ELLIPSE

OBJECTIVES

1. **If (-4, 1) and (6, 1) are the vertices of an ellipse and one of its foci lies on the line x - 2y = 2, then its eccentricity is**

   a) 2/7 b) 3/7 c) 4/7 d) 3/5

2. **The foci of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$ is**

   a) (5,-2), (-3,2) b) (5,2)(-3,2) c) (5,2),(3,2) d)(-5,-2),(-3,2)

3. **The equations of the directrices of the ellipse $25x^2 + 9y^2 - 150x - 90y + 225 = 0$ are**

   a) 4y+5 = 0, 4y-45 = 0 b) 4y+35 = 0, 4y - 15 = 0

   c) 4y+35 = 0, 4y-25 = 0 d) 4x-35 = 0, 4x + 35 = 0

4. **In an ellipse, the distance between the foci is 8 and the distance between the directrices is 25. Then the length of major axis is**

   a) $10\sqrt{2}$ b) $20\sqrt{2}$ c) $30\sqrt{2}$ d) $40\sqrt{2}$

5. **The distance between the foci of $4x^2 + y^2 - 16x - 6y - 39 = 0$ is**

   a) $3\sqrt{3}$ c) $8\sqrt{3}$ c) 8 d) 16

6. **The auxilary circle of the ellipse $9x^2 + 25y^2-18x-100y-116=0$**

   a) $(x+1)^2 + (y+2)^2 = 9$ b) $(x+1)^2+(y+1)^2=25$

   c) $(x-1)^2+(y-2)^2=9$ d) $(x-1)^2+(y-2)^2 = 25$

7. **The equation of the locus of the point of intersection of the perpendicular tangents to the ellipse $9x^2 + 16y^2 = 144$ is**

   a) $x^2 + y^2 = 5$ b) $x^2 + y^2 = 7$ c) $x^2 + y^2 = 25$ d) $x^2 + y^2 = 2$

8. **The radius of the director circle of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$ is**

   a) $\sqrt{34}$ b) $\sqrt{29}$ c) 5 d) 8

9. **The focus and the centre of an ellipse are (2, 3), (3, 4). Then the equation of the minor axis is**

   a) x + y = 5 b) x - y +1 = 0 c) x + y - 7 = 0 d) x + y + 7 = 0

10. **The equation** $2x^2 + 3y^2 = 30$ **represents**

    (a) A circle (b) An ellipse (c) A hyperbola (d) A parabola

11. The equation of the ellipse whose one of the vertices is (0,7) and the corresponding directrix is $y = 12$, is

(a) $95x^2 + 144y^2 = 4655$ (b) $144x^2 + 95y^2 = 4655$ (c) $95x^2 + 144y^2 = 13680$ (d) None of these

12. The lengths of major and minor axis of an ellipse are 10 and 8 respectively and its major axis along the *y*-axis. The equation of the ellipse referred to its centre as origin is

(a) $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (b) $\frac{x^2}{16} + \frac{y^2}{25} = 1$ (c) $\frac{x^2}{100} + \frac{y^2}{64} = 1$ (d) $\frac{x^2}{64} + \frac{y^2}{100} = 1$

13. Equation of the ellipse with eccentricity $\frac{1}{2}$ and foci at $(\pm 1, 0)$ is

(a) $\frac{x^2}{3} + \frac{y^2}{4} = 1$ (b) $\frac{x^2}{4} + \frac{y^2}{3} = 1$ (c) $\frac{x^2}{3} + \frac{y^2}{4} = \frac{4}{3}$ (d) None of these

14. The equation of ellipse whose distance between the foci is equal to 8 and distance between the directrix is 18, is

(a) $5x^2 - 9y^2 = 180$ (b) $9x^2 + 5y^2 = 180$

(c) $x^2 + 9y^2 = 180$ (d) $5x^2 + 9y^2 = 180$

15. The equation of an ellipse whose focus (–1, 1), whose directrix is $x - y + 3 = 0$ and whose eccentricity is $\frac{1}{2}$, is given by

(a) $7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0$ (b) $7x^2 - 2xy + 7y^2 - 10x + 10y + 7 = 0$

(c) $7x^2 - 2xy + 7y^2 - 10x - 10y - 7 = 0$ (d) $7x^2 - 2xy + 7y^2 + 10x + 10y - 7 = 0$

16. The equation of an ellipse whose eccentricity is 1/2 and the vertices are (4, 0) and (10, 0) is

(a) $3x^2 + 4y^2 - 42x + 120 = 0$ (b) $3x^2 + 4y^2 + 42x + 120 = 0$

(c) $3x^2 + 4y^2 + 42x - 120 = 0$ (d) $3x^2 + 4y^2 - 42x - 120 = 0$

17. The eccentricity of the ellipse $25x^2 + 16y^2 = 100$, is

(a) $\frac{5}{14}$ (b) $\frac{4}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

