DEFINITE INTEGRATION

OBJECTIVE PROBLEMS

1. $\int_0^{\pi/4} \tan^2 x\, dx =$

(a) $1-\frac{\pi}{4}$ (b) $1+\frac{\pi}{4}$ (c) $\frac{\pi}{4}-1$ (d) $\frac{\pi}{4}$

2. $\int_0^{\pi/2} e^x \sin x\, dx =$

(a) $\frac{1}{2}(e^{\pi/2}-1)$ (b) $\frac{1}{2}(e^{\pi/2}+1)$ (c) $\frac{1}{2}(1-e^{\pi/2})$ (d) $2(e^{\pi/2}+1)$

3. $\int_a^b \frac{\log x}{x} dx =$

(a) $\log\left(\frac{\log b}{\log a}\right)$ (b) $\log(ab)\log\left(\frac{b}{a}\right)$ (c) $\frac{1}{2}\log(ab)\log\left(\frac{b}{a}\right)$ (d) $\frac{1}{2}\log(ab)\log\left(\frac{a}{b}\right)$

4. $\int_1^2 \frac{1}{x^2} e^{\frac{-1}{x}} dx =$

(a) $\sqrt{e}+1$ (b) $\sqrt{e}-1$ (c) $\frac{\sqrt{e}+1}{e}$ (d) $\frac{\sqrt{e}-1}{e}$

5. $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$

(a) $\frac{\pi}{4}+\frac{1}{2}\log 2$ (b) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (c) $\frac{\pi}{2}+\log 2$ (d) $\frac{\pi}{2}-\log 2$

6. If $\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$, **then** $k =$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) None of these

7. $\int_0^1 \frac{dx}{[ax+b(1-x)]^2} =$

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) ab (d) $\frac{1}{ab}$

8. The value of integral $\int_{1/\pi}^{2/\pi} \frac{\sin(1/x)}{x^2} dx =$

(a) 2 (b) -1

(c) 0 (d) 1

9. The value of $\int_{-2}^{2}(ax^3+bx+c)$ **depends on**

(a) The value of a (b) The value of b

(c) The value of c (d)The values of a and b

10. $\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx =$

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{32}$

11. $\int_0^1 \frac{e^{-x}}{1+e^{-x}}dx =$

(a) $\log\left(\frac{1+e}{e}\right)-\frac{1}{e}+1$ (b) $\log\left(\frac{1+e}{2e}\right)-\frac{1}{e}+1$ (c) $\log\left(\frac{1+e}{2e}\right)+\frac{1}{e}-1$ (d)None of these

12. $\int_0^{\pi/2} \frac{\cos x}{1+\cos x+\sin x}dx =$

(a) $\frac{\pi}{4}+\frac{1}{2}\log 2$ (b) $\frac{\pi}{4}+\log 2$ (c) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (d) $\frac{\pi}{4}-\log 2$

13. The value of the definite integral

$\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$ for $0<\alpha<\pi$ is equal to

(a) $\sin\alpha$ (b) $\tan^{-1}(\sin\alpha)$ (c) $\alpha\sin\alpha$ (d) $\frac{\alpha}{2}(\sin\alpha)^{-1}$

14. The integral $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx =$

(a) π (b) 2π (c) 3π (d)None of these

15. $\int_0^1 (1-x)^9\,dx =$

(a) 1 (b) $\frac{1}{10}$ (c) $\frac{11}{10}$ (d)2

16. If $\int_0^1 x\log\left(1+\frac{x}{2}\right)dx = a+b\log\frac{2}{3}$, **then**

(a) $a=\frac{3}{2},\ b=\frac{3}{2}$ (b) $a=\frac{3}{4},\ b=-\frac{3}{4}$ (c) $a=\frac{3}{4},\ b=\frac{3}{2}$ (d) $a=b$

17. If $I_1=\int_e^{e^2}\frac{dx}{\log x}$ **and** $I_2=\int_1^2\frac{e^x}{x}dx$, **then**

(a) $I_1=I_2$ (b) $I_1>I_2$ (c) $I_1<I_2$ (d) None of these

18. The value of $\int_1^2 \log x\, dx$ is

(a) $\log 2/e$ (b) $\log 4$ (c) $\log 4/e$ (d) $\log 2$

19. $\int_0^1 \frac{e^x(x-1)}{(x+1)^3} dx =$

(a) $\frac{e}{4}$ (b) $\frac{e}{4}-1$ (c) $\frac{e}{4}+1$ (d) None of these

20. The value of $\int_0^{\pi/4} \frac{1+\tan x}{1-\tan x} dx$ is

(a) $-\frac{1}{2}\log 2$ (b) $\frac{1}{4}\log 2$ (c) $\frac{1}{3}\log 2$ (d)None of these

