INVERSE TRIGONOMETRIC FUNCTIONS

OBJECTIVES

1. $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right] =$

1) $\frac{2a}{b}$ 2) $\frac{2b}{a}$ 3) $\frac{a}{b}$ 4) $\frac{b}{a}$

2. If $\frac{\pi}{2}\le x\le\frac{3\pi}{2}$ **then** $\sin^{-1}(\sin x)=$

1) x 2) $-x$ 3) $\pi+x$ 4) $\pi-x$

3. If $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$ **, then** x **=**

(a)1 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d)None of these

4. If $\sin^{-1}x=\theta+\beta$ **and** $\sin^{-1}y=\theta-\beta$**, then** $1+xy=$

(a) $\sin^2\theta+\sin^2\beta$ (b) $\sin^2\theta+\cos^2\beta$ (c) $\cos^2\theta+\cos^2\beta$ (d) $\cos^2\theta+\sin^2\beta$

5. The value of $\sin^{-1}(\sin 10)$ **is**

(a)10 (b) $10-3\pi$ (c) $3\pi-10$ (d)None of these

6. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{\pi}{2}$**, then the value of** $x^2+y^2+z^2+2xyz$ **is equal to**

(a)0 (b) 1 (c) 2 (d) 3

7.Two angles of a triangle are $\cot^{-1}2$ **and** $\cot^{-1}3$ **then find the third angle is**

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{3\pi}{4}$ 4) $\frac{2\pi}{3}$

8.The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ **is**

1) $\pi-x$ 2) $2\pi-x$ 3) $\frac{x}{2}$ 4) $\frac{2\pi-x}{2}$

9. If $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]=\alpha$ then $\sin 2\alpha =$

1) x 2) x^2 3) $2x^2$ 4) None

10. $\sin[\cot^{-1}(\cos\tan^{-1}x)]=$

(a) $\frac{x}{\sqrt{x^2+2}}$ (b) $\frac{x}{\sqrt{x^2+1}}$ (c) $\frac{1}{\sqrt{x^2+2}}$ (d) $\sqrt{\frac{x^2+1}{x^2+2}}$

11. If $\sin(\cot^{-1}(x+1)=\cos(\tan^{-1}x)$, then $x=$

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c)0 (d) $\frac{9}{4}$

12. $\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]=$

(a)6/17 (b)17/6 (c)7/16 (d)16/7

13. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$**, then**

(a) $x^2+y^2+z^2+xyz=0$ (b) $x^2+y^2+z^2+2xyz=0$

(c) $x^2+y^2+z^2+xyz=1$ (d) $x^2+y^2+z^2+2xyz=1$

14. If $\sin^{-1}x=\frac{\pi}{5}$ for some $x\in[-1,1]$ then $\cos^{-1}x=$

1) $\frac{3\pi}{10}$ 2) $\frac{5\pi}{10}$ 3) $\frac{7\pi}{10}$ 4) $\frac{9\pi}{10}$

15. $\cot^{-1}(\sec x+\tan x)=$

1) $\frac{\pi}{4}-\frac{x}{2}$ 2) $\frac{\pi}{4}+\frac{x}{2}$ 3) $\pi-x$ 4) $\pi+x$

16. If the adjacent sides of a rectangle are in the ratio 3 : 1 then the acute angle between the diagonals is

1) $2\sin^{-1}\frac{3}{5}$ 2) $\sin^{-1}\frac{3}{5}$ 3) $\tan^{-1}\frac{3}{5}$ 4) None

17. **If** $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$, **then**

(a) $x+y+z-xyz=0$ (b) $x+y+z+xyz=0$

(c) $xy+yz+zx+1=0$ (d) $xy+yz+zx-1=0$

18. **If** $\tan^{-1}\frac{a+x}{a}+\tan^{-1}\frac{a-x}{a}=\frac{\pi}{6}$ **,then** $x^2=$

(a) $2\sqrt{3}a$ (b) $\sqrt{3}a$ (c) $2\sqrt{3}a^2$ (d)None of these

19. **If** $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$ **,then** $x=$

