\infty}\left(\frac{n}{n+y}\right)^{n}$ equals

(a) 0 (b) 1

(c) 1/*v* (d) e^{-y}

54. $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$ **equals**

(a) $\pi/2$ (b) 0

(c) $2/e$ (d) $-e/2$

55. $\lim_{x \to 0} \frac{a^{\sin x} - 1}{b^{\sin x} - 1} =$

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$

(c) $\frac{\log a}{\log b}$ (d) $\frac{\log b}{\log a}$

56. The value of $\lim_{x \to 0} \frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x}$ **is**

(a) 10/3 (b) 3/10

(c) 6/5 (d) 5/6

57. $\lim_{x \to \infty} \frac{\log x^n - [x]}{[x]}, n \in N,$ ($[x]$ **denotes greatest integer less than or equal to *x*)**

(a) Has value –1 (b) Has value 0

(c) Has value 1 (d) Does not exist

58. The value of $\lim_{x \to 0} \left[\frac{\sqrt{a+x} - \sqrt{a-x}}{x} \right]$ **is**

(a) 1 (b) 0

(c) $\sqrt{a}$ (d) $1/\sqrt{a}$

59. $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ is equal to

(a) 0 (b) 1

(c) –1 (d) $1/2$

60. $\lim_{x \to 0} \frac{4^x - 9^x}{x(4^x + 9^x)} =$

(a) $\log\left(\frac{2}{3}\right)$ (b) $\frac{1}{2}\log\left(\frac{3}{2}\right)$

(c) $\frac{1}{2}\log\left(\frac{2}{3}\right)$ (d) $\log\left(\frac{3}{2}\right)$

61. $\lim_{x \to 3} [x] =$**, (where [.] = greatest integer function)**

(a) 2 (b) 3

(c) Does not exist (d) None of these

62. $\lim_{x\to\pi/2} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x} =$

(a) $\log a$ (b) $\log 2$

(c) a (d) $\log x$

63. $\lim_{x\to 0} \frac{\sin(\pi \cos^2 x)}{x^2} =$

(a) $-\pi$ (b) π

(c) $\pi/2$ (d) 1

64. **If** $f(1)=1, f'(1)=2$ **, then** $\lim_{x\to 1} \frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ **is**

(a) 2 (b) 4

(c) 1 (d) 1/2

65. **If** $S_n = \sum_{k=1}^{n} a_k$ **and** $\lim_{n\to\infty} a_n = a,$ **then** $\lim_{n\to\infty} \frac{S_{n+1} - S_n}{\sqrt{\sum_{k=1}^{n} k}}$ **is equal to**

(a) 0 (b) a

(c) $\sqrt{2}a$ (d) $2a$

66. $\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x}$ is

(a) 0 (b) 1

(c) 2 (d) Non existent

67. **The value of** $\lim_{n\to\infty} \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n-1)(2n+1)}$ **is equal to**

(a) 1/2 (b) 1/3

(c) 1/4 (d) None of these

68. **The value of** $\lim_{n\to\infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)...\cos\left(\frac{x}{2^n}\right)$ **is**

(a) 1 (b) $\frac{\sin x}{x}$

(c) $\frac{x}{\sin x}$ (d) None of these

69. $\lim_{x\to 0}(1-ax)^{\frac{1}{x}} =$

(a) e (b) e^{-a}

(c) 1 (d) e^{a}

70. $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+.....+(x+100)^{10}}{x^{10}+10^{10}}$ **is equal to**

(a) 0 (b) 1

(c) 10 (d) 100

71. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, **then the values of *a* and *b* are**

(a) $a=1, b=2$ (b) $a=1, b\in R$

(c) $a\in R, b=2$ (d) $a\in R, b\in R$

72. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, **then the value of *k* is**

(a) 0 (b) $-\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

73. $\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+...+\frac{n}{1-n^2}\right]$ **is equal to**

(a) 0 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) None of these

74. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, \text{ when } [x]\neq 0\\ 0, \text{ when } [x]=0\end{cases}$ **where [*x*] is greatest integer function, then** $\lim_{x\to 0}f(x)=$

(a) –1 (b) 1

(c) 0 (d) None of these

75. $\lim_{n\to\infty}\left[\frac{1}{n^3+1}+\frac{4}{n^3+1}+\frac{9}{n^3+1}+........+\frac{n^2}{n^3+1}\right]=$