18. The equation of the ellipse whose one focus is at (4, 0) and whose eccentricity is 4/5, is

(a) $\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$ (b) $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ (c) $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$ (d) $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$

19. The distance between the foci of an ellipse is 16 and eccentricity is $\frac{1}{2}$. Length of the major axis of the ellipse is

(a) 8 (b) 64 (c) 16 (d) 32

20. If the distance between a focus and corresponding directrix of an ellipse be 8 and the eccentricity be 1/2, then length of the minor axis is

(a) 3 (b) $4\sqrt{2}$ (c) 6 (d) None of these

21. The length of the latus rectum of the ellipse $5x^2 + 9y^2 = 45$ is

(a) $\sqrt{5}/4$ (b) $\sqrt{5}/2$ (c) 5/3 (d) 10/3

22. Eccentricity of the ellipse whose latus rectum is equal to the distance between two focus points, is

(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{5}-1}{2}$ (c) $\frac{\sqrt{5}}{2}$ (d) $\frac{\sqrt{3}}{2}$

23. If the eccentricity of an ellipse be 5/8 and the distance between its foci be 10, then its latus rectum is

(a) 39/4 (b) 12 (c) 15 (d) 37/2

24. The equation of the ellipse whose centre is at origin and which passes through the points (–3, 1) and (2, –2) is

(a) $5x^2 + 3y^2 = 32$ (b) $3x^2 + 5y^2 = 32$ (c) $5x^2 - 3y^2 = 32$ (d) $3x^2 + 5y^2 + 32 = 0$

25. If the length of the major axis of an ellipse is three times the length of its minor axis, then its eccentricity is

(a)39/4 (b)12 (c) 15 (d) $e = \frac{2\sqrt{2}}{3}$

26. The latus rectum of an ellipse is 10 and the minor axis is equal to the distance between the foci. The equation of the ellipse is

(a) $x^2 + 2y^2 = 100$ (b) $x^2 + \sqrt{2}y^2 = 10$ (c) $x^2 - 2y^2 = 100$ (d) None of these

27. The locus of a variable point whose distance from (–2, 0) is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$, is

(a) Ellipse (b) Parabola (c) Hyperbola (d) None of these

28. The length of the latus rectum of an ellipse is $\frac{1}{3}$ of the major axis. Its eccentricity is

(a) $\frac{2}{3}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\frac{5\times4\times3}{7^3}$ (d) $\left(\frac{3}{4}\right)^4$

29. If the eccentricity of the two ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are equal, then the value of a/b is

(a) 5/13 (b) 6/13 (c) 13/5 (d) 13/6

30. *P* is any point on the ellipse $9x^2+36y^2=324$, whose foci are *S* and *S'*. Then $SP+S'P$ equals

(a) 3 (b) 12 (c) 36 (d) 324

31. The equation of the ellipse whose foci are $(\pm 5, 0)$ and one of its directrix is $5x=36$, is

(a) $\frac{x^2}{36}+\frac{y^2}{11}=1$ (b) $\frac{x^2}{6}+\frac{y^2}{\sqrt{11}}=1$ (c) $\frac{x^2}{6}+\frac{y^2}{11}=1$ (d) None of these

32. The equation $\frac{x^2}{2-r}+\frac{y^2}{r-5}+1=0$ represents an ellipse, if

(a) $r>2$ (b) $2<r<5$ (c) $r>5$ (d) None of these

33. If the distance between the foci of an ellipse be equal to its minor axis, then its eccentricity is

(a) 1/2 (b) $1/\sqrt{2}$ (c) 1/3 (d) $1/\sqrt{3}$

34. The equations of the directrices of the ellipse $16x^2+25y^2=400$ are

(a) $2x=\pm 25$ (b) $5x=\pm 9$ (c) $3x=\pm 10$ (d) None of these

35. If the latus rectum of an ellipse be equal to half of its minor axis, then its eccentricity is

(a) 3/2 (b) $\sqrt{3}/2$ (c)2/3 (d) $\sqrt{2}/3$

36. The distance between the foci of the ellipse $3x^2+4y^2=48$ is

(a) 2 (b) 4 (c) 6 (d) 8

37. The distance of the point 'θ' on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from a focus is

(a) $a(e+\cos\theta)$ (b) $a(e-\cos\theta)$ (c) $a(1+e\cos\theta)$ (d) $a(1+2e\cos\theta)$

38. If a bar of given length moves with its extremities on two fixed straight lines at right angles, then the locus of any point on bar marked on the bar describes a/an

(a) Circle (b) Parabola (c)Ellipse (d)Hyperbola

39. The centre of the ellipse $\frac{(x+y-2)^2}{9}+\frac{(x-y)^2}{16}=1$ is

(a) (0, 0) (b) (1, 1) (c) (1, 0) (d) (0, 1)