(a) $\pm\frac{1}{2}$ (b) $0,\frac{1}{2}$ (c) $0,-\frac{1}{2}$ (d) $0,\pm\frac{1}{2}$

20. **If** $\cot^{-1}\alpha+\cot^{-1}\beta=\cot^{-1}x$, **then** $x=$

(a) $\alpha+\beta$ (b) $\alpha-\beta$ (c) $\frac{1+\alpha\beta}{\alpha+\beta}$ (d) $\frac{\alpha\beta-1}{\alpha+\beta}$

21. **If** $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$, **then** $\cos^{-1}x+\cos^{-1}y=$

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

22. **If** $x^2+y^2+z^2=r^2$, **then** $\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan\left(\frac{zx}{yr}\right)=$

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) None of these

23. **If** $k\le\sin^{-1}x+\cos^{-1}x+\tan^{-1}x\le K$, **then**

(a) $k=0, K=\pi$ (b) $k=0, K=\frac{\pi}{2}$ (c) $k=\frac{\pi}{2}, K=\pi$ (d) None of these

24 **The value of** $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)=$

1) 14/15 2) 14/25 3) 4/15 4) 24/25

25. **If** $x=\tan 1$ **and** $y=\tan^{-1}1$ **then**

1) $x<y$ 2) $x=y$ 3) $x>y$ 4) None

26. $\sin^{-1}x > \cos^{-1}x$ **holds for**

1) $\forall x$ 2) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$ 3) $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ 4) None

27. **If** $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, **then the value of** $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)}$ **will be**

(a) $2abc$ (b) abc (c) $\frac{1}{2}abc$ (d) $\frac{1}{3}abc$

28. **If** $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$, **then the value of** ***q*** **is**

(a)1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

29. $\cot^{-1}[(\cos\alpha)^{1/2}] - \tan^{-1}[(\cos\alpha)^{1/2}] = x$, **then** $\sin x =$

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$ (c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

30. **If** $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, **then** $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} =$

(a)0 (b)1 (c) $\frac{1}{xyz}$ (d) xyz

31. $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right] =$

(a) $\frac{2a}{1+a^2}$ (b) $\frac{1-a^2}{1+a^2}$ (c) $\frac{2a}{1-a^2}$ (d)None of these

32. **The value of** $\cos^{-1}(\cos 12) - \sin^{-1}(\sin 14)$ **is**

(a) -2 (b) $8\pi - 26$ (c) $4\pi + 2$ (d)None of these

33. $\cos^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right)$ **is equal to**

(a) $\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$ (b) $2\tan^{-1}\left(2\tan\frac{x}{2}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(2\tan\frac{x}{2}\right)$ (d) $2\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$

34. If $\angle A = 90^o$ in the triangle ABC, then $\tan^{-1}\left(\frac{c}{a+b}\right) + \tan^{-1}\left(\frac{b}{a+c}\right) =$

(a) 0 (b) 1 (c) $\pi/4$ (d) $\pi/6$

35. The solution of $\sin^{-1} x - \sin^{-1} 2x = \pm\dfrac{\pi}{3}$ **is**

(a) $\pm\frac{1}{3}$ (b) $\pm\frac{1}{4}$ (c) $\pm\frac{\sqrt{3}}{2}$ (d) $\pm\frac{1}{2}$

36. $\tan\left[\dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right]+\tan\left[\dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right]=$

(a) $\frac{2a}{b}$ (b) $\frac{2b}{a}$ (c) $\frac{a}{b}$ (d) $\frac{b}{a}$

37. $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=$

(a) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$ (b) $\frac{\pi}{4}+\cos^{-1}x^2$ (c) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$ (d) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

38. The equation $\sin^{-1}x-\cos^{-1}x=\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ **has**

(a) No solution (b) Unique solution (c) Infinite number of solutions (d) None of these

39. $\sin\left\{\tan^{-1}\left(\dfrac{1-x^2}{2x}\right)+\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)\right\}$ **is equal to**