21. $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx =$

(a) $\log\frac{4}{3}$ (b) $\log\frac{1}{3}$ (c) $\log\frac{3}{4}$ (d) None of these

22. $\int_{\pi/4}^{\pi/2} e^x(\log \sin x + \cot x) dx =$

(a) $e^{\pi/4}\log 2$ (b) $-e^{\pi/4}\log 2$ (c) $\frac{1}{2}e^{\pi/4}\log 2$ (d) $-\frac{1}{2}e^{\pi/4}\log 2$

23. $\int_0^2 \sqrt{\frac{2+x}{2-x}}\, dx =$

(a) $\pi+2$ (b) $\pi+\frac{3}{2}$ (c) $\pi+1$ (d)None of these

24. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right) dx =$

(a) $\frac{e^2}{2}+e$ (b) $e-\frac{e^2}{2}$ (c) $\frac{e^2}{2}-e$ (d) None of these

25. $\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2} =$

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{1}{2}$ (d) None of these

26. $\int_0^{\pi/2} \frac{\sin x \cos x\, dx}{\cos^2 x + 3\cos x + 2} =$

(a) $\log\left(\frac{8}{9}\right)$ (b) $\log\left(\frac{9}{8}\right)$ (c) $\log(8\times 9)$ (d) None of these

27. $\int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx =$

(a) $\log_e\left(\frac{2}{3}\right)$ (b) $\log_e 3$ (c) $\frac{1}{2}\log_e\left(\frac{4}{3}\right)$ (d) $\log_e\left(\frac{4}{3}\right)$

28. The value of $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt$ **is**

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi}{4}$ (d) None of these

29. The value of $\int_0^1 \frac{x^4+1}{x^2+1} dx$ **is**

(a) $\frac{1}{6}(3\pi - 4)$ (b) $\frac{1}{6}(3 - 4\pi)$ (c) $\frac{1}{6}(3\pi + 4)$ (d) $\frac{1}{6}(3 + 4\pi)$

30. $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} =$

(a) πab (b) $\pi^2 ab$ (c) $\frac{\pi}{ab}$ (d) $\frac{\pi}{2ab}$

31. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ **is**

(a) 3 (b) 1 (c) 2 (d) 0

32. $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx =$

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) π

33. $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx =$

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) π

34. Let $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ **and** $I_2 = \int_1^2 \frac{dx}{x}$ **then**

(a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_1 = I_2$ (d) $I_1 > 2I_2$

35. $\int_0^{\pi/4} [\sqrt{\tan x} + \sqrt{\cot x}] dx$ **equals**

(a) $\sqrt{2}\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{\sqrt{2}}$ (d) 2π

36. $\int_0^{\pi/4}(\cos x-\sin x)dx+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx+\int_{2\pi}^{\pi/4}(\cos x-\sin x)dx$ **is equal to**

(a) $\sqrt{2}-2$ (b) $2\sqrt{2}-2$ (c) $3\sqrt{2}-2$ (d) $4\sqrt{2}-2$

37. $\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}=$

(a) $\frac{1}{2}\log\frac{5}{3}$ (b) $\frac{1}{3}\log\frac{5}{3}$ (c) $\frac{1}{2}\log\frac{3}{5}$ (d) $\frac{1}{5}\log\frac{3}{5}$

38. **If** $I_n=\int_0^{\pi/4}\tan^n\theta\,d\theta,$ **then** I_8+I_6 **equals**

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

39. **The value of** $\int_1^{e^2}\frac{dx}{x(1+\ln x)^2}$ **is**

(a) $2/3$ (b) $1/3$ (c) $3/2$ (d) $\ln 2$

40. . $\int_0^{\pi/2}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx=$

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

41. $\int_0^{\pi}x\,f(\sin x)dx=$

(a) $\pi\int_0^{\pi}f(\sin x)dx$ (b) $\frac{\pi}{2}\int_0^{\pi}f(\sin x)dx$ (c) $\frac{\pi}{2}\int_0^{\pi/2}f(\sin x)dx$ (d)None of these

