DIFFERENTIATION

OBJECTIVE PROBLEMS

1. $\frac{d}{dx}\log|x| = ..., (x \neq 0)$

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$

(c) x (d) $-x$

2. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\infty$, then $\frac{dy}{dx} =$

(a) y (b) $y-1$

(c) $y+1$ (d) None of these

3. $\frac{d}{dx}\left(\tan^{-1}\frac{\cos x}{1+\sin x}\right) =$

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) -1 (d) 1

4. $\frac{d}{dx}\tan^{-1}\left(\frac{ax-b}{bx+a}\right) =$

(a) $\frac{1}{1+x^2} - \frac{a^2}{a^2+b^2}$ (b) $\frac{-1}{1+x^2} - \frac{a^2}{a^2+b^2}$

(c) $\frac{1}{1+x^2} + \frac{a^2}{a^2+b^2}$ (d) None of these

5. **If** $y = b\cos\log\left(\frac{x}{n}\right)^n$**, then** $\frac{dy}{dx} =$

(a) $-n\, b\sin\log\left(\frac{x}{n}\right)^n$ (b) $n\, b\sin\log\left(\frac{x}{n}\right)^n$

(c) $\frac{-nb}{x}\sin\log\left(\frac{x}{n}\right)^n$ (d) None of these

6. If $x^{2/3}+y^{2/3}=a^{2/3}$**, then** $\frac{dy}{dx}=$

(a) $\left(\frac{y}{x}\right)^{1/3}$ (b) $-\left(\frac{y}{x}\right)^{1/3}$

(c) $\left(\frac{x}{y}\right)^{1/3}$ (d) $-\left(\frac{x}{y}\right)^{1/3}$

7. If $f(x)=x\tan^{-1}x$**, then** $f'(1)=$

(a) $1+\frac{\pi}{4}$ (b) $\frac{1}{2}+\frac{\pi}{4}$

(c) $\frac{1}{2}-\frac{\pi}{4}$ (d) 2

8. If $y=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....+\frac{x^n}{n!}$**, then** $\frac{dy}{dx}=$

(a) y (b) $y+\frac{x^n}{n!}$

(c) $y-\frac{x^n}{n!}$ (d) $y-1-\frac{x^n}{n!}$

9. $\frac{d}{dx}\log(\log x)=$

(a) $\frac{x}{\log x}$ (b) $\frac{\log x}{x}$

(c) $(x\log x)^{-1}$ (d) None of these

10. If $y=\tan^{-1}\frac{4x}{1+5x^2}+\tan^{-1}\frac{2+3x}{3-2x}$**, then** $\frac{dy}{dx}=$

(a) $\frac{1}{1+25x^2}+\frac{2}{1+x^2}$ (b) $\frac{5}{1+25x^2}+\frac{2}{1+x^2}$

(c) $\frac{5}{1+25x^2}$ (d) $\frac{1}{1+25x^2}$

11. If $y=x\left[\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)+\sin x\right]+\frac{1}{2\sqrt{x}}$ **, then** $\frac{dy}{dx}=$

(a) $(1+x)\cos x+(1-x)\sin x-\frac{1}{4x\sqrt{x}}$ (b) $(1-x)\cos x+(1+x)\sin x+\frac{1}{4x\sqrt{x}}$

(c) $(1+x)\cos x+(1+x)\sin x-\frac{1}{4x\sqrt{x}}$ (d) None of these

12. If $y = \sin^{-1}(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})$, **then** $\frac{dy}{dx} =$

(a) $\frac{-2x}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

(c) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x-x^2}}$ (d) None of these

13. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}} =$

(a) $\sec^2 x$ (b) $-\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\sec^2\left(\frac{\pi}{4} + x\right)$ (d) $\sec^2\left(\frac{\pi}{4} - x\right)$

14. $\frac{d}{dx}\log_7(\log_7 x) =$

(a) $\frac{1}{x\log_e x}$ (b) $\frac{\log_e 7}{x\log_e x}$

(c) $\frac{\log_7 e}{x\log_e x}$ (d) $\frac{\log_7 e}{x\log_7 x}$

15. $\frac{d}{dx}\left(\frac{\cot^2 x - 1}{\cot^2 x + 1}\right) =$

(a) $-\sin 2x$ (b) $2\sin 2x$

(c) $2\cos 2x$ (d) $-2\sin 2x$

16. $\frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right)$ is equal to

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) $\frac{1}{4}$

17. If $f(x) = \log_x(\log x)$, **then** $f'(x)$ **at** $x = e$ **is**

(a) e (b) $\frac{1}{e}$

(c) 1 (d) None of these

18. $\frac{d}{dx}\sqrt{\frac{1+\cos 2x}{1-\cos 2x}} =$

(a) $\sec^2 x$ (b) $-\text{cosec}^2 x$

(c) $2\sec^2 \frac{x}{2}$ (d) $-2\text{cosec}^2 \frac{x}{2}$

19. $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right] =$

(a) $-\frac{1}{2}$ (b) 0

(c) $\frac{1}{2}$ (d) 1

20. If $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$, then $\frac{dy}{dx} =$

(a) $\frac{4x}{1-x^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (b) $\frac{x}{(1-x^2)^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$

(c) $\frac{x}{(1-x^2)}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (d) $\frac{4x}{(1-x^2)^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$

21. If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then $\frac{dy}{dx} =$

(a) 0 (b) $\frac{1}{\sqrt{x}+1}$

(c) 1 (d) None of these

22. If $y = \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$, then $\frac{dy}{dx} =$

(a) $\frac{ay}{x\sqrt{a^2-x^2}}$ (b) $\frac{ay}{\sqrt{a^2-x^2}}$

(c) $\frac{ay}{x\sqrt{x^2-a^2}}$ (d) None of these

23. $\frac{d}{dx}\sin^{-1}(3x-4x^3) =$

(a) $\frac{3}{\sqrt{1-x^2}}$ (b) $\frac{-3}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{-1}{\sqrt{1-x^2}}$

24. $\frac{d}{dx}\tan^{-1}(\sec x+\tan x)=$

(a) 1 (b) 1/2

(c) $\cos x$ (d) $\sec x$

25. $\frac{d}{dx}\left(\frac{\log x}{\sin x}\right)=$

(a) $\frac{\frac{\sin x}{x}-\log x.\cos x}{\sin x}$ (b) $\frac{\frac{\sin x}{x}-\log x.\cos x}{\sin^2 x}$

(c) $\frac{\sin x-\log x.\cos x}{\sin^2 x}$ (d) $\frac{\frac{\sin x}{x}-\log x}{\sin^2 x}$

26. If $y=\cot^{-1}\left(\frac{1+x}{1-x}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$

(c) $\frac{2}{1+x^2}$ (d) $-\frac{2}{1+x^2}$

27. If $y=\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$, then $\frac{dy}{dx}=$

(a) $2x+\frac{2x^3}{\sqrt{x^4-1}}$ (b) $2x+\frac{x^3}{\sqrt{x^4-1}}$

(c) $x+\frac{2x^3}{\sqrt{x^4-1}}$ (d) None of these

28. $\frac{d}{dx}(e^x\log\sin 2x)=$

(a) $e^x(\log\sin 2x+2\cot 2x)$ (b) $e^x(\log\cos 2x+2\cot 2x)$

(c) $e^x(\log\cos 2x+\cot 2x)$ (d) None of these

29. If $y=t^{4/3}-3t^{-2/3}$, then $dy/dt=$

(a) $\frac{2t^2+3}{3t^{5/3}}$ (b) $\frac{2t^2+3}{t^{5/3}}$

(c) $\frac{2(2t^2+3)}{t^{5/3}}$ (d) $\frac{2(2t^2+3)}{3t^{5/3}}$

30. If $y = \sin(\sqrt{\sin x + \cos x})$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2}\frac{\cos\sqrt{\sin x + \cos x}}{\sqrt{\sin x + \cos x}}$