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

40. The value of $\cos^{-1}\left(\cos\dfrac{5\pi}{3}\right)+\sin^{-1}\left(\cos\dfrac{5\pi}{3}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$ (c) $\frac{10\pi}{3}$ (d) 0

41. The value of $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)-\sin^{-1}\left(\dfrac{1}{2}\right)$ **is**

(a) 45° (b) 90° (c) 15° (d) 30°

42. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$, **then** $xy+yz+zx=$

(a) 0 (b) 1 (c) 3 (d) –3

43. $\cos\left[\cos^{-1}\left(\frac{-1}{7}\right)+\sin^{-1}\left(\frac{-1}{7}\right)\right]=$

(a) $-1/3$ (b) 0 (c) $1/3$ (d) $4/9$

44. The value of $\tan\left[\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)\right]$ **is**

(a) $\frac{6}{17}$ (b) $\frac{6}{\sqrt{13}}$ (c) $\frac{\sqrt{13}}{5}$ (d) $\frac{17}{6}$

45. The value of $\tan\left(\tan^{-1}\frac{1}{2}-\tan^{-1}\frac{1}{3}\right)$ **is**

(a) $5/6$ (b) $7/6$ (c) $1/6$ (d) $1/7$

46. If $\cos(2\sin^{-1}x)=\frac{1}{9}$, **then** $x=$

(a) Only 2/3 (b) Only –2/3 (c) 2/3, –2/3 (d) Neither 2/3 nor –2/3

47. If $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, **then** x **=**

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these

48. If $2\cos^{-1}\sqrt{\frac{1+x}{2}}=\frac{\pi}{2}$, **then** $x=$

(a) 1 (b) 0 (c) –1/2 (d) 1/2

49. $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]=$

(a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3+\sqrt{5}}{2}$ (c) $\frac{2}{3-\sqrt{5}}$ (d) $\frac{2}{3+\sqrt{5}}$

50. $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=$

(a) $\cot^{-1}\sqrt{x}$ (b) $\tan^{-1}\sqrt{x}$ (c) $\tan^{-1}x$ (d) $\cot^{-1}x$

51. If $3\sin^{-1}\frac{2x}{1-x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$ **then** x **=**

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) None of these

52. The value of $\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)+\cos(\tan^{-1}2\sqrt{2})=$

(a) $\frac{16}{15}$ (b) $\frac{14}{15}$ (c) $\frac{12}{15}$ (d) $\frac{11}{15}$

53. $\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ **is equal to**

(a) $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ (b) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$

(c) $\tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$ (d)None of these

54. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ **is**

(a) Zero (b) One (c) Two (d) Infinite

55. The equation $2\cos^{-1}x+\sin^{-1}x=\frac{11\pi}{6}$ **has**

(a) No solution (b) Only one solution (c) Two solutions (d) Three solutions

56. If $A=\tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ **and** $B=\tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$**, then the value of A – B is**

1) 0° 2) 45° 3) 60° 4) 30°

57. The value of $\sin^{-1}(\sin 10)$ **is**

1) 10 2) $10-3\pi$ 3) $3\pi-10$ 4) None of these

58. $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ **is**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) π 4) $\frac{\pi}{3}$

59. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$**, then** $x+y+z$ **is equal to**

(a) xyz (b) 0 (c)1 (d)$2xyz$

60. $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right]=$

(a) $\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$ (b) $\cos^{-1}\left(\frac{a+b\cos\theta}{a\cos\theta+b}\right)$

(c) $\cos^{-1}\left(\frac{a\cos\theta}{a+b\cos\theta}\right)$ (d) $\cos^{-1}\left(\frac{b\cos\theta}{a\cos\theta+b}\right)$