40. The equation of the ellipse whose centre is (2, –3), one of the foci is (3, –3) and the corresponding vertex is (4, –3) is

(a) $\frac{(x-2)^2}{3}+\frac{(y+3)^2}{4}=1$ (b) $\frac{(x-2)^2}{4}+\frac{(y+3)^2}{3}=1$ (c) $\frac{x^2}{3}+\frac{y^2}{4}=1$ (d) None of these

41. The length of the axes of the conic $9x^2+4y^2-6x+4y+1=0$, are

(a) $\frac{1}{2}, 9$ (b) $3, \frac{2}{5}$ (c) $1, \frac{2}{3}$ (d) 3, 2

42. The eccentricity of the conic $4x^2+16y^2-24x-3y=1$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\sqrt{3}$

43. The equation of the tangent to the ellipse $x^2+16y^2=16$ making an angle of 60^o with *x*-axis is

(a) $\sqrt{3}x-y+7=0$ (b) $\sqrt{3}x-y-7=0$ (c) $\sqrt{3}x-y\pm 7=0$ (d) None of these

44. If $y=mx+c$ is tangent on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, then the value of *c* is

(a) 0 (b) $3/m$ (c) $\pm\sqrt{9m^2+4}$ (d) $\pm 3\sqrt{1+m^2}$

45. The equation of the tangents drawn at the ends of the major axis of the ellipse $9x^2+5y^2-30y=0$, are

(a) $y=\pm 3$ (b) $x=\pm\sqrt{5}$ (c) $y=0, y=6$ (d) None of these

46. The locus of the point of intersection of mutually perpendicular tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

(a) A straight line (b) A parabola (c)A circle (d) None of these

47. The equation of normal at the point (0, 3) of the ellipse $9x^2+5y^2=45$ is

(a) $y-3=0$ (b) $y+3=0$ (c) *x*-axis (d) *y*-axis

48. If the foci of an ellipse are $(\pm\sqrt{5}, 0)$ and its eccentricity is $\frac{\sqrt{5}}{3}$, then the equation of the ellipse is

(a) $9x^2+4y^2=36$ (b) $4x^2+9y^2=36$ (c) $36x^2+9y^2=4$ (d) $9x^2+36y^2=4$

49. An ellipse has *OB* as semi minor axis, *F* and *F′* its foci and the angle *FBF′* is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{3}}$ © $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

50. The value of λ, for which the line $2x-\frac{8}{3}\lambda y=-3$ is a normal to the conic $x^2+\frac{y^2}{4}=1$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{3}{8}$

51. The point (4, –3) with respect to the ellipse $4x^2+5y^2=1$

(a) Lies on the curve (b) Is inside the curve (c)Is outside the curve (d)Is focus of the curve

52. The sum of the focal distances of any point on the conic $\frac{x^2}{25}+\frac{y^2}{16}=1$ is

(a) 10 (b) 9 (c) 41 (d)18

53. If the line $x\cos\alpha+y\sin\alpha=p$ be normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then

(a) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=a^2-b^2$ (b) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=(a^2-b^2)^2$

(c) $p^2(a^2\sec^2\alpha+b^2\operatorname{cosec}^2\alpha)=a^2-b^2$ (d) $p^2(a^2\sec^2\alpha+b^2\operatorname{cosec}^2\alpha)=(a^2-b^2)^2$

54. Eccentric angle of a point on the ellipse $x^2+3y^2=6$ at a distance 2 units from the centre of the ellipse is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

55. The equations of the tangents of the ellipse $9x^2+16y^2=144$ which passes through the point (2, 3) is

(a) $y=3,\ x+y=5$ (b) $y=-3,\ x-y=5$ (c) $y=-4,\ x+y=3$ (d) $y=-4,\ x-y=3$

56. The eccentric angles of the extremities of latus recta of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are given by

(a) $\tan^{-1}\left(\pm\frac{ae}{b}\right)$ (b) $\tan^{-1}\left(\pm\frac{be}{a}\right)$ (c) $\tan^{-1}\left(\pm\frac{b}{ae}\right)$ (d) $\tan^{-1}\left(\pm\frac{a}{be}\right)$

57. In the ellipse, minor axis is 8 and eccentricity is $\frac{\sqrt{5}}{3}$. Then major axis is

(a) 6 (b) 12 (c) 10 (d)16

58.Latus rectum of ellipse $4x^2+9y^2-8x-36y+4=0$ is

(a) 8/3 (b) 4/3 (c) $\frac{\sqrt{5}}{3}$ (d)16/3

59. The sum of focal distances of any point on the ellipse with major and minor axes as 2*a* and 2*b* respectively, is equal to

(a) $2a$ (b) $\frac{2a}{b}$ (c) $\frac{2b}{a}$ (d) $\frac{b^2}{a}$

60. The centre of the ellipse $4x^2 + 9y^2 - 16x - 54y + 61 = 0$ **is**

(a) (1,3) (b) (2, 3) (c) (3, 2) (d) (3, 1)