(a) 1 (b) 2/3

(c) 1/3 (d) 0

76. If $\lim_{n\to\infty}\frac{1-(10)^n}{1+(10)^{n+1}}=\frac{-\alpha}{10}$, **then give the value of** α **is**

(a) 0 (b) –1

(c) 1 (d) 2

77. $\lim_{n\to\infty}\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^n}$ **equals**

(a) 2 (b) –1

(c) 1 (d) 3

78. The value of $\lim_{x \to 0} \frac{\int_0^x \cos t^2}{x} dt$ **is**

(a) 0 (b) 1

(c) -1 (d) None of these

79. If $\lim_{x \to 0} \frac{[(a-n)nx - \tan x]\sin nx}{x^2} = 0,$ **where *n* is non zero real number, then *a* is equal to**

(a) 0 (b) $\frac{n+1}{n}$

(c) n (d) $n + \frac{1}{n}$

80. $\lim_{x \to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1} =$

(a) $\log 2$ (b) $\log 4$

(c) $\log \sqrt{2}$ (d) None of these

81. The value of $\lim_{x \to 0} \frac{\log[1+x^3]}{\sin^3 x} =$

(a) 0 (b) 1

(c) 3 (d) None of these

LIMITS

HINTS AND SOLUTIONS

1. (a) Apply L-Hospital rule,

2. (c) Apply L-Hospital's rule.

3. (b) $\lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}$.

4. (a) $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{a}{b} \frac{\sin ax}{ax} \frac{bx}{\sin bx} = \frac{a}{b}$.

5. (c) Apply L-Hospital's rule .

6. (a) Apply L-Hospital's rule,

7. (b) Apply L-Hospital's rule,

8. (a) $\lim_{x\to 0+} \frac{x}{1+e^{-1/x}} = 0$ as $e^{-1/x} \to 0$ when $x \to 0^+$

9. (d) $\lim_{x\to 0} \frac{x(e^x-1)}{1-\cos x} = \lim_{x\to 0} \frac{2x(e^x-1)}{4.\sin^2 \frac{x}{2}}$

$$= 2\lim_{x\to 0}\left[\frac{(x/2)^2}{\sin^2 \frac{x}{2}}\right]\left(\frac{e^x-1}{x}\right) = 2.$$

10. (b) $\lim_{x\to 0} x \log \sin x = \lim_{x\to 0} \log(\sin x)^x = \log[\lim_{x\to 0} (\sin x)^x]$

$$= \log\left[\lim_{x\to 0}(1+\sin x-1)^{\frac{x(\sin x-1)}{\sin x-1}}\right]$$

$$= \log_e[e^{\lim_{x\to 0} x(\sin x-1)}] = \log_e 1.$$

11. (a) Apply the L-Hospital's rule, $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.

12. (a) Apply L-Hospital's rule

13. (a) $\lim_{x\to 0} \frac{x.2\sin^2 x}{x^2} = 2.\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \cdot \lim_{x\to 0} x = 0$.

14. (d) $f(x) = \left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$, then

$$\lim_{x\to 0+} f(x) = \lim_{h\to 0}\left(\frac{e^{1/h}-1}{e^{1/h}+1}\right) = \lim_{h\to 0} \frac{e^{1/h}\left(1-\frac{1}{e^{1/h}}\right)}{e^{1/h}\left(1+\frac{1}{e^{1/h}}\right)} = 1$$

Similarly $\lim_{x\to 0-} f(x) = -1$. Hence limit does not exist.

15. (d) Apply L-Hospital's rule.

16. (b) $\lim_{x\to 0} \frac{x\left(x - \frac{2\tan 2x}{2x}\right)}{\tan x} = -2.$

17. (b) $\lim_{x\to a} \frac{f(a)[g(x)-g(a)]-g(a)[f(x)-f(a)]}{[x-a]}$

$$= f(a)g'(a) - g(a)f'(a) = 2\times 2-(-1)(1) = 5.$$

18. (c) $\lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^2}{4} = \frac{1}{4}.$

19. (a) Apply L-Hospital's rule, $\lim_{x\to 1} \frac{\log x}{x-1} = \lim_{x\to 1} \frac{\frac{1}{x}}{1} = 1$

20. (a) Apply L-Hospital's rule two times.

21. (c) $\lim_{x\to a} \frac{\cos x - \cos a}{\cot x - \cot a} = \lim_{x\to a}\left(\frac{-\sin x}{-\text{cosec}^2 x}\right) = \lim_{x\to a} \sin^3 x = \sin^3 a$.