61. $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]=$

(a) $\pi-x$ (b) $2\pi-x$ (c) $\frac{x}{2}$ (d) $\pi-\frac{x}{2}$

62. If $\theta=\tan^{-1}a,\phi=\tan^{-1}b$ and $ab=-1$, then $\theta-\phi=$

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d)None of these

63. If $\tan(\cos^{-1}x)=\sin\left(\cot^{-1}\frac{1}{2}\right)$**, then** x **=**

(a) $\pm\frac{5}{3}$ (b) $\pm\frac{\sqrt{5}}{3}$ (c) $\pm\frac{5}{\sqrt{3}}$ (d)None of these

64. The value of $\sin\left(\cot^{-1}\left(\cos\left(\tan^{-1}x\right)\right)\right)$ **is**

1) $\sqrt{\frac{x^2+2}{x^2+1}}$ 2) $\sqrt{\frac{x^2+1}{x^2+2}}$ 3) $\frac{x}{\sqrt{x^2+2}}$ 4) $\frac{1}{\sqrt{x^2+2}}$

65. If $x\geq 1$**, then** $2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ **is equal to**

1) $4\tan^{-1}x$ 2) 0 3) $\frac{\pi}{2}$ 4) π

INVERSE TRIGONOMETRIC FUNCTIONS

HINTS AND SOLUTIONS

1.(b)

2.(d)

3. (c) We have $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]=\frac{1}{2}\theta$ (Putting $x=\tan\theta$)

$\Rightarrow \tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right]=\frac{\theta}{2}$

$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\theta}{2}\Rightarrow\frac{\pi}{4}-\theta=\frac{\theta}{2}$

$\Rightarrow \theta=\frac{\pi}{6}=\tan^{-1}x\Rightarrow x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$.

4. (b) Obviously $x=\sin(\theta+\beta)$ and $y=\sin(\theta-\beta)$

$\therefore 1+xy=1+\sin(\theta+\beta)\sin(\theta-\beta)$

$=1+\sin^2\theta-\sin^2\beta=\sin^2\theta+\cos^2\beta$.

5. (c) Since $3\pi<10<3\pi+\frac{\pi}{2}\Rightarrow 0<10-3\pi<\frac{\pi}{2}$

$\Rightarrow \frac{-\pi}{2}<3\pi-10<0\Rightarrow\sin^{-1}\{\sin(3\pi-10)\}=3\pi-10$.

6.(b) standard problem

7.(c)

8.(d)

9.(b)

10. (d) $\sin[\cot^{-1}(\cos\tan^{-1}x)]$

$=\sin\left[\cot^{-1}\left(\cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right]$

$=\sin\left[\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right]=\sin\left[\sin^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right]=\sqrt{\frac{1+x^2}{2+x^2}}$.

$$\sin[\cot^{-1}(x+1)] = \sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right) = \frac{1}{\sqrt{x^2+2x+2}}$$

11.(a) $\cos(\tan^{-1}x) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$

$$\frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow x^2+2x+2 = 1+x^2 \Rightarrow \quad x = -\frac{1}{2}.$$

12.(b) $\tan\left[\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right]$

$$= \tan\left[\tan^{-1}\frac{\sqrt{\left(1-\frac{16}{25}\right)}}{\frac{4}{5}} + \tan^{-1}\frac{2}{3}\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}.\frac{2}{3}}\right)\right] = \tan.\tan^{-1}\frac{17}{6} = \frac{17}{6}.$$

13.d) Put $x = y = z = \frac{1}{2}$,

14.(a)

15.(a)

16.(b)

17. (d) $x = y = z = \frac{1}{\sqrt{3}}$, so that

$$\tan^{-1}\frac{1}{\sqrt{3}} + \tan^{-1}\frac{1}{\sqrt{3}} + \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{2}$$

18.(c) Given equation is $\tan^{-1}\frac{a+x}{a} + \tan^{-1}\frac{a-x}{a} = \frac{\pi}{6}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a+x}{a}+\frac{a-x}{a}}{1-\frac{a+x}{a}.\frac{a-x}{a}}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{2a^2}{x^2} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow x^2 = 2\sqrt{3}a^2.$$

19.(d) $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}3x$

$$\Rightarrow \tan^{-1}(x-1) + \tan^{-1}(x) = \tan^{-1}3x - \tan^{-1}(x+1)$$

$$\Rightarrow \tan^{-1}\left[\frac{(x-1)+x}{1-(x-1)(x)}\right] = \tan^{-1}\left[\frac{3x-(x+1)}{1+3x(x+1)}\right]$$

$\Rightarrow \frac{2x-1}{1-x^2+x} = \frac{2x-1}{1+3x^2+3x}$

$\Rightarrow (1-x^2+x)(2x-1) = (1+3x^2+3x)(2x-1)$

On simplification $x = 0, \pm\frac{1}{2}$.