61. The eccentricity of the ellipse $9x^2 + 5y^2 - 18x - 2y - 16 = 0$ **is**

(a) 1/2 (b) 2/3 (c) 1/3 (d) ¾

62. An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 *cm* and 4 *cm*, the necessary length of the string and the distance between the pins respectively in *cm*, are

(a) $6, 2\sqrt{5}$ (b) $6, \sqrt{5}$ (c) $4, 2\sqrt{5}$ (d) None of these

63. The line $y = mx + c$ **is a normal to the ellipse** $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$**, if** $c =$

(a) $-(2am + bm^2)$ (b) $\frac{(a^2+b^2)m}{\sqrt{a^2+b^2m^2}}$ (c) $-\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$ (d) $\frac{(a^2-b^2)m}{\sqrt{a^2+b^2}}$

64. The equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **at the point** $(a\cos\theta, b\sin\theta)$ **is**

(a) $\frac{ax}{\sin\theta} - \frac{by}{\cos\theta} = a^2 - b^2$ (b) $\frac{ax}{\sin\theta} - \frac{by}{\cos\theta} = a^2 + b^2$

(c) $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$ (d) $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 + b^2$.

65. If the normal at the point $P(\theta)$ **to the ellipse** $\frac{x^2}{14} + \frac{y^2}{5} = 1$ **intersects it again at the point** $Q(2\theta)$**, then** $\cos\theta$ **is equal to**

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

66. The foci of the ellipse $25(x+1)^2 + 9(y+2)^2 = 225$ **are at**

(a) (–1, 2) and (–1, –6) (b) (–1, 2) and (6, 1)

(c) (1, –2) and (1, –6) (d)(–1, –2) and (1, 6)

67. If $P \equiv (x, y)$**,** $F_1 \equiv (3, 0)$**,** $F_2 \equiv (-3, 0)$ **and** $16x^2 + 25y^2 = 400$**, then** $PF_1 + PF_2$ **equals**

(a) 8 (b) 6 (c)10 (d)12

68. The angle between the pair of tangents drawn to the ellipse $3x^2 + 2y^2 = 5$ **from the point (1, 2), is**

(a) $\tan^{-1}\left(\frac{12}{5}\right)$ (b) $\tan^{-1}(6\sqrt{5})$ (c) $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$ (d) $\tan^{-1}(12\sqrt{5})$

69. If the line $y = mx + c$ **touches the ellipse** $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$**, then** $c =$

(a) $\pm\sqrt{b^2m^2 + a^2}$ (b) $\pm\sqrt{a^2m^2 + b^2}$ (c) $\pm\sqrt{b^2m^2 - a^2}$ (d) $\pm\sqrt{a^2m^2 - b^2}$

70. The locus of the point of intersection of the perpendicular tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ **is**

(a) $x^2 + y^2 = 9$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 13$ (d) $x^2 + y^2 = 5$

ELLIPSE

HINTS AND SOLUTIONS

1. d. 2a=10 and (h+ae,1) is a point on the line.

2. b

3. a

4. a 2ae=8 and 2a/e=25.

5. b 2ae

6. d $(x+h)^2+(y+k)^2=a^2$

7. c director circle.

8. a

9. c minor axis is the line perpendicular to $\overline{cs}$ passes through c.

10. (b) $\frac{x^2}{(30/2)}+\frac{y^2}{(30/3)}=1 \Rightarrow \frac{x^2}{15}+\frac{y^2}{10}=1$.

11. (b) Vertex (0,7), directrix $y=12$, $\therefore$ $b=7$

Also $\frac{b}{e}=12 \Rightarrow e=\frac{7}{12}, a=7\sqrt{\frac{95}{144}}$

Hence equation of ellipse is $144x^2+95y^2=4655$.

12. (b) Here given that $2b=10, 2a=8 \Rightarrow b=5, a=4$

Hence the required equation is $\frac{x^2}{16}+\frac{y^2}{25}=1$.

13. (b) Given that, $e=\frac{1}{2}$ and $(\pm ae, 0)=(\pm 1, 0)$

$\Rightarrow ae=1 \Rightarrow a=2$. Now $b^2=a^2(1-e^2)$

$\Rightarrow b^2=4\left(1-\frac{1}{4}\right) \Rightarrow b^2=3$

Hence, equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$.

14. (d) $2ae=8, \frac{2a}{e}=18 \Rightarrow a=\sqrt{4\times 9}=6$

$e=\frac{2}{3}, b=6\sqrt{1-\frac{4}{9}}=\frac{6}{3}\sqrt{5}=2\sqrt{5}$

Hence the required equation is $\frac{x^2}{36}+\frac{y^2}{20}=1$

i.e., $5x^2+9y^2=180$.