(b) $\frac{\cos\sqrt{\sin x + \cos x}}{\sqrt{\sin x + \cos x}}$

(c) $\frac{1}{2}\frac{\cos\sqrt{\sin x + \cos x}}{\sqrt{\sin x + \cos x}}.(\cos x - \sin x)$

(d) None of these

31. $\frac{d}{dx}\log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) =$

(a) $\operatorname{cosec} x$ (b) $-\operatorname{cosec} x$

(c) $\sec x$ (d) $-\sec x$

32. $\frac{d}{dx}\left[\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right] =$

(a) $\sec x$ (b) $\operatorname{cosec} x$

(c) $\operatorname{cosec}\frac{x}{2}$ (d) $\sec\frac{x}{2}$

33. If $y = (1 + x^{1/4})(1 + x^{1/2})(1 - x^{1/4})$, then $\frac{dy}{dx} =$

(a) 1 (b) – 1

(c) x (d) $\sqrt{x}$

34. If $y = \tan^{-1}\left(\frac{\sqrt{a} - \sqrt{x}}{1 + \sqrt{ax}}\right)$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{1}{(1+x)\sqrt{x}}$

(c) $-\frac{1}{2(1+x)\sqrt{x}}$ (d) None of these

35. $\frac{d}{dx}e^{x\sin x} =$

(a) $e^{x\sin x}(x\cos x + \sin x)$ (b) $e^{x\sin x}(\cos x + x\sin x)$

(c) $e^{x\sin x}(\cos x + \sin x)$ (d) None of these

36. $\frac{d}{dx}\left[\log\sqrt{\sin\sqrt{e^x}}\right] =$

(a) $\frac{1}{4}e^{x/2}\cot(e^{x/2})$ (b) $e^{x/2}\cot(e^{x/2})$

(c) $\frac{1}{4}e^x\cot(e^x)$ (d) $\frac{1}{2}e^{x/2}\cot(e^{x/2})$

37. If $y = \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$, then $\frac{dy}{dx} =$

(a) $\frac{-8}{(e^{2x}-e^{-2x})^2}$ (b) $\frac{8}{(e^{2x}-e^{-2x})^2}$

(c) $\frac{-4}{(e^{2x}-e^{-2x})^2}$ (d) $\frac{4}{(e^{2x}-e^{-2x})^2}$

38. If $y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$, then $\frac{dy}{dx}$ is equal to

(a) 0 (b) $-\frac{1}{2}$

(c) 1/2 (d) 1

39. $\frac{d}{dx}\{\log(\sec x+\tan x)\} =$

(a) $\cos x$ (b) $\sec x$

(c) $\tan x$ (d) $\cot x$

40. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx} =$

(a) 0 (b) 1

(c) 2 (d) 3

41. If $y = \sqrt{\frac{1+e^x}{1-e^x}}$, then $\frac{dy}{dx} =$

(a) $\frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$ (b) $\frac{e^x}{(1-e^x)\sqrt{1-e^x}}$

(c) $\frac{e^x}{(1-e^x)\sqrt{1+e^{2x}}}$ (d) $\frac{e^x}{(1-e^x)\sqrt{1+e^x}}$

42. If $f(2)=4$, $f'(2)=1$ then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}=$

(a) 1 (b) 2

(c) 3 (d) –2

43. $\frac{d}{dx}\left(\cos^{-1}\sqrt{\frac{1+\cos x}{2}}\right)=$

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) None of these

44. $\frac{d}{dx}[\tan^{-1}(\cot x)+\cot^{-1}(\tan x)]=$

(a) 0 (b) 1

(c) – 1 (d) – 2

45. The value of $\frac{d}{dx}[|x-1|+|x-5|]$ **at** $x=3$ **is**

(a) – 2 (b) 0

(c) 2 (d) 4

46. If $y=\sin^{-1}\sqrt{(1-x)}+\cos^{-1}\sqrt{x}$, then $\frac{dy}{dx}=$

(a) $\frac{1}{\sqrt{x(1-x)}}$ (b) $\frac{-1}{\sqrt{x(1-x)}}$

(c) $\frac{1}{\sqrt{x(1+x)}}$ (d) None of these

47. If $y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$**, then** $\frac{dy}{dx}=$

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) 3 (d) 1

48. The derivative of $f(x)=x|x|$ **is**

(a) $2x$ (b) – $2x$

(c) $2x^2$ (d) $2|x|$

49. $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x+2}{x-2}\right)^{3/4}\right\}\right]$ equals

(a) $\frac{x^2-7}{x^2-4}$ (b) 1

(c) $\frac{x^2+1}{x^2-4}$ (d) $e^x\frac{x^2-1}{x^2-4}$

50. If $y=\log_{\cos x}\sin x$ **, then** $\frac{dy}{dx}$ **is equal to**

(a) $\frac{\cot x\log\cos x+\tan x\log\sin x}{(\log\cos x)^2}$

(b) $\frac{\tan x\log\cos x+\cot x\log\sin x}{(\log\cos x)^2}$

(c) $\frac{\cot x\log\cos x+\tan x\log\sin x}{(\log\sin x)^2}$

(d) None of these

51. If $f(x)=\cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$, **then the value of** $f'(e)=$

(a) 1 (b) $1/e$

(c) $2/e$ (d) $\frac{2}{e^2}$

52. If $y\sqrt{x^2+1}=\log\left\{\sqrt{x^2+1}-x\right\}$, then $(x^2+1)\frac{dy}{dx}+xy+1=$

(a) 0 (b) 1

(c) 2 (d) None of these

53. If $f(x)$ **is a differentiable function, then** $\lim_{x\to a}\frac{af(x)-xf(a)}{x-a}$ **is**

(a) $af'(a)-f(a)$ (b) $af(a)-f'(a)$

(c) $af'(a)+f(a)$ (d) $af(a)+f'(a)$

54. If $y=\tan^{-1}(\sec x-\tan x)$ **then** $\frac{dy}{dx}=$

(a) 2 (b) –2

(c) 1/2 (d) –1/2

55. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right]=$

(a) $-\frac{1}{1+x^2}$ (b) $\frac{1}{1+a^2}-\frac{1}{1+x^2}$

(c) $\frac{1}{1+\left(\frac{a-x}{1+ax}\right)^2}$ (d) $\frac{-1}{\sqrt{1-\left(\frac{a-x}{1+ax}\right)^2}}$

56. $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$ **equals to**

(a) 1 (b) $\frac{x^2+1}{x^2-4}$

(c) $\frac{x^2-1}{x^2-4}$ (d) $e^x\frac{x^2-1}{x^2-4}$

57. If $y=\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$ **then** $\frac{dy}{dx}=$

(a) 2 (b) – 1

(c) $\frac{a}{b}$ (d) 0

58. If $\sin y+e^{-x\cos y}=e$, **then** $\frac{dy}{dx}$ **at** $(1,\pi)$ **is**

(a) $\sin y$ (b) $-x\cos y$

(c) e (d) $\sin y-x\cos y$

59. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]=$

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$

(c) $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1-x)\sqrt{x}}$

60. If $y=\sqrt{\sin x+y}$, **then** $\frac{dy}{dx}$ **equals to**

(a) $\frac{\sin x}{2y-1}$ (b) $\frac{\cos x}{2y-1}$

(c) $\frac{\sin x}{2y+1}$ (d) $\frac{\cos x}{2y+1}$

61. If $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$, **then** $\frac{dy}{dx}$ **is**

(a) $1/2$ (b) $\pi/4$

(c) 0 (d) 1

62. If $\sin y = x\sin(a+y)$, then $\frac{dy}{dx} =$

(a) $\frac{\sin^2(a+y)}{\sin(a+2y)}$ (b) $\frac{\sin^2(a+y)}{\cos(a+2y)}$

(c) $\frac{\sin^2(a+y)}{\sin a}$ (d) $\frac{\sin^2(a+y)}{\cos a}$

63. If $3\sin(xy) + 4\cos(xy) = 5$, then $\frac{dy}{dx} =$

(a) $-\frac{y}{x}$ (b) $\frac{3\sin(xy) + 4\cos(xy)}{3\cos(xy) - 4\sin(xy)}$

(c) $\frac{3\cos(xy) + 4\sin(xy)}{4\cos(xy) - 3\sin(xy)}$ (d) None of these

64. If $f(x) = \frac{1}{1-x}$**, then the derivative of the composite function** $f[f\{f(x)\}]$ **is equal to**

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

65. If $x^3 + 8xy + y^3 = 64$ **,then** $\frac{dy}{dx} =$

(a) $-\frac{3x^2 + 8y}{8x + 3y^2}$ (b) $\frac{3x^2 + 8y}{8x + 3y^2}$

(c) $\frac{3x + 8y^2}{8x^2 + 3y}$ (d) None of these

66. If $\cos(x+y) = y\sin x$, **then** $\frac{dy}{dx} =$

(a) $-\frac{\sin(x+y) + y\cos x}{\sin x + \sin(x+y)}$ (b) $\frac{\sin(x+y) + y\cos x}{\sin x + \sin(x+y)}$