20.(d) $\cot^{-1}\alpha + \cot^{-1}\beta = \cot^{-1}x$

$\Rightarrow \cot^{-1}\left(\frac{\alpha\beta-1}{\alpha+\beta}\right) = \cot^{-1}x \Rightarrow x = \frac{\alpha\beta-1}{\alpha+\beta}$.

21.(b) $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3} \Rightarrow \frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.

22.(c) verification

23.(a) We have $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x = \frac{\pi}{2} + \tan^{-1}x$

Since $\frac{-\pi}{2} \le \tan^{-1}x \le \frac{\pi}{2} \Rightarrow 0 \le \frac{\pi}{2} + \tan^{-1}x \le \pi$

$\therefore\ K = \pi, k = 0$.

24.(a)

25.c)

26.(c)

27.(a) standard problem

28.(d) $\alpha = \cos^{-1}\sqrt{p};\ \ \beta = \cos^{-1}\sqrt{1-p}$

and $\gamma = \cos^{-1}\sqrt{1-q}$ or $\cos\alpha = \sqrt{p}$; $\cos\beta = \sqrt{1-p}$

and $\cos\gamma = \sqrt{1-q}$.

Therefore $\sin\alpha = \sqrt{1-p},\ \ \sin\beta = \sqrt{p}$ and $\sin\gamma = \sqrt{q}$.

$\alpha+\beta+\gamma = \frac{3\pi}{4}$ or $\alpha+\beta = \frac{3\pi}{4} - \gamma$ or $\cos(\alpha+\beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$

$\Rightarrow \cos\alpha\ \cos\beta - \sin\alpha\sin\beta =$

$\cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$

$$\Rightarrow \sqrt{p}\sqrt{1-p}-\sqrt{1-p}\sqrt{p} = -\left(\frac{1}{\sqrt{2}}\sqrt{1-q}-\frac{1}{\sqrt{2}}.\sqrt{q}\right)$$

$$\Rightarrow 0=\sqrt{1-q}-\sqrt{q} \Rightarrow 1-q=q \Rightarrow q=\frac{1}{2}.$$

29.(a) $\tan^{-1}\left[\frac{1}{\sqrt{\cos\alpha}}\right]-\tan^{-1}\left[\sqrt{\cos\alpha}\right]=x$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{\sqrt{\cos\alpha}}-\sqrt{\cos\alpha}}{1+\frac{\sqrt{\cos\alpha}}{\sqrt{\cos\alpha}}}\right]=x \Rightarrow \tan x=\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

$$\therefore \sin x=\frac{1-\cos\alpha}{1+\cos\alpha}=\frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}=\tan^2\left(\frac{\alpha}{2}\right).$$

30.(b) $\tan^{-1}(x)+\tan^{-1}(y)+\tan^{-1}(z)=\pi$

$$\Rightarrow \tan^{-1}x+\tan^{-1}y=\pi-\tan^{-1}z$$

$$\Rightarrow \frac{x+y}{1-xy}=-z \Rightarrow x+y=-z+xyz$$

$$\Rightarrow x+y+z=xyz$$

Dividing by *xyz*, we get

$$\frac{1}{yz}+\frac{1}{xz}+\frac{1}{xy}=1.$$

31.(c) $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

$$=\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right] \text{(Let } a=\tan\theta\text{)}$$

$$=\tan\left[\frac{1}{2}\sin^{-1}(\sin 2\theta)+\frac{1}{2}\cos^{-1}(\cos 2\theta)\right]$$

$$=\tan(2\theta)=\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{2a}{1-a^2}$$