42. $\int_0^{\pi/2}\frac{\cos x-\sin x}{1+\sin x\cos x}dx=$

(a) 2 (b) -2 (c)0 (d)None of these

43. $\int_{-\pi/2}^{\pi/2}\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta=$

(a) 0 (b) 1 (c)2 (d)None of these

44 .Assume that f **is continuous everywhere, then** $\frac{1}{c}\int_{ac}^{bc}f\left(\frac{x}{c}\right)dx=$

(a) $\int_a^b f\left(\frac{x}{c}\right)dx$ (b) $\frac{1}{c}\int_a^b f(x)dx$ (c) $\int_a^b f(x)dx$ (d)None of these

45. **If n is a positive integer and $[x]$ is the greatest integer not exceeding x, then** $\int_0^n \{x-[x]\}\,dx$ **equals**

(a) $n^2/2$ (b) $n(n-1)/2$ (c) $n/2$ (d) $\frac{n^2}{2}-n$

46. $\int_{-1}^{1} x|\,x\,|\,dx =$

(a) 1 (b) 0 (c) 2 (d) -2

47. $\int_0^{\pi/2} \log \sin x\,dx =$

(a) $-\left(\frac{\pi}{2}\right)\log 2$ (b) $\pi\log\frac{1}{2}$ (c) $-\pi\log\frac{1}{2}$ (d) $\frac{\pi}{2}\log 2$

48. $\int_{-1}^{1} x^{17}\cos^4 x\,dx =$

(a) -2 (b) -1 (c) 0 (d) 2

49. $\int_0^{\pi/2} |\sin x - \cos x|\,dx =$

(a) 0 **(b)** $2(\sqrt{2}-1)$ **(c)** $\sqrt{2}-1$ **(d)** $2(\sqrt{2}+1)$

50. The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x\,dx$ **is**

(a) 3/2 (b) –8/3 (c) 3/8 (d) 8/3

51. $\int_{-2}^{2} |1-x^2|\,dx =$

(a) 2 (b) 4 (c) 6 (d) 8

52. **For any integer** n, **the integral**

$\int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x\,dx =$

(a) -1 (b) 0 (c) 1 (d) π

53. **The value of the integral** $I=\int_0^1 x(1-x)^n\,dx$ **is**

(a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$ (c) $\frac{1}{n+1}-\frac{1}{n+2}$ (d) $\frac{1}{n+1}+\frac{1}{n+2}$

54. $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx =$

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{3\pi^2}{2}$ (d) $\frac{\pi^2}{3}$

55 **If** $f(a+b-x) = f(x),$ **then** $\int_a^b x\, f(x)\, dx =$

(a) $\frac{a+b}{2}\int_a^b f(b-x)\, dx$ (b) $\frac{a+b}{2}\int_a^b f(x)\, dx$ (c) $\frac{b-a}{2}\int_a^b f(x)\, dx$ (d) None of these

56. **If** $f(x)$ **is a continuous periodic function with period** $T,$ **then the integral** $I = \int_a^{a+T} f(x)\, dx$ **is**

(a) Equal to $2a$ (b) Equal to $3a$ (c) Independent of a (d) None of these

57. **The value of** $\int_{-1}^{1} (\sqrt{1+x+x^2} - \sqrt{1-x+x^2})\, dx$ **is**

(a) 0 (b) 1 (c) -1 (d) None of these

58. $\int_{1/e}^{e} |\log x|\, dx =$

(a) $1-\frac{1}{e}$ (b) $2\left(1-\frac{1}{e}\right)$ (c) $e^{-1}-1$ (d) None of these

59. **The value of** $\int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi} d\phi,$ **is**

(a) $\pi \tan\frac{\pi}{8}$ (b) $\log\tan\frac{\pi}{8}$ (c) $\tan\frac{\pi}{8}$ (d) None of these

60. **The value of** $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ **is**

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$

61. **The value of** $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ **is**

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) None of these

62. **The value of** $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ **is**

(a) 1 (b) 0 (c) -1 (d) None of these

63. **The value of** $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx$ **is**

(a) 0 (b) $2\int_0^1 \frac{\sin x}{3-|x|} dx$ (c) $2\int_0^1 \frac{-x^2}{3-|x|} dx$ (d) $2\int_0^1 \frac{\sin x - x^2}{3-|x|} dx$