22. (a) $\lim_{x\to\infty} \frac{(a^2-b^2)}{(c^2-d^2)} \frac{\left[\sqrt{1+\frac{c^2}{x^2}}+\sqrt{1+\frac{d^2}{x^2}}\right]}{\left[\sqrt{1+\frac{a^2}{x^2}}+\sqrt{1+\frac{b^2}{x^2}}\right]} = \frac{a^2-b^2}{c^2-d^2}.$

23. (b) $\lim_{x\to 0} \frac{x.(2^x-1)}{1-\cos x} = \lim_{x\to 0} \frac{2^x-1}{x}.\frac{x^2}{1-\cos x}$

$$= \log 2 . \lim_{x\to 0} \frac{x^2}{2\sin^2 \frac{x}{2}} = (\log 2).2 = 2\log 2 = \log 4 .$$

24. (a) $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x\to 0} \frac{\sin x - \sin x \cos x}{x^3 \cos x}$

$$= \lim_{x\to 0} \frac{\sin x \left(2\sin^2 \frac{x}{2}\right)}{x^3 \cos x} = \lim_{x\to 0}\left[\frac{\sin x}{x}.\frac{2}{\cos x}.\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2}.\frac{1}{4}\right] = \frac{1}{2}.$$

25. (b) $\lim_{x\to 2} \frac{x^n - 2^n}{x-2} = n.2^{n-1} \Rightarrow n.2^{n-1} = 80 \Rightarrow n = 5$.

26. (d) put $\pi - 2x = \theta \Rightarrow x = \frac{\pi}{2} - \frac{\theta}{2}$

27. (a) Apply L-Hospital's rule.

28. (a) Apply L-Hospital's rule

29. (c) Apply L-Hospital's rule

30. (c) Apply L-Hospital's rule

31. (c) $\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}=\lim_{x\to 0}\frac{m\sin mx}{n\sin nx}=\lim_{x\to 0}\frac{m^2\cos mx}{n^2\cos nx}=\frac{m^2}{n^2}.$

32. (b) Let $\sin^{-1}x=y\Rightarrow x=\sin y$

So $\lim_{y\to 0}\frac{\sqrt{1+\sin y}-\sqrt{1-\sin y}}{y}=1$

33. (c) Apply L-Hospital's rule,

34. (c) $\lim_{x\to 2-}\frac{|x-2|}{x-2}=\lim_{h\to 0}\frac{|2-h-2|}{2-h-2}=-1$

and $\lim_{x\to 2+}\frac{|x-2|}{x-2}=\lim_{h\to 0}\frac{|2+h-2|}{2+h-2}=1$

Hence limit does not exist.

35. (b) $\lim_{\theta\to 0}\frac{\sin 3\theta-\sin\theta}{\sin\theta}=\lim_{\theta\to 0}\frac{\sin 3\theta}{\sin\theta}-\lim_{\theta\to 0}\frac{\sin\theta}{\sin\theta}$

$$=\frac{3}{1}-1=2.$$

36. (d) $\lim_{x\to\infty}x(a^{1/x}-1)=\lim_{x\to\infty}\left[\frac{a^{1/x}-1}{1/x}\right]$

$$=\lim_{x\to\infty}\frac{[e^{\log_e a^{1/x}}-1]}{1/x}=\log_e a=-\log_e\frac{1}{a}.$$

37. (d) $\lim_{x\to 2}f(x)=\lim_{x\to 2}\frac{\sin(e^{x-2}-1)}{\log(x-1)}$

$=\lim_{t\to 0}\frac{\sin(e^t-1)}{\log(1+t)}$, {Putting $x=2+t$}

$=\lim_{t\to 0}\frac{\sin(e^t-1)}{e^t-1}.\frac{e^t-1}{t}.\frac{t}{\log(1+t)}$

$=\lim_{t\to 0}\frac{\sin(e^t-1)}{e^t-1}.\left(\frac{1}{1!}+\frac{t}{2!}+...\right)\times\left[\frac{1}{\left(1-\frac{1}{2}t+\frac{1}{3}t^2-...\right)}\right]$

38. (d) $\lim_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{x}=\lim_{x\to 0}\frac{e^{\alpha x}-1-e^{\beta x}+1}{x}$

$=\alpha\lim_{x\to 0}\frac{e^{\alpha x}-1}{\alpha x}-\beta\lim_{x\to 0}\frac{e^{\beta x}-1}{\beta x}=\alpha.1-\beta.1=\alpha-\beta.$

39. (d) $\lim_{x\to 1-}f(x)=1=\lim_{x\to 1+}f(x)$.