32.(a) $\cos^{-1}(\cos 12)-\sin^{-1}(\sin 14) \Rightarrow 12-14=-2.$

33.(d) $x=\frac{\pi}{2}$ then $\cos x=0$

$$\cos^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right)=\cos^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{4}{3}\right)$$

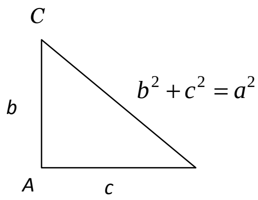
Put $x=\frac{\pi}{2}$ in $2\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$

We get $2\tan^{-1}\left(\frac{1}{2}\tan\frac{\pi}{4}\right)$

$$= 2\tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{2.\frac{1}{2}}{1-\frac{1}{4}}\right) = \tan^{-1}\left(\frac{4}{3}\right).$$

34.(c) $\angle A = 90^{o}$

$$\tan^{-1}\left(\frac{c}{a+b}\right)+\tan^{-1}\left(\frac{b}{a+c}\right)$$



$$= \tan^{-1}\left[\frac{\frac{c}{a+b}+\frac{b}{a+c}}{1-\left(\frac{c}{a+b}\right)\left(\frac{b}{a+c}\right)}\right]$$

$$= \tan^{-1}\left[\frac{ca+c^2+ab+b^2}{a^2+ab+ca+bc-bc}\right]$$

$$= \tan^{-1}\left[\frac{a^2+ab+ca}{a^2+ab+ca}\right] = \tan^{-1}(1) = \frac{\pi}{4}.$$

35.(d) $\sin^{-1}2x = \sin^{-1}x - \sin^{-1}\frac{\sqrt{3}}{2}$

$$\sin^{-1}2x = \sin^{-1}\left(x\sqrt{\left(1-\frac{3}{4}\right)}-\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$$

$$2x = \left(\frac{x}{2}-\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$$

$$\frac{\sqrt{3}}{2}\sqrt{1-x^2} = \frac{x}{2}-2x = \frac{-3x}{2}$$

$$\frac{3(1-x^2)}{4} = \frac{9x^2}{4}$$

$$\Rightarrow 3-3x^2 = 9x^2 \Longrightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}.$$

36.(b) $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]$

Let $\frac{1}{2}\cos^{-1}\frac{a}{b} = \theta \Rightarrow \cos 2\theta = \frac{a}{b}$

$$\tan\left[\frac{\pi}{4}+\theta\right]+\tan\left[\frac{\pi}{4}-\theta\right]$$

$$= \frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta} = \frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{(1-\tan^2\theta)}$$

$$= \frac{1+\tan^2\theta+2\tan\theta+1+\tan^2\theta-2\tan\theta}{(1+\tan^2\theta)}$$

$$= \frac{2(1+\tan^2\theta)}{1-\tan^2\theta} = 2\sec 2\theta = \frac{2}{\cos 2\theta}$$

$$= \frac{2}{a/b} = \frac{2b}{a}.$$

37.(a) $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

$$= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right] = \tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right]$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4}+\theta\right) = \frac{\pi}{4}+\theta = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2.$$

38.(b) $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

But $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\therefore\ \sin^{-1}x = \frac{\pi}{3}$ and $\cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow\ x = \frac{\sqrt{3}}{2}$ is the unique solution.

39.(b) $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$

Put $x = \tan\theta$ we get,

$$\sin\left[\tan^{-1}\left(\frac{1-\tan^2\theta}{2\tan\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right]$$

$$= \sin[\tan^{-1}(\cot 2\theta)+\cos^{-1}(\cos 2\theta)]$$

$$= \sin[\tan^{-1}\tan(\pi/2-2\theta)+\cos^{-1}\cos 2\theta]$$

$$= \sin\frac{\pi}{2} = 1.$$

40.(a) $\cos^{-1}\left[\cos\frac{5\pi}{3}\right]+\sin^{-1}\left[\frac{\cos 5\pi}{3}\right] = \frac{\pi}{2}$

41.(d) $\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]-\sin^{-1}\left[\frac{1}{2}\right] = 60^o - 30^o = 30^o$.