64. If $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$, **then**

(a) $f(2a-x) = -f(x)$ (b) $f(2a-x) = f(x)$ (c) $f(a-x) = -f(x)$ (d) $f(a-x) = f(x)$

65. $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ equals

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

66. The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ **is**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π

67. The value of $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ **is**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5

68. The value of $\int_0^{\sqrt{2}} [x^2] dx$, **where [.] is the greatest integer function**

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$ (c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$

69. $\int_0^{2\pi} (\sin x + |\sin x|)\, dx =$

(a) 0 (b) 4 (c) 8 (d) 1

70. $\int_{-1}^{1} \log(x + \sqrt{x^2+1}) dx =$

(a) 0 (b) log 2 (c) $\log\frac{1}{2}$ (d) None of these

71. If [*x*] denotes the greatest integer less than or equal to *x*, then the value of $\int_1^5 [|x-3|]\, dx$ **is**

(a) 1 (b) 2 (c) 4 (d) 8

72. The value of $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ **is**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5

73. Suppose *f* is such that $f(-x) = -f(x)$ **for every real *x* and** $\int_0^1 f(x)dx = 5$, **then** $\int_{-1}^{0} f(t)dt =$

(a) 10 (b) 5 (c) 0 (d) – 5

74. $\int_0^{2a} f(x)dx =$

(a) $2\int_0^a f(x)dx$ (b) 0 (c) $\int_0^a f(x)dx + \int_0^a f(2a-x)dx$ (d) $\int_0^a f(x)dx + \int_0^{2a} f(2a-x)dx$

75. $\int_{-2}^{2} |[x]|dx =$

(a) 1 (b) 2 (c) 3 (d) 4

76. $\int_0^{1000} e^{x-[x]}dx$ is

(a) $e^{1000}-1$ (b) $\frac{e^{1000}-1}{e-1}$ (c) $1000(e-1)$ (d) $\frac{e-1}{1000}$

77. The value of $\int_a^b \frac{x}{|x|}dx,\ a<b<0$ **is**

(a) $-(|a|+|b|)$ (b) $|b|-|a|$ (c) $|a|-|b|$ (d) $|a|+|b|$

78. The value of $\int_{-2}^{2}\left[p\ln\left(\frac{1+x}{1-x}\right)+q\ln\left(\frac{1-x}{1+x}\right)^{-2}+r\right]dx$ **depends on**

(a) The value of p (b) The value of q

(c) The value of r (d)The value of p and q

79. If f is continuous function, then

(a) $\int_{-3}^{5} f(x)dx = \int_{-6}^{10} f(x/2)]dx$ (b) $\int_{-3}^{5} 2f(x)dx = \int_{-6}^{10} f(x-1)dx$

(c) $\int_{-3}^{5} f(x)dx = \int_{-4}^{4} f(x-1)dx$ (d) $\int_{-3}^{5} f(x)dx = \int_{-2}^{6} f(x-1)dx$

80. The integral value $\int_{-2}^{0}\left[x^3+3x^2+3x+3+(x+1)\cos(x+1)\right]dx$ **is**

(a) 2 (b) 4 (c) 0 (d) 8

81. If $\int_0^{\pi} xf(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$ **, then A is**

(a) 2π (b) π (c) $\frac{\pi}{4}$ (d) 0

82. If $g(x)=\int_0^x \cos^4 t\,dt,$ **then** $g(x+\pi)$ **equals**

(a) $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ (c) $g(x)g(\pi)$ (d) $g(x)/g(\pi)$

83. $\int_0^{\pi/2}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta =$

(a) $\pi\log 2$ (b) $\frac{\pi}{\log 2}$ (c) π (d) None of these

84. Let a, b, c **be non-zero real numbers such that** $\int_0^3 (3ax^2+2bx+c)dx = \int_1^3 (3ax^2+2bx+c)dx$, **then**

(a) $a+b+c=3$ (b) $a+b+c=1$ (c) $a+b+c=0$ (d) $a+b+c=2$

85. The function $L(x)=\int_1^x \frac{dt}{t}$ **satisfies the equation**

(a) $L(x+y)=L(x)+L(y)$ (b) $L\left(\frac{x}{y}\right)=L(x)+L(y)$ (c) $L(xy)=L(x)+L(y)$ (d) None of these