40. (d) Standard formula.

41. (a) $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx} = \lim_{x\to\infty}\left\{\left(1+\frac{1}{a+bx}\right)^{a+bx}\right\}^{\frac{c+dx}{a+bx}} = e^{d/b}$

$\left\{\because \lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{a+bx} = e \text{ and } \lim_{x\to\infty}\frac{c+dx}{a+bx} = \frac{d}{b}\right\}$.

42. (c) $\lim_{x\to 0} 2\sin a.\frac{(\cos x-1)}{x\sin x} = -2\sin a.\frac{(1-\cos x)}{x^2}.\left(\frac{x}{\sin x}\right)$

$= \lim_{x\to 0} -2\sin a.\frac{2\sin^2(x/2)}{4\left(\frac{x}{2}\right)^2\left(\frac{\sin x}{x}\right)} = -\sin a$.

43. (a) $\lim_{x\to 0+} x^m(\log x)^n = \lim_{x\to 0+}\frac{(\log x)^n}{x^{-m}}$

44. (d) Apply L-Hospital's rule two times.

45. (b) $\lim_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right] = \lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$

$= \lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}} = \lim_{x\to\infty}\frac{\sqrt{1+x^{-1/2}}}{\sqrt{1+\sqrt{x^{-1}+x^{-3/2}}}+1} = \frac{1}{2}$.

46. (c) $\lim_{x\to\pi/2}\{(1-\sin x)\tan x\} = \lim_{x\to\pi/2}\frac{\sin x-\sin^2 x}{\cos x}$

Apply L-Hospital's rule,

47. (c) Given limit $= \lim_{x\to 0}[(1+\tan x)^{\text{cosec } x}\times 1/(1+\sin x)^{\text{cosec } x}]$

$= \lim_{x\to 0}[\{(1+\tan x)^{\cot x}\}^{\sec x}\times\{1/(1+\sin x)^{\text{cosec } x}\}]$

$= e^{\sec 0}.\frac{1}{e} = e.\frac{1}{e} = 1.$

48. (b) Put $\cos^{-1}x = y$. So if $x\to -1, y\to\pi$

$\therefore \lim_{x\to -1}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}} = \lim_{y\to\pi}\frac{\sqrt{\pi}-\sqrt{y}}{\sqrt{1+\cos y}}$

49. (a) Apply L-Hospital's rule to find both the limits.

50. (c) $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b} = \lim_{x\to\infty}\left(1+\frac{a-b}{x+b}\right)^{x+b} = \lim_{x\to\infty}\left\{\left(1+\frac{a-b}{x+b}\right)^{\frac{x+b}{a-b}}\right\}^{a-b} = e^{a-b}$.

51. (a) L-Hospital's rule, we get

52. (d)

53. (d) $\lim_{n\to\infty}\left(\frac{n}{n+y}\right)^n = \lim_{n\to\infty}\left(\frac{1}{1+\frac{y}{n}}\right)^n = \lim_{n\to\infty}\left(1+\frac{y}{n}\right)^{-n} = \lim_{n\to\infty}\left[\left(1+\frac{y}{n}\right)^n\right]^{-1} = e^{-y}$.

54. (d) $(1+x)^{\frac{1}{x}} = e^{\frac{1}{x}[\log(1+x)]}$

$$= e^{\frac{1}{x}\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+....\right)} = e^{\left(1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+....\right)}$$

$$= e.e^{\left(-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+....\right)}$$

$$= e\left[\frac{\left(-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+...\right)}{1!}+\frac{\left(-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+...\right)^2}{2!}+...\right]$$

$$= \left[e-\frac{ex}{2}+\frac{11e}{24}x^2+...+...\right]$$

$$\therefore \lim_{x\to 0}\frac{(1+x)^{1/x}-e}{x} = \lim_{x\to 0}\left[\frac{e-\frac{ex}{2}-\frac{11e}{24}x^2+...e}{x}\right]$$

$$\Rightarrow \lim_{x\to 0}\left(-\frac{e}{2}-\frac{11e}{24}x+...\right) = -\frac{e}{2}.$$