(c) $\frac{y\cos x - \sin(x+y)}{\sin x - \sin(x+y)}$ (d) None of these

67. If sin(x+y)=log(x+y), then $\frac{dy}{dx}$ =

(a) 2 (b) – 2

(c) 1 (d) –1

68. If $\sin y = x\cos(a+y)$, then $\frac{dy}{dx} =$

(a) $\frac{\cos^2(a+y)}{\cos a}$ (b) $\frac{\cos(a+y)}{\cos^2 a}$

(c) $\frac{\sin^2(a+y)}{\sin a}$ (d) None of these

69. Let f and g be differentiable functions satisfying $g'(a)=2,\ g(a)=b$ and $fog = I$ (identity function). Then $f'(b)$ is equal to

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{2}{3}$ (d) None of these

70. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x)=\frac{1}{1+x^3}$. Then $g'(x)$ is equal to

(a) $\frac{1}{1+(g(x))^3}$ (b) $\frac{1}{1+(f(x))^3}$

(c) $1+(g(x))^3$ (d) $1+(f(x))^3$

71. If $x = a(t+\sin t)$ and $y = a(1-\cos t)$, then $\frac{dy}{dx}$ equals

(a) $\tan(t/2)$ (b) $\cot(t/2)$

(c) $\tan 2t$ (d) $\tan t$

72. $x\sqrt{1+y}+y\sqrt{1+x}=0$ **, then** $\frac{dy}{dx} =$

(a) $1+x$ (b) $(1+x)^{-2}$

(c) $-(1+x)^{-1}$ (d) $-(1+x)^{-2}$

73. If $x = a(t-\sin t)$ and $y = a(1-\cos t)$, then $\frac{dy}{dx} =$

(a) $\tan\left(\frac{t}{2}\right)$ (b) $-\tan\left(\frac{t}{2}\right)$ (c) $\cot\left(\frac{t}{2}\right)$ (d) $-\cot\left(\frac{t}{2}\right)$

74. If $x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$, then $\frac{dy}{dx}=$

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$

(c) $\frac{-x}{y}$ (d) $\frac{x}{y}$

75. If $x^2+y^2=t-\frac{1}{t},\ x^4+y^4=t^2+\frac{1}{t^2}$, then $x^3y\frac{dy}{dx}=$

(a) 1 (b) 2

(c) 3 (d) 4

76. If $x=a\cos^3\theta, y=a\sin^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=$

(a) $\tan^2\theta$ (b) $\sec^2\theta$

(c) $\sec\theta$ (d) $|\sec\theta|$

77. If $x^p y^q=(x+y)^{p+q}$, then $\frac{dy}{dx}=$

(a) $\frac{y}{x}$ (b) $-\frac{y}{x}$

(c) $\frac{x}{y}$ (d) $-\frac{x}{y}$

78. The first derivative of the function $\left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right)+x^x\right]$ **with respect to *x* at *x* = 1 is**

(a) $\frac{3}{4}$ (b) 0

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

79. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+.....\infty}}}$ **, then** $\frac{dy}{dx}=$

(a) $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$

(c) $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

80. If $x^y = y^x$, **then** $\frac{dy}{dx} =$

(a) $\frac{y(x\log_e y + y)}{x(y\log_e x + x)}$ (b) $\frac{y(x\log_e y - y)}{x(y\log_e x - x)}$

(c) $\frac{x(x\log_e y - y)}{y(y\log_e x - x)}$ (d) $\frac{x(x\log_e y + y)}{y(y\log_e x + x)}$

81. If $y = \log x^x$, **then** $\frac{dy}{dx} =$

(a) $x^x(1+\log x)$ (b) $\log(ex)$

(c) $\log\left(\frac{e}{x}\right)$ (d) None of these

82. If $y = x^{(x^x)}$, **then** $\frac{dy}{dx} =$

(a) $y[x^x(\log ex).\log x + x^x]$

(b) $y[x^x(\log ex).\log x + x]$

(c) $y[x^x(\log ex).\log x + x^{x-1}]$

(d) $y[x^x(\log_e x).\log x + x^{x-1}]$

83. If $x^y = e^{x-y}$, **then** $\frac{dy}{dx} =$

(a) $\log x.[\log(ex)]^{-2}$ (b) $\log x.[\log(ex)]^2$

(c) $\log x.(\log x)^2$ (d) None of these

84. If $y = e^{x+e^{x+e^{x+....\infty}}}$ **, then** $\frac{dy}{dx} =$

(a) $\frac{y}{1-y}$ (b) $\frac{1}{1-y}$

(c) $\frac{y}{1+y}$ (d) $\frac{y}{y-1}$

85. If $x = \sin^{-1}(3t - 4t^3)$ and $y = \cos^{-1}\sqrt{(1-t^2)}$, then $\frac{dy}{dx}$ is equal to

(a) 1/2 (b) 2/5

(c) 3/2 (d) 1/3

86. If $y=(\sin x)^{(\sin x)^{(\sin x)......\infty}}$, then $\frac{dy}{dx}=$

(a) $\frac{y^2\cot x}{1-y\log\sin x}$ (b) $\frac{y^2\cot x}{1+y\log\sin x}$

(c) $\frac{y\cot x}{1-y\log\sin x}$ (d) $\frac{y\cot x}{1+y\log\sin x}$

87. If $y^x+x^y=a^b$, then $\frac{dy}{dx}=$

(a) $-\frac{yx^{y-1}+y^x\log y}{xy^{x-1}+x^y\log x}$ (b) $\frac{yx^{y-1}+y^x\log y}{xy^{x-1}+x^y\log x}$

(c) $-\frac{yx^{y-1}+y^x}{xy^{x-1}+x^y}$ (d) $\frac{yx^{y-1}+y^x}{xy^{x-1}+x^y}$

88. If $y=x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+......\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{2xy}{2y-x^2}$ (b) $\frac{xy}{y+x^2}$