15. (a) Let any point on it be (x,y), then $\frac{\sqrt{(x+1)^2}+\sqrt{(y-1)^2}}{\left|\frac{x-y+3}{\sqrt{2}}\right|}=\frac{1}{2}$

Squaring and simplifying, we get

$7x^2+2xy+7y^2+10x-10y+7=0$.

16. (a) Major axis $=6=2a \Rightarrow a=3$

$e=\frac{1}{2} \Rightarrow b=3\sqrt{1-\frac{1}{4}}=\frac{3\sqrt{3}}{2}$. Also centre is (7, 0)

Equation is $\frac{(x-7)^2}{9}+\frac{y^2}{(27/4)}=1$

$\Rightarrow 3x^2+4y^2-42x+120=0$.

17. (c) $\frac{x^2}{4}+\frac{y^2}{(25/4)}=1$. Here, $a=2, b=5/2$

$\therefore b>a$, therefore $a^2=b^2(1-e^2)$

$\Rightarrow 4=\frac{25}{4}(1-e^2) \Rightarrow \frac{16}{25}=1-e^2 \Rightarrow e^2=1-\frac{16}{25}=\frac{9}{25}$,

$\therefore e=\frac{3}{5}$.

18. (b) Here $ae=4$ and $e=\frac{4}{5} \Rightarrow a=5$

Now $b^2=a^2(1-e^2) \Rightarrow b^2=25\left(1-\frac{16}{25}\right)=9$

Hence equation of the ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$.

19. (d) Given, distance between the foci $=2ae=16$ and eccentricity of ellipse (e) $=1/2$. We know that length of the major axis of the ellipse $=2a=\frac{2ae}{e}=\frac{16}{1/2}=32$.

20. (d) $\frac{a}{e}-ae=8$. Also $e=\frac{1}{2} \Rightarrow a=\frac{8e}{(1-e^2)}=\frac{8.4}{2(3)}=\frac{16}{3}$

$\therefore b = \frac{16}{3}\sqrt{\left(1-\frac{1}{4}\right)} = \frac{16}{3}\frac{\sqrt{3}}{2} = \frac{8\sqrt{3}}{3}$

Hence the length of minor axis is $\frac{16\sqrt{3}}{3}$.

21. (d) Here the ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$.

Latus rectum $=\frac{2b^2}{a}=\frac{2\cdot 5}{3}=\frac{10}{3}$.

22. (b) $\frac{2b^2}{a}=2ae \Rightarrow b^2=a^2e$ or $e=\frac{b^2}{a^2}$

Also $e=\sqrt{1-\frac{b^2}{a^2}}$ or $e^2=1-e$ or $e^2+e-1=0$

Therefore $e=\frac{-1\pm\sqrt{5}}{2}$. As $e<1$, $\therefore$ $e=\frac{\sqrt{5}-1}{2}$.

23. (a) $a=\frac{10}{\frac{2.5}{8}}=8$, $b=8\sqrt{1-\frac{25}{64}}=8\frac{\sqrt{39}}{8}$

Latus rectum $=\frac{2b^2}{a}=\frac{2\times 39}{8}=\frac{39}{4}$.

24. (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Since it passes through (–3, 1) and (2, –2), so $\frac{9}{a^2}+\frac{1}{b^2}=1$ and $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}\Rightarrow a^2=\frac{32}{3}$, $b^2=\frac{32}{5}$

Hence required equation of ellipse is $3x^2+5y^2=32$.

25. (d) Major axis = 3(Minor axis)

$\Rightarrow 2a=3(2b)\Rightarrow a^2=9b^2=9a^2(1-e^2)\Rightarrow e=\frac{2\sqrt{2}}{3}$.

26. (a) Given $\frac{2b^2}{a}=10$ and $2b=2ae$

Also $b^2=a^2(1-e^2) \Rightarrow e^2=(1-e^2) \Rightarrow e=\frac{1}{\sqrt{2}}$

$\Rightarrow b=\frac{a}{\sqrt{2}}$ or $b=5\sqrt{2}$, $a=10$

Hence equation of ellipse is $\frac{x^2}{(10)^2}+\frac{y^2}{(5\sqrt{2})^2}=1$

27. (a) Let point P (x_1,y_1)

So, $\sqrt{(x_1+2)^2+y_1^2}=\frac{2}{3}\left(x_1+\frac{9}{2}\right)$

$\Rightarrow (x_1+2)^2+y_1^2=\frac{4}{9}\left(x_1+\frac{9}{2}\right)^2$

$\Rightarrow 9[x_1^2+y_1^2+4x_1+4]=4\left(x_1^2+\frac{81}{4}+9x_1\right)$

$\Rightarrow 5x_1^2+9y_1^2=45 \Rightarrow \frac{x_1^2}{9}+\frac{y_1^2}{5}=1$,

Locus of (x_1, y_1) is $\frac{x^2}{9}+\frac{y^2}{5}=1$, which is equation of an ellipse.