86. The value of $\int_{-2}^{3} |1-x^2|\, dx$ **is**

(a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

87. If $\int_{\sin x}^{1} t^2 f(t)\,dt = 1-\sin x$, $x \in \left(0,\frac{\pi}{2}\right)$ **then** $f\left(\frac{1}{\sqrt{3}}\right)$ **equal to**

(a) 3 (b) $\frac{1}{3}$ c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

88. The value of the integral $\sum_{k=1}^{n}\int_0^1 f(k-1+x)dx$ **is**

(a) $\int_0^1 f(x)dx$ (b) $\int_0^2 f(x)dx$ (c) $\int_0^n f(x)dx$ (d) $n\int_0^1 f(x)dx$

89. $\int_0^\infty \frac{xdx}{(1+x)(1+x^2)} =$

(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) 1

90. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x(\sin x+\cos x)dx =$

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$ (c) $\frac{6}{15}$ (d) $\frac{8}{15}$

91. The greatest value of the function $F(x)=\int_1^x |t|dt$ **on the interval** $\left[-\frac{1}{2},\frac{1}{2}\right]$ **is given by**

(a) $\frac{3}{8}$ (b) $-\frac{1}{2}$ (c) $-\frac{3}{8}$ (d) $\frac{2}{5}$

92. The value of the integral $\int_{-1}^{1}\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)dx$ **is**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$ (d) None of these

93. $\int_0^{\infty} \frac{x\,dx}{(1+x)(1+x^2)} =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d)None of these

94. The points of intersection of $F_1(x) = \int_2^x (2t-5)dt$ **and** $F_2(x) = \int_0^x 2t\,dt,$ **are**

(a) $\left(\frac{6}{5}, \frac{36}{25}\right)$ (b) $\left(\frac{2}{3}, \frac{4}{9}\right)$ (c) $\left(\frac{1}{3}, \frac{1}{9}\right)$ (d) $\left(\frac{1}{5}, \frac{1}{25}\right)$

95. $\int_0^{\infty} \frac{x \ln x\,dx}{(1+x^2)^2}$ **is equal to**

(a) 0 (b) 1 (c) ∞ (d)None of these

96. $\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]dx$ **is equal to**

(a) 0 (b) π (c) $\pi/2$ (d) $\pi/4$

97. If $f(x) = \int_{x^2}^{x^4} \sin\sqrt{t}\,dt,$ **then** $f'(x)$ **equals**

(a) $\sin x^2 - \sin x$ (b) $4x^3 \sin x^2 - 2x \sin x$ (c) $x^4 \sin x^2 - x \sin x$ (d) None of these

98. $\int_0^{\infty} \frac{dx}{\left(x+\sqrt{x^2+1}\right)^3} =$

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d)None of these

99. $\int_0^a x^4\sqrt{a^2-x^2}\,dx =$

(a) $\frac{\pi}{32}$ (b) $\frac{\pi}{32}a^6$ (c) $\frac{\pi}{16}a^6$ (d) $\frac{\pi}{8}a^6$

100. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right); x > 0$ **. If** $\int_1^4 \frac{3}{x}e^{\sin x^3}dx = F(k) - F(1)$**, then one of the possible value of *k*, is**

(a) 15 (b) 16 (c)63 (d)64

101. $\int_0^a x^2(a^2-x^2)^{3/2}dx =$

(a) $\frac{\pi a^6}{32}$ (b) $\frac{2a^5}{15}$ (c) $\frac{a^6}{32}$ (d)None of these

102 .The value of $\lim_{n\to\infty}\left[\frac{n}{1+n^2}+\frac{n}{4+n^2}+\frac{n}{9+n^2}+....+\frac{1}{2n}\right]$ **is equal to**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c)1 (d)None of these

103. $\lim_{n\to\infty}\frac{1^{99}+2^{99}+3^{99}+.......n^{99}}{n^{100}}=$

(a) $\frac{9}{100}$ (b) $\frac{1}{100}$ (c) $\frac{1}{99}$ (d) $\frac{1}{101}$

104. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}=$

(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$ (c) $\frac{1}{p}-\frac{1}{p-1}$ (d) $\frac{1}{P+2}$