(c) $\frac{xy}{y-x^2}$ (d) $\frac{2xy}{2+\frac{x^2}{y}}$

89. If $2^x+2^y=2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x=y=1$ is

(a) 0 (b) – 1

(c) 1 (d) 2

90. If $x^m y^n=2(x+y)^{m+n}$, the value of $\frac{dy}{dx}$ is

(a) $x+y$ (b) x/y (c) y/x (d) $-y/x$

91. If $y=\sqrt{x}^{\sqrt{x}^{\sqrt{x}....\infty}}$, then $\frac{dy}{dx}=$

(a) $\frac{y^2}{2x-2y\log x}$ (b) $\frac{y^2}{2x+\log x}$

(c) $\frac{y^2}{2x+2y\log x}$ (d) None of these

92. If $x = e^{y+e^{y+\ldots \text{to} \infty}}$**,** $x > 0$, **then** $\frac{dy}{dx}$ **is**

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$

(c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

93. If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, **then** $f'(1)$ **is equal to**

(a) – 1 (b) 1

(c) $\log 2$ (d) $-\log 2$

94. $\frac{d}{dx}\cos^{-1}\frac{x-x^{-1}}{x+x^{-1}} =$

(a) $\frac{1}{1+x^2}$ (b) $\frac{-1}{1+x^2}$

(c) $\frac{2}{1+x^2}$ (d) $\frac{-2}{1+x^2}$

95. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dy}{dx} =$

(a) $\sqrt{\frac{1-x^2}{1-y^2}}$ (b) $\sqrt{\frac{1-y^2}{1-x^2}}$

(c) $\sqrt{\frac{x^2-1}{1-y^2}}$ (d) $\sqrt{\frac{y^2-1}{1-x^2}}$

96. $\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\} =$

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$

(c) $-\frac{2}{1+x^2}$ (d) $\frac{2}{1+x^2}$

97. If $y = \cos^{-1}\left(\frac{3\cos x + 4\sin x}{5}\right)$**, then** $\frac{dy}{dx} =$

(a) 0 (b) 1

(c) -1 (d) $\frac{1}{2}$

98. $\frac{d}{dx}\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} =$

(a) $\frac{a}{a^2+x^2}$ (b) $\frac{-a}{a^2+x^2}$

(c) $\frac{1}{a\sqrt{a^2-x^2}}$ (d) $\frac{1}{\sqrt{a^2-x^2}}$

99. Let $3f(x)-2f(1/x)=x$, **then** $f'(2)$ **is equal to**

(a) $2/7$ (b) $1/2$

(c) 2 (d) $7/2$

100. $\frac{d}{dx}\left(\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}\right)$ **is equal to**

(a) $\frac{1}{1+x^2}$ (b) $\frac{1}{2(1+x^2)}$

(c) $\frac{x^2}{2\sqrt{1+x^2}(\sqrt{1+x^2}-1)}$ (d) $\frac{2}{1+x^2}$

101. If $u=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ and $v=2\tan^{-1}x$, then $\frac{du}{dv}$ is equal to

(a) 4 (b) 1

(c) 1/4 (d) –1/4

102. If $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$**, then** $\frac{dy}{dx}$ **equals**

(a) $\frac{2}{1-x^2}$ (b) $\frac{1}{1+x^2}$

(c) $\pm\frac{2}{1+x^2}$ (d) $-\frac{2}{1+x^2}$

103. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ ***w.r.t.*** $\cot^{-1}\left(\frac{1-3x^2}{3x-x^2}\right)$ **is**

(a) 1 (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

104. Differential coefficient of $\frac{\tan^{-1}x}{1+\tan^{-1}x}$ *w.r.t.* $\tan^{-1}x$ is

(a) $\frac{1}{1+\tan^{-1}x}$ (b) $\frac{-1}{1+\tan^{-1}x}$

(c) $\frac{1}{(1+\tan^{-1}x)^2}$ (d) $\frac{-1}{2(1+\tan^{-1}x)^2}$

105. If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2} - m^2y =$

(a) $m^2(ae^{mx} - be^{-mx})$ (b) 1

(c) 0 (d) None of these

106. If $y = ax^{n+1} + bx^{-n}$, then $x^2\frac{d^2y}{dx^2} =$

(a) $n(n-1)y$ (b) $n(n+1)y$

(c) ny (d) n^2y

107. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ *w.r.t.* $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

(a) –1 (b) 1

(c) 2 (d) 4

108. The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}x$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 1 (d) None of these

109. $\frac{d^2x}{dy^2}$ is equal to

(a) $\frac{1}{(dy/dx)^2}$ (b) $\frac{(d^2y/dx^2)}{(dy/dx)^2}$

(c) $\frac{d^2y}{dx^2}$ (d) $\frac{(-d^2y/dx^2)}{(dy/dx)^2}$

110. If $f(x)$ is a differentiable function and $f''(0) = a$ then $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

(a) $3a$ (b) $2a$

(c) $5a$ (d) $4a$

111. If $e^y + xy = e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$

(c) $\frac{1}{e^3}$ (d) None of these

112. If $y = \left(x + \sqrt{1 + x^2}\right)^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$

(c) $-y$ (d) $2x^2y$

113. If $\sqrt{(1 - x^6)} + \sqrt{(1 - y^6)} = a^3(x^3 - y^3)$, then $\frac{dy}{dx} =$

(a) $\frac{x^2}{y^2}\sqrt{\frac{1 - x^6}{1 - y^6}}$ (b) $\frac{y^2}{x^2}\sqrt{\frac{1 - y^6}{1 - x^6}}$

(c) $\frac{x^2}{y^2}\sqrt{\frac{1 - y^6}{1 - x^6}}$ (d) None of these

114. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then

(a) $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$

(b) $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = x^2(y^2 + 4)$

(c) $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = (y^2 + 4)$

(d) None of these

115. If $y = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log(x + \sqrt{x^2 + a^2})$, then $\frac{dy}{dx} =$

(a) $\sqrt{x^2 + a^2}$ (b) $\frac{1}{\sqrt{x^2 + a^2}}$

(c) $2\sqrt{x^2 + a^2}$ (d) $\frac{2}{\sqrt{x^2 + a^2}}$

116. If $y^2 = p(x)$ **is a polynomial of degree three, then** $2\frac{d}{dx}\left\{y^3 . \frac{d^2y}{dx^2}\right\}$ **=**

(a) $p'''(x) + p'(x)$ (b) $p''(x).p'''(x)$

(c) $p(x).p'''(x)$ (d) Constant

117. If $f(x+y) = f(x).f(y)$ **for all** ***x*** **and** ***y*** **and** $f(5) = 2$**,** $f'(0) = 3$**, then** $f'(5)$ **will be**

(a) 2 (b) 4

(c) 6 (d) 8

118. Let $f:(0,+\infty) \rightarrow R$ **and** $F(x) = \int_0^x f(t)\,dt$ **.**

If $F(x^2) = x^2(1+x)$, then $f(4)$ equals

(a) $\frac{5}{4}$ (b) 7

(c) 4 (d) 2

DIFFERENTIATION

HINTS AND SOLUTIONS

1. (a) $\log|x| = \log x$, if $x > 0 = \log(-x)$, if $x < 0$

Hence $\frac{d}{dx}\{\log|x|\} = \frac{1}{x}$, if $x > 0$

$$= \left(\frac{1}{-x}\right)(-1) = \frac{1}{x}, \text{if } x < 0$$

Thus $\frac{d}{dx}\{\log|x|\} = \frac{1}{x}$, if $x \neq 0$.

2. (a) $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\infty \Rightarrow y = e^x$

Differentiating with respect to *x*, we get $\frac{dy}{dx} = e^x = y$.

3. (a) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)\right]$

$$= \frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\right)\right]$$

$$= \frac{d}{dx}\left[\tan^{-1}\left(\frac{1 - \tan\left(\frac{x}{2}\right)}{1 + \tan\left(\frac{x}{2}\right)}\right)\right] = \frac{d}{dx}\left[\tan^{-1}\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] = -\frac{1}{2}$$

4. (d) $\frac{d}{dx}\tan^{-1}\left(\frac{ax - b}{bx + a}\right) = \frac{1}{1 + \left(\frac{ax - b}{bx + a}\right)^2} \cdot \frac{d}{dx}\left(\frac{ax - b}{bx + a}\right)$

5. (c) $\frac{dy}{dx} = -b \sin \log\left(\frac{x}{n}\right)^n \frac{1}{(x/n)^n} \frac{n}{n}\left(\frac{x}{n}\right)^{n-1} = -\frac{nb}{x} \sin \log\left(\frac{x}{n}\right)^n$.

6. (b) $x^{2/3} + y^{2/3} = a^{2/3}$

$\Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$ or $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{-1/3} = -\left(\frac{y}{x}\right)^{1/3}$.

7. (b) $f(x) = x\tan^{-1} x$

Differentiating *w.r.tx*, we get $f'(x) = x\dfrac{1}{1+x^2} + \tan^{-1} x$

Now put $x = 1$, then $f'(1) = \dfrac{1}{2} + \tan^{-1}(1) = \dfrac{\pi}{4} + \dfrac{1}{2}$.