55. (c) $\lim_{x\to 0}\frac{a^{\sin x}-1}{b^{\sin x}-1} = \lim_{x\to 0}\frac{a^{\sin x}-1}{\sin x}\times\frac{\sin x}{b^{\sin x}-1}$

$$= \log_e a\times\frac{1}{\log_e b} = \frac{\log a}{\log b}.$$

56. (a) $\lim_{x\to 0}\frac{(1-\cos 2x)\sin 5x}{x^2\sin 3x} = \lim_{x\to 0}\frac{2\sin^2 x\sin 5x}{x^2\sin 3x}$

$$= \lim_{x\to 0}\left(\frac{2\sin^2 x}{x^2}\right)\frac{\left(\frac{\sin 5x}{x}\right)}{\left(\frac{\sin 3x}{x}\right)}$$

$$= \lim_{x\to 0}2\left(\frac{\sin x}{x}\right)^2\times\frac{5\lim_{x\to 0}\left(\frac{\sin 5x}{5x}\right)}{3\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)} = \frac{2\times 5}{3} = \frac{10}{3}.$$

57. (a) $\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]} = \lim_{x\to\infty}\frac{\log x^n}{[x]} - \lim_{x\to\infty}\frac{[x]}{[x]} = 0-1 = -1.$

58. (d) $\lim_{x\to 0}\left[\frac{\sqrt{a+x}-\sqrt{a-x}}{x}\right] = \lim_{x\to 0}\left[\frac{(\sqrt{a+x}-\sqrt{a-x})(\sqrt{a+x}+\sqrt{a-x})}{x(\sqrt{a+x}+\sqrt{a-x})}\right]$

$$= \lim_{x\to 0}\left[\frac{2x}{x(\sqrt{a+x}+\sqrt{a-x})}\right] = \frac{2}{\sqrt{a}+\sqrt{a}} = \frac{1}{\sqrt{a}}.$$

59. (d) Applying L-Hospital's rule,

60. (a) L-Hospital's rule,

61. (c) $\lim_{h\to 0^+}[3+h]=3$ and $\lim_{h\to 0^-}[3-h]=2$

$\therefore \lim_{x\to 3}[x]$ does not exist.

62. (a) $\lim_{x\to\pi/2}\left(\frac{a^{\cot x}-a^{\cos x}}{\cot x-\cos x}\right)=\lim_{x\to\pi/2}a^{\cos x}\left(\frac{a^{\cot x-\cos x}-1}{\cot x-\cos x}\right)$

$=a^{\cos(\pi/2)}\lim_{x\to\pi/2}\left(\frac{a^{\cot x-\cos x}-1}{\cot x-\cos x}\right)=1.\log a=\log a$.

63. (b) Limit $=\lim_{x\to 0}\left(\frac{\cos(\pi\cos^2 x).\pi.2\cos x(-\sin x)}{2x}\right)$

$=\lim_{x\to 0}\pi\cos(\pi\cos^2 x).\cos x.\left(\frac{-\sin x}{x}\right)$

64. (a) Applying L-Hospital's rule

65. (a) We have $\lim_{n\to\infty}\frac{S_{n+1}-S_n}{\sqrt{\sum_{k=1}^{n}k}}=\lim_{n\to\infty}\frac{a_{n+1}}{\sqrt{\frac{n(n+1)}{2}}}=0$

66. (c) Applying L-Hospital's rule,

67. (a) $\lim_{n\to\infty}\frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+....+\left(\frac{1}{(2n-1)}-\frac{1}{(2n+1)}\right)\right]$

$=\lim_{n\to\infty}\frac{1}{2}\left[1-\frac{1}{2n+1}\right]=\frac{1}{2}$.

68. (b) We know that

$\cos A\cos 2A\cos 4A....\cos 2^{n-1}A=\frac{\sin 2^n A}{2^n\sin A}$

Taking $A=\frac{x}{2^n}$, we get

$\cos\left(\frac{x}{2^n}\right)\cos\left(\frac{x}{2^{n-1}}\right)...\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{2}\right)=\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)}$

$\therefore \lim_{n\to\infty}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)...\cos\left(\frac{x}{2^{n-1}}\right)\cos\left(\frac{x}{2^n}\right)$

$=\lim_{n\to\infty}\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)}=\lim_{n\to\infty}\frac{\sin x}{x}\frac{(x/2^n)}{\sin(x/2^n)}=\frac{\sin x}{x}$.

69. (b) $\lim_{x\to 0}[1+(-a)x]^{1/x}=e^{-a}$.