105. $\lim_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n}$ equals

(a) e (b) $1/e$ (c) $\pi/4$ (d) $4/\pi$

106. $\int_0^\infty \frac{\log(1+x^2)}{1+x^2}dx=$

(a) $\pi\log\frac{1}{2}$ (b) $\pi\log 2$ (c) $2\pi\log\frac{1}{2}$ (d) $2\pi\log 2$

107. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+.....+\frac{1}{n}\sec^2 1\right]$ **equals**

(a) $\tan 1$ (b) $\frac{1}{2}\tan 1$ (c) $\frac{1}{2}\sec 1$ (d) $\frac{1}{2}\operatorname{cosec} 1$

108. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{\sqrt{n^2+n}}+\frac{1}{\sqrt{n^2+2n}}+.....+\frac{1}{\sqrt{n^2+(n-1)n}}\right]$ **is equal to**

(a) $2+2\sqrt{2}$ (b) $2\sqrt{2}-2$ (c) $2\sqrt{2}$ (d)2

109. $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}$ **equals**

(a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$ (c) $-1+\sqrt{2}$ (d) $1+\sqrt{2}$

110. The value of integral $\int_0^1\frac{x^b-1}{\log x}dx$ **is**

(a) $\log b$ (b) $2\log(b+1)$ (c) $3\log b$ (d)None of these

111. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+.....\frac{1}{2n}\right]=$

(a)0 (b) $\log_e 4$ (c) $\log_e 3$ (d) $\log_e 2$

DEFINITE INTEGRATION

HINTS AND SOLUTIONS

1. (a) $\int_0^{\pi/4} \tan^2 x dx = \int_0^{\pi/4} (\sec^2 x - 1) dx$

$= \int_0^{\pi/4} \sec^2 x dx - \int_0^{\pi/4} 1 dx = [\tan x]_0^{\pi/4} - [x]_0^{\pi/4} = 1 - \frac{\pi}{4}$.

2. (b) Let $I = \int_0^{\pi/2} e^x \sin x \, dx$

$= -[e^x \cos x]_0^{\pi/2} + \int_0^{\pi/2} e^x \cos x \, dx$

$= -[e^x \cos x]_0^{\pi/2} + [e^x \sin x]_0^{\pi/2} - \int_0^{\pi/2} e^x \sin x \, dx$

$\therefore 2I = [e^x(\sin x - \cos x)]_0^{\pi/2} = (e^{\pi/2} + 1)$

3. (c) Let $I = \int_a^b \frac{1}{x} \log x \, dx = (\log x \log x)_a^b - \int_a^b \frac{1}{x} \log x \, dx$

$\Rightarrow 2I = [(\log x)^2]_a^b \Rightarrow I = \frac{1}{2}[(\log b)^2 - (\log a)^2]$

$= \frac{1}{2}[(\log b + \log a)(\log b - \log a)] = \frac{1}{2} \log(ab) \log\left(\frac{b}{a}\right)$.

4. (d) Put $t = -\frac{1}{x} \Rightarrow dt = \frac{1}{x^2} dx$, then it reduces to $\int_{-1}^{-1/2} e^t dt = [e^t]_{-1}^{-1/2} = e^{-1/2} - e^{-1} = \frac{\sqrt{e} - 1}{e}$.

5. (b) $I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1 - x^2)^{3/2}} dx$

Put $\sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1 - x^2}} dx = dt$ and $x = \sin t$

Also $t = 0$ to $\frac{\pi}{4}$ as $x = 0$ to $\frac{1}{\sqrt{2}}$

$\Rightarrow I = \int_0^{\pi/4} t . \sec^2 t \, dt = \frac{\pi}{4} - \frac{1}{2} \log 2$.

6. (b) $\int_0^k \frac{1}{2 + 8x^2} dx = \frac{1}{2} \int_0^k \frac{dx}{1 + (2x)^2} = \frac{1}{4} \int_0^{2k} \frac{dt}{1 + t^2}$

$= \frac{1}{4} | \tan^{-1} t |_0^{2k} = \frac{1}{4} \tan^{-1} 2k$.

Comparing it with the given value, we get $\tan^{-1} 2k = \frac{\pi}{4} \Rightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$.