8. (c) $y = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + ... + \dfrac{x^n}{n!}$

$$\Rightarrow \frac{dy}{dx} = 0 + 1 + x + \frac{x^2}{2!} + ... + \frac{x^{n-1}}{(n-1)!}$$

$$\Rightarrow \frac{dy}{dx} + \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} \Rightarrow \frac{dy}{dx} = y - \frac{x^n}{n!}.$$

9. (c) $\dfrac{d}{dx}\log(\log x) = \dfrac{1}{x}.\dfrac{1}{\log x} = (x\log x)^{-1}$.

10. (c) $y = \tan^{-1}\dfrac{4x}{1+5x^2} + \tan^{-1}\dfrac{2+3x}{3-2x}$

$$= \tan^{-1}\frac{5x - x}{1 + 5x.x} + \tan^{-1}\frac{\frac{2}{3} + x}{1 - \frac{2}{3}.x}$$

$$= \tan^{-1} 5x - \tan^{-1} x + \tan^{-1}\frac{2}{3} - \tan^{-1} x$$

11. (a) $y = x\left[\left(\cos\dfrac{x}{2} + \sin\dfrac{x}{2}\right)\left(\cos\dfrac{x}{2} - \sin\dfrac{x}{2}\right) + \sin x\right] + \dfrac{1}{2\sqrt{x}}$

$$\Rightarrow y = x(\cos x + \sin x) + \frac{1}{2\sqrt{x}}$$

12. (c) Putting $x = \sin A$ and $\sqrt{x} = \sin B$

13. (b) $y = \sqrt{\dfrac{1-\sin 2x}{1+\sin 2x}} = \dfrac{\cos x - \sin x}{\cos x + \sin x}$

14. (c) $\dfrac{d}{dx}[\log_7(\log_7 x)] = \dfrac{d}{dx}\left(\dfrac{\log_e(\log_7 x)}{\log_e 7}\right)$

$$= \frac{1}{x\log_e x}.\frac{1}{\log_e 7} = \frac{\log_7 e}{x\log_e x}.$$

15. (d) $\dfrac{d}{dx}\left[\dfrac{\cot^2 x - 1}{\cot^2 x + 1}\right] = \dfrac{d}{dx}\left[\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}\right]$

$= \frac{d}{dx}[\cos 2x] = -2\sin 2x$.

16. (a) Let $y = \tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}} = \tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{4}}{2\sin^2\frac{x}{4}}}$

$y = \tan^{-1}\cot\frac{x}{4} = \tan^{-1}\tan\left(\frac{\pi}{2}-\frac{x}{4}\right) = \frac{\pi}{2}-\frac{x}{4}$

$\therefore \frac{dy}{dx} = -\frac{1}{4}$.

17. (b) $f(x) = \log_x(\log x) = \frac{\log(\log x)}{\log x}$

$\Rightarrow f'(x) = \frac{\frac{1}{x}-\frac{1}{x}\log(\log x)}{(\log x)^2} \Rightarrow f'(e) = \frac{\frac{1}{e}-0}{1} = \frac{1}{e}$.

18. (b) $\frac{d}{dx}\sqrt{\frac{1+\cos 2x}{1-\cos 2x}} = \frac{d}{dx}\cot x = -\text{cosec}^2 x$.

19. (c) $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right] = \frac{d}{dx}\left[\tan^{-1}\tan\frac{x}{2}\right] = \frac{1}{2}$.

20. (d) $\frac{dy}{dx} = \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{(1-x^2)2x+(1+x^2)2x}{(1-x^2)^2}\right]$

$= \frac{4x}{(1-x^2)^2}\cos\left(\frac{1+x^2}{1-x^2}\right)$.

21. (a) $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$

$= \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \frac{\pi}{2}$

22. (a) $y = \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}} \Rightarrow y = \frac{(\sqrt{a+x}-\sqrt{a-x})^2}{(a+x)-(a-x)}$

$\Rightarrow y = \frac{(a+x)+(a-x)-2(\sqrt{a^2-x^2})}{2x}$

$= \frac{2a-2\sqrt{a^2-x^2}}{2x}$ or $y = \frac{a-\sqrt{a^2-x^2}}{x}$

23. (a) Put $x = \sin\theta$, we get $\frac{d}{dx}\sin^{-1}(3x-4x^3)$

$$= \frac{d}{dx}\sin^{-1}(\sin 3\theta) = \frac{3}{\sqrt{1-x^2}}.$$

24. (b) $\frac{d}{dx}\tan^{-1}(\sec x + \tan x) = \frac{d}{dx}\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

$$= \frac{d}{dx}\tan^{-1}\left(\frac{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right) = \frac{d}{dx}\left(\frac{\pi}{4}+\frac{x}{2}\right) = \frac{1}{2}.$$

25. (b) $\frac{d}{dx}\left(\frac{\log x}{\sin x}\right) = \frac{\frac{\sin x}{x} - \log x . \cos x}{\sin^2 x}$.

26. (b) $y = \cot^{-1}\left(\frac{1+x}{1-x}\right)$

$$\frac{dy}{dx} = -\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\left[\frac{(1-x)+(1+x)}{(1-x)^2}\right]$$

27. (a) Rationalizing, $y = \frac{2x^2+2\sqrt{x^4-1}}{2} = x^2 + (x^4-1)^{1/2}$

28. (a) $\frac{d}{dx}(e^x \log \sin 2x) = e^x \log\sin 2x + 2e^x \frac{1}{\sin 2x}\cos 2x$

$$= e^x \log \sin 2x + e^x 2\cot 2x \;= e^x(\log\sin 2x + 2\cot 2x).$$

29. (d) $y = t^{4/3} - 3t^{-2/3}$

$$\therefore \frac{dy}{dt} = \frac{4}{3}t^{1/3} + 3\times\frac{2}{3}t^{-5/3} = \frac{4t^2+6}{3t^{5/3}} = \frac{2(2t^2+3)}{3t^{5/3}}.$$

30. (c) $y = \sin(\sqrt{\sin x + \cos x})$

$$\frac{dy}{dx} = \frac{1}{2}\frac{\cos(\sqrt{\sin x + \cos x})}{\sqrt{\sin x + \cos x}}(\cos x - \sin x).$$

31. (c) $\frac{d}{dx}\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\frac{1}{2}$

32. (b) $\frac{d}{dx}\left[\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right] = \frac{d}{dx}\left[\log\left(\tan\frac{x}{2}\right)\right] = \text{cosec}\, x$.

33. (b) $y = (1+x^{1/4})(1+x^{1/2})(1-x^{1/4})$

$\Rightarrow y = (1 + x^{1/4})(1 - x^{1/4})(1 + x^{1/2})$

$= (1 - x^{1/2})(1 + x^{1/2}) = 1 - x \Rightarrow \frac{dy}{dx} = -1$.

34. (c) $y = \tan^{-1}\sqrt{a} - \tan^{-1}\sqrt{x}$

Differentiating *w.r.t .x*, we get, $\frac{dy}{dx} = -\frac{1}{(1+x)} \cdot \frac{1}{2\sqrt{x}}$.

35. (a) Let $y = e^{x\sin x} \Rightarrow \log y = x\sin x$

$\therefore \frac{1}{y}\frac{dy}{dx} = \sin x + x\cos x$ or $\frac{dy}{dx} = e^{x\sin x}(\sin x + x\cos x)$.

36. (a) $\frac{d}{dx}[\log\sqrt{\sin\sqrt{e^x}}] = \frac{d}{dx}\left[\frac{1}{2}\log(\sin\sqrt{e^x})\right]$

$= \frac{1}{2}\cot\sqrt{e^x}\,\frac{1}{2\sqrt{e^x}}e^x = \frac{1}{4}e^{x/2}\cot(e^{x/2})$

37. (a) $y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

$\therefore \frac{dy}{dx} = \frac{(e^{2x} - e^{-2x})2(e^{2x} - e^{-2x}) - (e^{2x} + e^{-2x})2(e^{2x} + e^{-2x})}{(e^{2x} - e^{-2x})^2}$

38. (b) $y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}} = \tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}}$

$= \tan^{-1}\cot\frac{x}{2} = \tan^{-1}\tan\left(\frac{\pi}{2} - \frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2}$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$.