42.(c) $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$

$\because 0\le\cos^{-1}x\le\pi$

$\therefore 0\le\cos^{-1}y\le\pi$ and $0\le\cos^{-1}z\le\pi$

Here $\cos^{-1}x=\cos^{-1}y=\cos^{-1}z=\pi$

$\Rightarrow x=y=z=\cos\pi=-1$

$\therefore\ xy+yz+zx=(-1)(-1)+(-1)(-1)+(-1)(-1)$

$=1+1+1=3$.

43.(b) $\cos\left\{\cos^{-1}\left(\frac{-1}{7}\right)+\sin^{-1}\left(\frac{-1}{7}\right)\right\}=\cos\frac{\pi}{2}=0$.

44.(d) $\tan\left[\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)\right]$

$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)=\tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}.\frac{2}{3}}\right)$

$=\tan\left[\tan^{-1}\frac{17}{12}\times\frac{12}{6}\right]=\frac{17}{6}$.

45.(d) $\tan\left[\tan^{-1}\frac{1}{2}-\tan^{-1}\frac{1}{3}\right]=\tan\left[\tan^{-1}\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{6}}\right]$

$=\tan\tan^{-1}\left(\frac{1}{6}\times\frac{6}{7}\right)=\frac{1}{7}$.

46.(c) $\cos(2\sin^{-1}x)=\frac{1}{9}$

$\Rightarrow\cos(\sin^{-1}2x\sqrt{1-x^2})=\frac{1}{9}$

$\Rightarrow\cos(\cos^{-1}\sqrt{1-4x^2+4x^4})=\frac{1}{9}$

$\Rightarrow 1-2x^2=\frac{1}{9}\Rightarrow 2x^2=1-\frac{1}{9}=\frac{8}{9}$

$\Rightarrow x^2=\frac{4}{9}\Rightarrow x=\pm\frac{2}{3}$.

4.7(b) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$

$\Rightarrow\tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}(2\operatorname{cosec}x)$

$\frac{2\cos x}{\sin^2 x}=2\operatorname{cosec}x\Rightarrow 2\cos x=2\sin x$

or $\sin x = \cos x$

$\Rightarrow x = \frac{\pi}{4}$.

48.(b) $2\cos^{-1}\sqrt{\left(\frac{1+x}{2}\right)} = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\sqrt{\left(\frac{1+x}{2}\right)} = \frac{\pi}{4} \Rightarrow \cos\frac{\pi}{4} = \frac{\sqrt{1+x}}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{1+x}}{\sqrt{2}} \Rightarrow 1 = \sqrt{1+x} \Rightarrow x = 0$.

49. (d) $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$

$\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3} = \theta \Rightarrow \cos 2\theta = \frac{\sqrt{5}}{3}$

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \frac{\sqrt{5}}{3} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \sqrt{5} + \sqrt{5}\tan^2\theta = 3 - 3\tan^2\theta$

$\Rightarrow (\sqrt{5}+3)\tan^2\theta = 3-\sqrt{5} \Rightarrow \tan^2\theta = \frac{3-\sqrt{5}}{3+\sqrt{5}}$

$\Rightarrow \tan^2\theta = \frac{(3-\sqrt{5})^2}{4} \Rightarrow \tan\theta = \frac{3-\sqrt{5}}{2}$

50.(b) Let $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

Now, $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \frac{1}{2}\cos^{-1}\cos 2\theta = \frac{2\theta}{2} = \theta = \tan^{-1}\sqrt{x}$.

51.(b) $3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$

Put $x = \tan\theta$

52.(b) $\sin\left[2\tan^{-1}\left(\frac{1}{3}\right)\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \sin\left[\tan^{-1}\frac{2/3}{1-1/9}\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \sin[\tan^{-1} 3/4] + \cos[\tan^{-1} 2\sqrt{2}]$

$= \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$.