70. (d) $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+......+(x+100)^{10}}{x^{10}+10^{10}}$

$$= \lim_{x\to\infty} \frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+\ldots+\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left[1+\frac{10^{10}}{x^{10}}\right]} = 100 .$$

71. (b) Since, $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)=e^2$

$$\therefore \lim_{x\to\infty}\left[\left(1+\frac{ax+b}{x^2}\right)^{\frac{x^2}{ax+b}}\right]^{\frac{2(ax+b)}{x}} = e^2$$

$$\Rightarrow \lim_{x\to\infty} e^{\frac{2(ax+b)}{x}} = e^2 \Rightarrow \lim_{x\to\infty}\frac{2(ax+b)}{x}=2 \Rightarrow 2a=2 \Rightarrow a=1$$

Thus $a=1$ and $b\in R$.

72. (c) $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$

By L-Hospital's rule, $\lim_{x\to 0}\frac{\frac{1}{3+x}+\frac{1}{3-x}}{1}=k \Rightarrow \frac{2}{3}=k$.

73. (b) $\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots..+\frac{n}{1-n^2}\right]$

$$= \lim_{n\to\infty}\frac{\Sigma n}{1-n^2}=\frac{1}{2}\lim_{n\to\infty}\frac{n^2+n}{1-n^2}=-\frac{1}{2}.$$

74. (d) In closed interval of $x = 0$ at right hand side $[x] = 0$ and at left hand side $[x]=-1$. Also [0]=0.

Therefore function is defined as $f(x)=\begin{cases}\frac{\sin[x]}{[x]} & (-1\le x<0)\\ 0 & (0\le x<1)\end{cases}$

$\therefore$ Left hand limit $=\lim_{x\to 0-}f(x)=\lim_{x\to 0-}\frac{\sin[x]}{[x]}$

$$=\frac{\sin(-1)}{-1}=\sin 1^c$$

Right hand limit = 0. Hence limit doesn't exist.

75. (c) Given limit $=\lim_{n\to\infty}\frac{1^2+2^2+3^2+\ldots..+n^2}{1+n^3}=\lim_{n\to\infty}\frac{\Sigma n^2}{1+n^3}$

$$=\lim_{n\to\infty}\frac{1}{6}\frac{n(n+1)(2n+1)}{1+n^3}=\lim_{n\to\infty}\frac{1}{6}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{\left(\frac{1}{n^3}+1\right)}$$

$$=\frac{1}{6}.1.\frac{2}{(1)}=\left(\frac{1}{3}\right).$$

76. (c) $\lim_{n\to\infty} \frac{1-(10)^n}{1+(10)^{n+1}} = \lim_{n\to\infty} \frac{(10)^n\left[\left(\frac{1}{10}\right)^n - 1\right]}{(10)^{n+1}\left(1+\frac{1}{10^{n+1}}\right)} = -\frac{1}{10}$

$\therefore \alpha = 1$.

77. (c) $y = \lim_{n\to\infty} \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.......+\frac{1}{2^n} = \lim_{n\to\infty} \frac{1}{2}\frac{\left[1-\left(\frac{1}{2}\right)^n\right]}{\left(1-\frac{1}{2}\right)}$

$\lim_{n\to\infty}\left[1-\frac{1}{2^n}\right] = 1-0 = 1$

78. (b) $\lim_{x\to 0} \frac{\int_0^x \cos t^2 dt}{x}$

Applying L- Hospital rule, we get

$\lim_{x\to 0} \frac{\int_0^x \cos t^2 dt}{x} = \lim_{x\to 0} \frac{\cos x^2}{1} = 1$.

79. (d) $\lim_{x\to 0} n\frac{\sin nx}{nx}.\lim_{x\to 0}\left((a-n)n - \frac{\tan x}{x}\right) = 0$

$\Rightarrow n((a-n)n-1) = 0 \Rightarrow (a-n)n = 1 \Rightarrow a = n+\frac{1}{n}$.

80. (b) $\lim_{x\to 0} \frac{2^x - 1}{(1+x)^{1/2}-1} = \lim_{x\to 0} \frac{2^x \log 2}{\frac{1}{2}(1+x)^{-1/2}}$

$= 2\log 2 = \log 4.$

81. (b) $\lim_{x\to 0} \frac{\log(1+x^3)}{\sin^3 x} = \lim_{x\to 0} \frac{3x^2/(1+x^3)}{3\sin^2 x \cos x}$

$= \lim_{x\to 0}\left[\frac{1}{1+x^3}\left(\frac{x}{\sin x}\right)^2.\frac{1}{\cos x}\right] = \frac{1}{1+0}.(1)^2.\frac{1}{1} = 1$.