39. **(b)** $\frac{d}{dx}\{\log(\sec x + \tan x)\} = \frac{\sec x\tan x + \sec^2 x}{\sec x + \tan x} = \sec x$.

40. (a) $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

Or $y = \cos^{-1}\frac{x-1}{x+1} + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\therefore y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$

41. (a) $y = \sqrt{\frac{1+e^x}{1-e^x}}$ or $y^2 = \frac{1+e^x}{1-e^x}$

$$2y\frac{dy}{dx} = \frac{(1-e^x)e^x + (1+e^x)e^x}{(1-e^x)^2} = \frac{2e^x}{(1-e^x)^2}$$

42. (b) Given $f(2) = 4,\ f'(2) = 1$

$$\therefore \lim_{x\to 2}\frac{xf(2) - 2f(x)}{x-2} = \lim_{x\to 2}\frac{xf(2) - 2f(2) + 2f(2) - 2f(x)}{x-2}$$

$$= \lim_{x\to 2}\frac{(x-2)f(2)}{x-2} - \lim_{x\to 2}\frac{2f(x) - 2f(2)}{x-2}$$

$$= f(2) - 2\lim_{x\to 2}\frac{f(x) - f(2)}{x-2} = f(2) - 2f'(2) = 4 - 2(1) = 4 - 2 = 2$$

Aliter: Applying *L*-Hospital rule, we get $\lim_{x\to 2}\frac{f(2) - 2f'(2)}{1} = 2$

43. (b) $\frac{d}{dx}\left(\cos^{-1}\sqrt{\frac{1+\cos x}{2}}\right) = \frac{d}{dx}\left[\cos^{-1}\left(\cos\frac{x}{2}\right)\right] = \frac{1}{2}$.

44. (d) $\frac{d}{dx}[\tan^{-1}(\cot x) + \cot^{-1}(\tan x)]$

$$= \frac{1(-\text{cosec}^2 x)}{1+\cot^2 x} - \frac{1(\sec^2 x)}{1+\tan^2 x} = -1 - 1 = -2$$

45. (b) $f(x) = |x-1| + |x-5|$

$$f(x) = \begin{cases} -(x-1) - (x-5), & x < 1 \\ (x-1) - (x-5), & 1 < x < 5 \\ x - 1 + x - 5, & x > 5 \end{cases}$$

$$f(x) = \begin{cases} 6 - 2x, & x < 1 \\ 4, & 1 < x < 5 \\ 2x - 6, & x > 5 \end{cases}$$

46. (b) $\sin^{-1}\sqrt{1-x} = \sin^{-1}\sqrt{1-(\sqrt{x})^2} = \cos^{-1}\sqrt{x}$

$\therefore y = 2\cos^{-1}\sqrt{x}$ or $\frac{dy}{dx} = 2.\frac{-1}{\sqrt{1-x}}.\frac{1}{2\sqrt{x}}$

47. (a) $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

$$= \cot^{-1}\left[\frac{2 + 2\cos x}{2\sin x}\right] = \cot^{-1}\left[\frac{1+\cos x}{\sin x}\right]$$

$= \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2}$

48. (d) $f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & x > 0 \end{cases} \Rightarrow f'(x) = \begin{cases} -2x, & x < 0 \\ 2x, & x > 0 \end{cases}$

$\therefore f'(x) = 2|x|$.

49. (a) $y = \log e^x + \frac{3}{4}\log\frac{x+2}{x-2} = x + \frac{3}{4}\log\frac{x+2}{x-2}$

$\Rightarrow y = x + \frac{3}{4}[\log(x+2) - \log(x-2)]$

50. (a) We have $y = \log_{\cos x}\sin x = \frac{\log \sin x}{\log \cos x}$

$\therefore \frac{dy}{dx} = \frac{\cot x . \log\cos x + (\log\sin x)\tan x}{(\log\cos x)^2}$.

51. (b) $f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right] = 2\tan^{-1}(\log x)$

52. (a) $y\sqrt{x^2+1} = \log\left\{\sqrt{x^2+1} - x\right\}$

53. (a) $\lim_{x\to a}\frac{af(x) - xf(a)}{x-a} \Rightarrow \lim_{x\to a}\frac{af(x) - xf(a) + af(a) - af(a)}{x-a}$

$\Rightarrow \lim_{x\to a}\frac{a[f(x) - f(a)] - f(a)[x-a]}{x-a}$

$\Rightarrow \lim_{x\to a}\frac{a[f(x) - f(a)]}{x-a} - \lim_{x\to a} f(a) \Rightarrow af'(a) - f(a)$.

54. (b) $y = \tan^{-1}(\sec x - \tan x)$

$\frac{dy}{dx} = \frac{1}{1+(\sec x - \tan x)^2}(\sec x \tan x - \sec^2 x)$

$\frac{dy}{dx} = \frac{\cos^2 x . \sec^2 x(\sin x - 1)}{(1-\sin x)^2 + \cos^2 x}$

$\frac{dy}{dx} = \frac{\sin x - 1}{1 - 2\sin x + \sin^2 x + \cos^2 x} = \frac{\sin x - 1}{2(1-\sin x)} = -\frac{1}{2}$.

55. (a) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right]$.

$= \frac{d}{dx}[\tan^{-1} a - \tan^{-1} x] = 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$.

56. (c) Let $y = \left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right] = \log e^x + \log\left(\frac{x-2}{x+2}\right)^{3/4}$

$\Rightarrow y = x + \frac{3}{4}[\log(x-2) - \log(x+2)]$

57. (b) $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

Let $a = r\sin\theta$ and $b = r\cos\theta$

$\therefore y = \tan^{-1}\left[\frac{r\sin(\theta - x)}{r\cos(\theta - x)}\right]$

$y = \theta - x$; $y = \tan^{-1}\left(\frac{a}{b}\right) - x$

58. (c) $\sin y + e^{-x\cos y} = e,$

$\Rightarrow \cos y\frac{dy}{dx} + e^{-x\cos y}\left\{(-x)\left(-\sin y\frac{dy}{dx}\right) + \cos y(-1)\right\} = 0$

$\Rightarrow \cos y\frac{dy}{dx} + x\sin y\; e^{-x\cos y}\frac{dy}{dx} - \cos y e^{-x\cos y} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{\cos y\; e^{-x\cos y}}{\cos y + x\sin y\; e^{-x\cos y}}$

59. (e) $\frac{d}{dx}\left(\tan^{-1}\frac{(\sqrt{x}(3-x)}{1-3x}\right)$

Put $\sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$\frac{d}{dx}\left(\tan^{-1}\frac{(\tan\theta(3-\tan^2\theta)}{1-3\tan^2\theta}\right)$

$\frac{d}{dx}\left(\tan^{-1}\frac{(3\tan\theta - \tan^3\theta)}{1-3\tan^2\theta}\right)$

$\frac{d}{dx}(\tan^{-1}(\tan 3\theta) = \frac{d}{dx}(3\theta)$

$\frac{d}{dx}(3.\tan^{-1}\sqrt{x}) = \frac{3}{2\sqrt{x}(1+x)}$.

60. (b) $y = \sqrt{\sin x + y}, \Rightarrow y^2 = \sin x + y$

61. (d) $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right] = \tan^{-1}\left[\frac{1+\tan x}{1-\tan x}\right]$

$= \tan^{-1}\left[\frac{\tan(\pi/4) + \tan x}{1 - \tan(\pi/4)\tan x}\right] = \tan^{-1}\tan(\pi/4 + x)$

62. (c) $\sin y = x\sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$

$$\Rightarrow 1 = \frac{\cos y.\frac{dy}{dx}.\sin(a+y) - \sin y\cos(a+y)\frac{dy}{dx}}{\sin^2(a+y)}$$

$$= \frac{\frac{dy}{dx}.\sin(a+y-y)}{\sin^2(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}.$$

63. (a) $\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y} = -\frac{3y\cos(xy) - 4y\sin(xy)}{3x\cos(xy) - 4x\sin(xy)} = -\frac{y}{x}$.