53.(a) We have $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$

$$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$= \sum_{m=1}^{n}[\tan^{-1}(m^2+m+1)-\tan^{-1}(m^2-m+1)]$$

$$= (\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}7-\tan^{-1}3)+$$

$$(\tan^{-1}13-\tan^{-1}7)+\dots\dots+[\tan^{-1}(n^2+n+1)-\tan^{-1}(n^2-n+1)]$$

$$= \tan^{-1}(n^2+n+1)-\tan^{-1}1 = \tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right).$$

54.(c) $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$\tan^{-1}\sqrt{x(x+1)}$ is defined when

$x(x+1)\geq 0$ …..(i)

$\sin^{-1}\sqrt{x^2+x+1}$ is defined when

$0\leq x(x+1)+1\leq 1$ or $0\leq x(x+1)\leq 0$ …..(ii)

From (i) and (ii), $x(x+1)=0$

Or $x=0$ and –1.

55.(a) $2\cos^{-1}x+\sin^{-1}x=\frac{11\pi}{6}$

$$\Rightarrow \cos^{-1}x+(\cos^{-1}x+\sin^{-1}x)=\frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1}x+\frac{\pi}{2}=\frac{11\pi}{6}$$

$\Rightarrow \cos^{-1}x=4\pi/3$ Which is not possible as $\cos^{-1}x\in[0,\pi]$.

56.(d)

57.(c)

58.(c)

59.(a) $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$

$$\Rightarrow \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]=\pi$$

$$\Rightarrow x+y+z-xyz=0$$

$\Rightarrow x+y+z=xyz$.

60.(a) $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right]=\cos^{-1}\left[\frac{1-\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}\right]\left(\because 2\tan^{-1}x=\cos^{-1}\frac{1-x^2}{1+x^2}\right)$

61.(d) Put $x=\frac{\pi}{4}$

$$\cot^{-1}\left[\frac{\sqrt{\sqrt{2}-1}+\sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1}-\sqrt{\sqrt{2}+1}}\right]$$

$$=\cot^{-1}\left[\frac{\sqrt{2}-1+\sqrt{2}+1+2\sqrt{2-1}}{\sqrt{2}-1-\sqrt{2}-1}\right]$$

$$=\cot^{-1}\left[\frac{2\sqrt{2}+2}{-2}\right]=\cot^{-1}(-1-\sqrt{2})=157.5°.$$

62.(c) $\theta=\tan^{-1}a$ and $\phi=\tan^{-1}b$, $ab=-1$.

$$\Rightarrow \tan\theta\tan\phi=-1\Rightarrow\tan\theta=-\cot\phi \Rightarrow\theta-\phi=\frac{\pi}{2}.$$

63.(b) Given that $\tan\{\cos^{-1}(x)\}=\sin\left(\cot^{-1}\frac{1}{2}\right)$

Let $\cot^{-1}\frac{1}{2}=\phi\Rightarrow\frac{1}{2}=\cot\phi$

$$\Rightarrow\sin\phi=\frac{1}{\sqrt{1+\cot^2\phi}}=\frac{2}{\sqrt{5}}$$

Let $\cos^{-1}x=\theta\Rightarrow\sec\theta=\frac{1}{x}\Rightarrow\tan\theta=\sqrt{\sec^2\theta-1}$

$$\Rightarrow\tan\theta=\sqrt{\frac{1}{x^2}-1}\Rightarrow\tan\theta=\frac{\sqrt{1-x^2}}{x}$$

So, $\tan\{\cos^{-1}(x)\}=\sin\left(\cot^{-1}\frac{1}{2}\right)$

$$\Rightarrow\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)=\sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow\frac{\sqrt{1-x^2}}{x}=\frac{2}{\sqrt{5}}\Rightarrow\sqrt{(1-x^2)5}=2x$$

$$\Rightarrow x=\pm\frac{\sqrt{5}}{3}.$$

64.(b)

65.(d)