28. (b) Latus rectum $=1/3$ (Major axis)

$\Rightarrow \frac{2b^2}{a}=\frac{2a}{3} \Rightarrow a^2=3b^2=3a^2(1-e^2) \Rightarrow e=\sqrt{\frac{2}{3}}$.

29. (c) In the first case, eccentricity $e=\sqrt{1-(25/169)}$

In the second case, $e'=\sqrt{1-(b^2/a^2)}$

According to the given condition,

$\sqrt{1-b^2/a^2}=\sqrt{1-(25/169)}$

$\Rightarrow b/a=5/13$, $(\because a>0, b>0)$

$\Rightarrow a/b=13/5.$

30. (b) $SP+S'P=2a=2.6=12$.

31. (a) Foci $(\pm 5, 0)\equiv(\pm ae, 0)$. Directrix $\left(x=\frac{36}{5}\right)\equiv x=\frac{a}{e}$

So, $\frac{a}{e}=\frac{36}{5}, ae=5 \Rightarrow a=6$ and $e=\frac{5}{6}$

Therefore, $b=6\sqrt{1-\frac{25}{36}}=6\frac{\sqrt{11}}{6}=\sqrt{11}$

Hence equation is $\frac{x^2}{36}+\frac{y^2}{11}=1$.

32. (b) $\frac{x^2}{2-r}+\frac{y^2}{r-5}+1=0 \Rightarrow \frac{x^2}{r-2}+\frac{y^2}{5-r}=1$

Hence $r>2$ and $r<5 \Rightarrow 2<r<5.$

33. (b) Foci are $(\pm ae, 0)$. Therefore according to the condition, $2ae=2b$ or $ae=b$

Also, $b^2=a^2(1-e^2) \Rightarrow e^2=(1-e^2) \Rightarrow e=\frac{1}{\sqrt{2}}$.

34. (d) $\frac{x^2}{25}+\frac{y^2}{16}=1 \Rightarrow e=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

Therefore, directrices are $x \pm \frac{5}{3/5}=0$ or $3x \pm 25=0$.

35. (b) $\frac{2b^2}{a}=b \Rightarrow \frac{b}{a}=\frac{1}{2} \Rightarrow \frac{b^2}{a^2}=\frac{1}{4}$

Hence $e=\sqrt{1-\frac{b^2}{a^2}}=\frac{\sqrt{3}}{2}$.

36. (b) $\frac{x^2}{(48/3)}+\frac{y^2}{(48/4)}=1$

$a^2=16, b^2=12 \Rightarrow e=\sqrt{1-\frac{b^2}{a^2}}=\frac{1}{2}$

Distance is $2ae=2\cdot 4\cdot\frac{1}{2}=4$.

37. (c) Focal distance of any point $P\,(x,y)$ on the ellipse is equal to $SP=a+ex$. Here $x=a\cos\theta$

Here $SP=a+ae\cos\theta=a(1+e\cos\theta)$.

38. (c) It is obvious.

39. (b) The centre of the given ellipse is the point of intersection of the lines $x+y-2=0$ and $x-y=0$ *i.e.*,(1,1).

40. (b) Foci $=(3,-3) \Rightarrow ae=3-2=1$

Vertex $=(4,-3) \Rightarrow a=4-2=2 \Rightarrow e=\frac{1}{2}$

$\Rightarrow b=a\sqrt{\left(1-\frac{1}{4}\right)}=\frac{2}{2}\sqrt{3}=\sqrt{3}$

Therefore, equation of ellipse with centre $(2,-3)$ is

$\frac{(x-2)^2}{4}+\frac{(y+3)^2}{3}=1$.

41. (c) Given that, the equation of conic

$9x^2+4y^2-6x+4y+1=0$

$\Rightarrow (3x-1)^2+(2y+1)^2=1 \Rightarrow \frac{\left(x-\frac{1}{3}\right)^2}{\frac{1}{9}}+\frac{(y+1)^2}{\frac{1}{2}}=1$.

Here $a=\frac{1}{3}$, $b=\frac{1}{2}$; $\therefore$ $2a=\frac{2}{3}$, $2b=1$.

Length of axes are $\left(1,\frac{2}{3}\right)$.

42. (a) Given equation of conic is $4x^2 + 16y^2 - 24x - 3y = 1$

$\Rightarrow (2x-6)^2 + (4y-4)^2 = 53$

$\Rightarrow 4(x-3)^2 + 16(y-1)^2 = 53$

$\Rightarrow \frac{(x-3)^2}{53/4} + \frac{(y-1)^2}{53/16} = 1$

$\therefore\ e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{53/16}{53/4}} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$.

43. (c) $m = \tan 60° = \sqrt{3}$. Tangent is $y = \sqrt{3}x \pm \sqrt{1+3\cdot 16} \Rightarrow y = \sqrt{3}x \pm 7$.