64. (c) $f(x) = \frac{1}{1-x} \Rightarrow f\{f(x)\} = \frac{1-x}{-x}$

$$\Rightarrow f[f\{f(x)\}] = \frac{-x}{-x-1+x} = x$$

$\therefore$ Derivative of $f[f\{f(x)\}] = 1$.

65. (a) $x^3 + 8xy + y^3 = 64 \Rightarrow 3x^2 + 8\left(y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$

66. (a) $\cos(x+y) = (y\sin x)$

$$\Rightarrow -\sin(x+y)\left(1 + \frac{dy}{dx}\right) = y\cos x + \sin x\frac{dy}{dx}$$

67. (d) It is implicit function, so

$$\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y} = -\frac{\cos(x+y) - \frac{1}{x+y}}{\cos(x+y) - \frac{1}{x+y}} = -1.$$

68. (a) $x = \frac{\sin y}{\cos(a+y)}$. Find $\frac{dx}{dy}$ and then $\frac{dy}{dx}$.

69. (a) $fog = I \Rightarrow fog(x) = x$ for all x

$\Rightarrow f'(g(x))g'x = 1$ for all x

$$\Rightarrow f'(g(a)) = \frac{1}{g'(a)} = \frac{1}{2} \Rightarrow f'(b) = \frac{1}{2} \qquad (\because g(a) = b).$$

70. (c) Since $g(x)$ is the inverse of $f(x)$, therefore

$$f(x) = y \Leftrightarrow g(y) = x$$

Now, $g'(f(x)) = \frac{1}{f'(x)}, \forall x \Rightarrow g'(f(x)) = 1 + x^3, \ \forall x$

$\Rightarrow g'(y) = 1 + (g(y))^3$ [Using $f(x) = y \Leftrightarrow x = g(y)$]

$\Rightarrow g'(x) = 1 + (g(x))^3$

71. (a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{d}{dt}[a(1-\cos t)]}{\frac{d}{dt}[a(t+\sin t)]}$

72. (d) $x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow (x-y)(x+y+xy) = 0 \Rightarrow x+y+xy = 0$, $\{\because x \neq y\}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

73. (c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

74. (c) $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

Put $t = \tan\theta$ in both the equations

75. (a) $x^4 + y^4 = \left(t - \frac{1}{t}\right)^2 + 2 = (x^2+y^2)^2 + 2$

$\Rightarrow x^2y^2 = -1 \Rightarrow y^2 = -\frac{1}{x^2}$

76. (d) $\sqrt{1+\tan^2\theta} = |\sec\theta|$.

77. (a) Taking $\log$ both sides, $p\log x + q\log y = (p+q)\log(x+y)$

$\Rightarrow \frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1 + \frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = \frac{y}{x}$.

78. (a) $f(x) = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right] + x^x$

$f(x) = \frac{\pi}{2} - \sqrt{\frac{1+x}{2}} + x^x$

$\therefore f'(x) = -\frac{1}{\sqrt{2}}.\frac{1}{2\sqrt{1+x}} + x^x(1+\log x)$

$f'(1) = -\frac{1}{4} + 1 = \frac{3}{4}$.

79. (c) $y=\sqrt{\log x+y} \Rightarrow y^2=\log x+y$

$$\Rightarrow 2y\frac{dy}{dx}=\frac{1}{x}+\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{1}{x(2y-1)}$$

80. (b) $x^y=y^x \Rightarrow y\log_e x=x\log_e y$

81. (b) $y=\log x^x=x\log x$

Differentiating *w.r.t. x*, we get

$$\frac{dy}{dx}=(1+\log x)=\log e+\log x=\log(ex)$$

82. (c) $y=x^{(x^x)} \Rightarrow \log y=x^x\log x$

83. (a) $x^y=e^{x-y} \Rightarrow y\log x=x-y \Rightarrow y=\frac{x}{1+\log x}$

84. (a) $y=e^{x+y} \Rightarrow \log y=(x+y)\log e$

85. (d) $y=\cos^{-1}\sqrt{1-t^2}=\sin^{-1}t$

and $x=\sin^{-1}(3t-4t^3)=3\sin^{-1}t$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{1}{\sqrt{1-t^2}}\right)}{3\left(\frac{1}{\sqrt{1-t^2}}\right)} \Rightarrow \frac{dy}{dx}=\frac{1}{3}.$$

86. (a) $y=(\sin x)^{(\sin x)^{(\sin x)\ldots\ldots\infty}}$

$$\Rightarrow y=(\sin x)^y \Rightarrow \log_e y=y\log\sin x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{dy}{dx}[\log\sin x+y\cot x]$$

$$\therefore \frac{dy}{dx}=\frac{y^2\cot x}{1-y\log\sin x}.$$

87. (a) $x^y+y^x=a^b$; Let $x^y=u$ and $y^x=v$

$$\Rightarrow u+v=a^b \Rightarrow \frac{du}{dx}+\frac{dv}{dx}=0$$

88. (a) $y=x^2+\frac{1}{y} \Rightarrow y^2=x^2y+1$

$$\Rightarrow 2y\frac{dy}{dx}=y.2x+x^2\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{2xy}{2y-x^2}.$$

89. (b) $2^x + 2^y = 2^{x+y}$; Differentiating *w.r.t. x*, we get

$$2^x(\log 2) + 2^y(\log 2)\frac{dy}{dx} = 2^{(x+y)}.(\log 2)\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow 2^x + 2^y\frac{dy}{dx} = 2^{x+y} + 2^{x+y}\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}(2^y - 2^{x+y}) = 2^{x+y} - 2^x \Rightarrow \frac{dy}{dx} = \frac{2^{x+y}-2^x}{2^y - 2^{x+y}}.$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=y=1} = \frac{2^2-2}{2-2^2} = \frac{2}{-2} = -1.$$

90. (c) $x^m y^n = 2(x+y)^{m+n} \Rightarrow m\log x + n\log y = \log 2 + (m+n)\log(x+y)$

Differentiating both sides *w.r.t. x*,

91. (d) $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}\,.....\infty}} \Rightarrow y = (\sqrt{x})^y$

$$\Rightarrow \log y = y\log x^{1/2} = \frac{1}{2}y\log x$$

92. (c) $x = e^{y+e^{y+....to\,\infty}}$, $x > 0$, $x = e^{y+x}$

Taking log to the both sides, $\log x = (y+x)$

93. (a) $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$; Put $x^x = \tan\theta$

94. (d) Putting $x = \cot\theta$

95. (b) Putting $x = \sin\theta$ and $y = \sin\phi$

96. (d) $\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$

Let $\frac{1-x^2}{1+x^2} = \cos\theta \Rightarrow 1 - x^2 = (1+x^2)\cos\theta$

$$\Rightarrow -x^2(1+\cos\theta) = \cos\theta - 1$$

$$\Rightarrow \quad x^2 = \frac{1-\cos\theta}{1+\cos\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan^2\frac{\theta}{2}$$

Or $x = \tan\frac{\theta}{2}$ or $\theta = 2\tan^{-1}x$

97. (b) $y = \cos^{-1}\left[\frac{3}{5}\cos x - \frac{4}{5}\sin x\right]$

Putting $\frac{3}{5} = r\cos\theta, \frac{4}{5} = r\sin\theta \Rightarrow r = 1$

$\Rightarrow y = \cos^{-1}[\cos\theta\cos x - \sin\theta\sin x] = \theta + x \Rightarrow \frac{dy}{dx} = 1$.

98. (d) $\frac{d}{dx}\tan^{-1}\frac{x}{\sqrt{a^2 - x^2}}$

Putting $x = a\sin\theta,$

99. (b) $3f(x) - 2f(1/x) = x$ .....(i)

Let $1/x = y$, then $3f(1/y) - 2f(y) = 1/y$

$\Rightarrow -2f(y) + 3f(1/y) = 1/y$

$\Rightarrow -2f(x) + 3f(1/x) = 1/x$ .....(ii)

From 3 × (i) + 2 × (ii),

$9f(x) - 6f(1/x) - 4f(x) + 6f(1/x) = 3x + 2/x$

$5f(x) = 3x + \frac{2}{x} \Rightarrow f(x) = \frac{1}{5}\left[3x + \frac{2}{x}\right]$

100. (b) Let $y = \tan^{-1}\frac{\sqrt{1+x^2} - 1}{x}$

Put $x = \tan\theta,$ then $y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}\right)$

$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$y = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \tan^{-1}\tan\frac{\theta}{2}$

$y = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$, $(\because \theta = \tan^{-1}x)$.

101. (c) $u=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ and $v=2\tan^{-1}x$

Put $x=\tan\theta$ in u and v;

$u=\tan^{-1}\left\{\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right\}$ and $v=2\theta$

$u=\tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\}$ and $v=2\theta$

$u=\tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\}$ and $v=2\theta$

$u=\theta/2$ and $v=2\theta$; $\therefore \frac{du}{dv}=\frac{du/d\theta}{dv/d\theta}=\frac{1/2}{2}=\frac{1}{4}$.

102. (c) $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$\therefore y=\sin^{-1}\cos 2\theta=\frac{\pi}{2}\pm 2\theta$

103. (c) Let $y_1=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=2\tan^{-1}x$,

$\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)=3\tan^{-1}x \Rightarrow \frac{dy_1}{dy_2}=\frac{\left(\frac{dy_1}{dx}\right)}{\left(\frac{dy_2}{dx}\right)}=\frac{\left(\frac{2}{1+x^2}\right)}{\left(\frac{3}{1+x^2}\right)}=\frac{2}{3}$

104. (c) The differential coefficient of $\frac{\tan^{-1}x}{1+\tan^{-1}x}$ with respect to $\tan^{-1}x=\frac{\frac{d}{dx}\left(\frac{\tan^{-1}x}{1+\tan^{-1}x}\right)}{\frac{d}{dx}(\tan^{-1}x)}$

105. (c) $y=ae^{mx}+be^{-mx}$; $\therefore \frac{dy}{dx}=ame^{mx}-mbe^{-mx}$

Again $\frac{d^2y}{dx^2}=am^2e^{mx}+m^2be^{-mx}$

$\Rightarrow \frac{d^2y}{dx^2}=m^2(ae^{mx}+be^{-mx}) \Rightarrow \frac{d^2y}{dx^2}=m^2y$

Or $\frac{d^2y}{dx^2}-m^2y=0$.

106. (b) $y = ax^{n+1} + bx^{-n} \Rightarrow \frac{dy}{dx} = (n+1)ax^n - nbx^{-n-1}$

$\Rightarrow \frac{d^2y}{dx^2} = n(n+1)ax^{n-1} + n(n+1)bx^{-n-2}$

$\Rightarrow x^2 \frac{d^2y}{dx^2} = n(n+1)y$.

107. (b) Let $p = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

and $q = \cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1}x$; $\therefore \frac{dp}{dq} = \frac{dp/dx}{dq/dx} = 1$.

108. (a) Let $y_1 = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $y_2 = \tan^{-1}x$

Now $\frac{dy_1}{dx} = \frac{d}{dx}\left[\tan^{-1}\tan\frac{\theta}{2}\right]$, [By putting $x = \tan\theta$]

$\Rightarrow \frac{dy_1}{dx} = \frac{d}{dx}\left[\tan^{-1}\tan\frac{\theta}{2}\right] = \frac{1}{2(1+x^2)}$ & $\frac{dy_2}{dx} = \frac{1}{1+x^2}$

Hence $\frac{dy_1}{dy_2} = \frac{1}{2}$.

109. (d) $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right) = \frac{-1}{\left(\frac{dy}{dx}\right)^2}.\frac{d^2y}{dx^2}$.

110. (a) $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$

Using L-Hospital's rule twice, we get

$\lim_{x\to 0}\frac{2f''(x)-3.2.2f''(2x)+4.4f''(4x)}{2} = 3a$

111. (b) We have $e^y + xy = e$. Differentiating *w.r.t.x*, we get $e^y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$ .....(i)

Differentiating *w.r.t .x*, we get

$e^y\frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} + x\frac{d^2y}{dx^2} = 0$ .....(ii)

112. (a) $y=(x+\sqrt{1+x^2})^n \Rightarrow \frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{x}{\sqrt{1+x^2}}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{n(x+\sqrt{1+x^2})^n}{\sqrt{1+x^2}}$

$\Rightarrow (\sqrt{1+x^2})\frac{dy}{dx}=n(x+\sqrt{1+x^2})^n$

113. (c) Put $x^3=\sin\theta, y^3=\sin\phi$

$\therefore \sqrt{1-x^6}+\sqrt{1-y^6}=a^3(x^3-y^3)$

$\Rightarrow \cos\theta+\cos\phi=a^3(\sin\theta-\sin\phi)$

Or $2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}=2a^3\sin\frac{\theta-\phi}{2}\cos\frac{\theta+\phi}{2}$

Or $\cos\frac{\theta+\phi}{2}\left[\cos\frac{\theta-\phi}{2}-a^3\sin\frac{\theta-\phi}{2}\right]=0$

If $\cos\frac{\theta+\phi}{2}=0$, then $\frac{\theta+\phi}{2}=\frac{\pi}{2}$

$\therefore \theta=\pi-\phi$ or $\sin\theta=\sin\phi$ or $x=y$

114. (a) $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{n\sec^n\theta\tan\theta+n\cos^{n-1}\theta\sin\theta}{\sec\theta\tan\theta+\sin\theta}$

$=\frac{n(\sec^n\theta+\cos^n\theta)}{\sec\theta+\cos\theta}$ (Dividing N^r and D^r by $\tan\theta$)

$\Rightarrow \left(\frac{dy}{dx}\right)^2=\frac{n^2(\sec^n\theta+\cos^n\theta)^2}{(\sec\theta+\cos\theta)^2}$

$=\frac{n^2[(\sec^n\theta-\cos^n\theta)^2+4\sec^n\theta\cos^n\theta]}{(\sec\theta-\cos\theta)^2+4\sec\theta.\cos\theta}=\frac{n^2(y^2+4)}{x^2+4}$

$\Rightarrow (x^2+4)\left(\frac{dy}{dx}\right)^2=n^2(y^2+4)$.

115. (a) standard problem

116. (c) $2y\frac{dy}{dx}=p'(x) \Rightarrow 2\frac{dy}{dx}=\frac{p'(x)}{y} \Rightarrow 2\frac{d^2y}{dx^2}=\frac{yp''(x)-p'(x)y'}{y^2}$

$\Rightarrow 2y^3\frac{d^2y}{dx^2}=y^2p''(x)-y\frac{dy}{dx}p'(x)=p(x)p''(x)-\frac{1}{2}\{p'(x)\}^2$

$\Rightarrow 2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)=p'(x)p''(x)+p(x)p'''(x)-p'(x)p''(x)$

$= p(x)p'''(x)$.

117. (c) Let $x = 5,\ y = 0 \Rightarrow f(5+0) = f(5).f(0)$

$\Rightarrow f(5) = f(5)f(0) \Rightarrow f(0) = 1$

Therefore, $f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$

$= \lim_{h\to 0} \frac{f(5)f(h) - f(5)}{h} = \lim_{h\to 0} 2\left[\frac{f(h)-1}{h}\right]$, $\{\because f(5) = 2\}$

$= 2 \lim_{h\to 0} .\left[\frac{f(h) - f(0)}{h}\right] = 2 \times f'(0) = 2 \times 3 = 6$.

118. (c) $x^2(1+x) = \int_0^{x^2} f(t)dt.$

Differentiating $w.r.t\ .x$, $2x(1+x) + x^2 = f(x^2).2x$

$\Rightarrow f(x^2) = 1 + x + \frac{x}{2}, x > 0$

Putting $x = 2,\ f(4) = 1 + 2 + \frac{2}{2} = 4.$