44. (c) Here, $a = 3, b = 2$. $\therefore$ By formula, $c^2 = b^2 + a^2m^2$

$\therefore\ c^2 = 4 + 9m^2$; $\therefore\ c = \pm\sqrt{9m^2+4}$.

45. (c) $9x^2 + 5(y^2 - 6y) = 0$

$\Rightarrow 9x^2 + 5(y^2 - 6y + 9) = 45 \Rightarrow \frac{x^2}{5} + \frac{(y-3)^2}{9} = 1$

$\because a^2 < b^2$, so axis of ellipse on y-axis.

At y axis, put $x = 0$, so we can obtained vertex.

Then $0 + 5y^2 - 30y = 0 \Rightarrow y = 0,\ y = 6$

Therefore, tangents of vertex $y = 0,\ y = 6$.

46. (c) It is a fundamental concept.

47. (d) For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, equation of normal at point (x_1, y_1),

$\Rightarrow \frac{(x-x_1)a^2}{x_1} = \frac{(y-y_1)b^2}{y_1}$; $\therefore (x_1, y_1) \equiv (0,3),\ a^2 = 5,\ b^2 = 9$

$\Rightarrow \frac{(x-0)}{0}\ 5 = \frac{(y-3).9}{3}$ or $x = 0$

48. (b) $\because ae = \pm\sqrt{5} \Rightarrow a = \pm\sqrt{5}\left(\frac{3}{\sqrt{5}}\right) = \pm 3 \Rightarrow a^2 = 9$

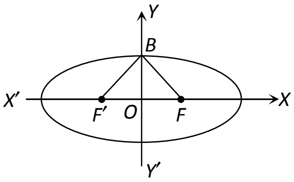
$\therefore b^2 = a^2(1-e^2) = 9\left(1-\frac{5}{9}\right) = 4$

Hence, equation of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

49. (c) $\angle F'BF = 90°$, $F'B \perp FB$

i.e., slope of $(F'B)$ × Slope of $(F'B) = -1$

$\Rightarrow \frac{b}{ae} \times \frac{b}{-ae} = -1$, $b^2 = a^2e^2$ .....(i)



We know that $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{a^2e^2}{a^2}}=\sqrt{1-e^2}$

$e^2=1-e^2$, $2e^2=1$, $e^2=\frac{1}{2}$, $e=\frac{1}{\sqrt{2}}$.

50. (d) We know that the equation of the normal at point $(a\cos\theta, b\sin\theta)$ on the curve $x^2+\frac{y^2}{4}=1$ is given by

$ax\sin\theta-by\operatorname{cosec}\theta=a^2-b^2$ .....(i)

Comparing equation (i) with $2x-\frac{8}{3}\lambda y=-3$. We get,

$a\sin\theta=2$, $b\operatorname{cosec}\theta=\frac{8}{3}\lambda$ or $ab=\frac{16}{3}\lambda$ .....(ii)

$\because a=1, b=2$; $\therefore 2=\frac{16}{3}\lambda$ or $\lambda=3/8$

51. (c) Given ellipse is $\frac{x^2}{1/4}+\frac{y^2}{1/5}=1$

$\therefore \frac{16}{1/4}+\frac{9}{1/5}-1=64+45-1>0$

Point (4, –3) lies outside the ellipse.

52. (a) For any point *P* on the ellipse have focus *S* and *S′* $SP+S'P=2a$

$\therefore$ For $\frac{x^2}{25}+\frac{y^2}{16}=1$

Sum of focal distances = 2 × 5 = 10.

53. (d) The equation of any normal is $ax\sec\phi-by\operatorname{cosec}\phi=a^2-b^2$ .....(i)

Given line $x\cos\alpha+y\sin\alpha=p$ comparing and eliminating ϕ

$\Rightarrow p^2(b^2\operatorname{cosec}^2\alpha+a^2\sec^2\alpha)=(a^2-b^2)^2$.

54. (a) Let the eccentric angle of the point be θ, then its co-ordinates are $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$.

Hence $6\cos^2\theta+2\sin^2\theta=4$ or $\cos^2\theta=\frac{1}{2}$

Hence, $\cos\theta=\pm\frac{1}{\sqrt{2}}$, $\therefore \theta=\frac{\pi}{4}$ or $\frac{3\pi}{4}$.

55. (a) The tangent will be $y-3=m(x-2) \Rightarrow y-mx=3-2m$.

But it is tangent to the given ellipse, therefore $m=0,-1$. Hence tangents are $y=3$ and $x+y=5$.

56. (c) Coordinates of any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angle is θ are $(a\cos\theta, b\sin\theta)$.

The coordinates of the end points of latus recta are $\left(ae, \pm\frac{b^2}{a}\right)$. $\therefore a\cos\theta = ae$ and $b\sin\theta = \pm\frac{b^2}{a}$

$\Rightarrow \tan\theta = \pm\frac{b}{ae} \Rightarrow \theta = \tan^{-1}\left(\pm\frac{b}{ae}\right)$.

57. (b) Given, minor axis of ellipse $(2b) = 8$ or $b = 4$ and eccentricity $(e) = \sqrt{5}/3$. We know that in an ellipse, $e^2 = 1 - \frac{b^2}{a^2}$ or $\frac{5}{9} = 1 - \frac{16}{a^2}$ or $\frac{16}{a^2} = 1 - \frac{5}{9}$ or $a^2 = \frac{16\times 9}{4} = 36$ or $a = 6$. We also know that major axis of the ellipse $= 2a = 2\times 6 = 12$.

58. (a) The ellipse is $4(x-1)^2 + 9(y-2)^2 = 36$

Therefore, latus rectum $= \frac{2b^2}{a} = \frac{2.4}{3} = \frac{8}{3}$.

59. (a) Sum of focal distances of a point in an ellipse is always equal to length of major axis of that ellipse. It is a property of ellipse.

60. (b) $4(x-2)^2 + 9(y-3)^2 = 36$

Hence the centre is (2, 3).

61. (b) Given equation can be written as

$\frac{(x-1)^2}{5} + \frac{(y-2)^2}{9} = 1$; $e = \sqrt{\frac{b^2 - a^2}{b^2}} = \sqrt{\frac{9-5}{9}} = \frac{2}{3}$.

62. (d) Given $2a = 6,\ 2b = 4$ *i.e.*, $a = 3,\ b = 2$

$e^2 = 1 - \frac{b^2}{a^2} = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$

Distance between the pins $= 2ae = 2\sqrt{5}\,cm$

Length of string $= 2a + 2ae = 6 + 2\sqrt{5}\,cm$.

63. (c) As we know that the line $lx + my + n = 0$ is normal to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$. But in this condition, we have to replace *l* by *m*, *m* by -1 and *n* by *c*, then the required condition is $c = \pm\frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$.

64. (c) $ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$.

65. (b) The normal at $P(a\cos\theta, b\sin\theta)$ is $ax\sec\theta - by\ \mathrm{cosec}\,\theta = a^2 - b^2$, where $a^2 = 14,\ b^2 = 5$

It meets the curve again at $Q(2\theta)$ *i.e.*, $(a\cos 2\theta, b\sin 2\theta)$.

$\therefore \frac{a}{\cos\theta}a\cos 2\theta - \frac{b}{\sin\theta}(b\sin 2\theta) = a^2 - b^2$

$\Rightarrow \frac{14}{\cos\theta}\cos 2\theta - \frac{5}{\sin\theta}(\sin 2\theta) = 14 - 5$

$\Rightarrow 18\cos^2\theta - 9\cos\theta - 14 = 0$

$\Rightarrow (6\cos\theta - 7)(3\cos\theta + 2) = 0 \Rightarrow \cos\theta = -\frac{2}{3}$.

66. (a) $\frac{(x+1)^2}{\frac{225}{25}} + \frac{(y+2)^2}{\frac{225}{9}} = 1$

$a = \sqrt{\frac{225}{25}} = \frac{15}{5}, b = \sqrt{\frac{225}{9}} = \frac{15}{3} \Rightarrow e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

Focus $= \left(-1, -2 \pm \frac{15}{3}.\frac{4}{5}\right) = (-1, -2 \pm 4) = (-1, 2); (-1, -6)$.

67. (c) Equation of the curve is $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$

$\Rightarrow -5 \le x \le 5, -4 \le y \le 4$

$PF_1 + PF_2 = \sqrt{[(x-3)^2 + y^2]} + \sqrt{[(x+3)^2 + y^2]}$

$= \sqrt{(x-3)^2 + \frac{400 - 16x^2}{25}} + \sqrt{(x+3)^2 + \frac{400 - 16x^2}{25}}$

$= \frac{1}{5}\left\{\sqrt{(9x^2 + 625 - 150x)} + \sqrt{(9x^2 + 625 + 150x)}\right\}$

$= \frac{1}{5}\left\{\sqrt{(3x-25)^2} + \sqrt{(3x+25)^2}\right\} = \frac{1}{5}\{25 - 3x + 3x + 25\}$

$= 10$

68. (c) $SS_1 = T^2$

$\tan\theta = 2\frac{\sqrt{h^2 - ab}}{a+b}, a = 9$, $b = -4$ and $h = -12$.

69. (a) Since, here a and b are interchanged.

70. (c) The locus of point of intersection of two perpendicular tangents drawn on the ellipse is $x^2 + y^2 = a^2 + b^2$, which is called 'director- circle'.

Given ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$, $\therefore$ Locus is $x^2 + y^2 = 13$.