7. (d) Let $I=\int_0^1 \frac{dx}{[(a-b)x+b]^2}$

Put $t=(a-b)x+b \Rightarrow dt=(a-b)dx$

As $x=1 \Rightarrow t=a$ and $x=0 \Rightarrow t=b$, then

$$I=\frac{1}{a-b}\int_b^a \frac{1}{t^2}dt=\frac{1}{(a-b)}\left[-\frac{1}{t}\right]_b^a=\frac{1}{(a-b)}\left(\frac{a-b}{ab}\right)=\frac{1}{ab}.$$

8. (d) Put $t=\frac{1}{x} \Rightarrow dt=-\frac{1}{x^2}dx$ as $t=\frac{\pi}{2}$ and π

$$\therefore \int_{1/\pi}^{2/\pi} \frac{\sin\left(\frac{1}{x}\right)}{x^2}dx=-\int_{\pi/2}^{\pi} \sin t\, dt=-[\cos t]_{\pi/2}^{\pi}$$

$$=-\left[\cos\pi-\cos\left(\frac{\pi}{2}\right)\right]=1.$$

9. (c) $\int_{-2}^{2}(ax^3+bx+c)dx=\left[\frac{ax^4}{4}+\frac{bx^2}{2}+cx\right]_{-2}^{2}=4c.$

10. (d) Put $t=\tan^{-1}x \Rightarrow dt=\frac{1}{1+x^2}dx$, then

$$\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx=\int_0^{\pi/4} t\,dt=\left[\frac{t^2}{2}\right]_0^{\pi/4}=\frac{\pi^2}{32}.$$

11. (b) Put $1+e^{-x}=t \Rightarrow -e^{-x}dx=dt$, then we have

$$I=\int_2^{1+\frac{1}{e}} \frac{(t-1)(-dt)}{t}=\int_2^{1+\frac{1}{e}}\left(\frac{1}{t}-1\right)dt$$

$$=[\log_e t-t]_2^{1+\frac{1}{e}}=\log_e\left(1+\frac{1}{e}\right)-\left(1+\frac{1}{e}\right)-\log_e 2+2$$

$$=\log_e\left(\frac{e+1}{2e}\right)-\frac{1}{e}+1.$$

12. (c) $\int_0^{\pi/2} \frac{\cos x}{1+\cos x+\sin x}dx$

$$=\int_0^{\pi/2} \frac{\cos^2(x/2)-\sin^2(x/2)}{2\cos^2(x/2)+2\sin(x/2)\cos(x/2)}dx$$

$$=\frac{1}{2}\int_0^{\pi/2} \frac{1-\tan^2(x/2)}{1+\tan(x/2)}dx=\frac{1}{2}\int_0^{\pi/2}\left[1-\tan\left(\frac{x}{2}\right)\right]dx$$

$$\frac{\pi}{4}+\log\frac{1}{\sqrt{2}}=\frac{\pi}{4}-\frac{1}{2}\log 2.$$

13. (d) $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}=\int_0^1 \frac{dx}{(x+\cos\alpha)^2+1-\cos^2\alpha}$

$$=\int_0^1 \frac{dx}{(x+\cos\alpha)^2+\sin^2\alpha}=\left[\frac{1}{\sin\alpha}\tan^{-1}\frac{x+\cos\alpha}{\sin\alpha}\right]_0^1$$

$= \frac{1}{\sin\alpha}\left(\tan^{-1}\cot\frac{\alpha}{2} - \tan^{-1}\cot\alpha\right) = \frac{\alpha}{2}.\frac{1}{\sin\alpha}$.

14. (b) $I = \int_{-1}^{3}\left\{\tan^{-1}\left(\frac{x}{x^2+1}\right) + \tan^{-1}\left(\frac{x^2+1}{x}\right)\right\}dx$

$= \int_{-1}^{3}\left\{\tan^{-1}\left(\frac{x}{x^2+1}\right) + \cot^{-1}\left(\frac{x}{x^2+1}\right)\right\}dx$

$= \int_{-1}^{3}\frac{\pi}{2}dx = 2\pi$.

15. (b) Required value $= \left[\frac{-(1-x)^{10}}{10}\right]_0^1 = \frac{1}{10}$.

16. (c) Integrate it by parts taking $\log\left(1+\frac{x}{2}\right)$ as first function $= \left[\log\left(1+\frac{x}{2}\right)\frac{x^2}{2}\right]_0^2 - \int_0^1 \frac{1}{1+\frac{x}{2}}\frac{1}{2}\frac{x^2}{2}dx$

$= \frac{1}{2}\log\frac{3}{2} - \frac{1}{2}\int_0^1\frac{x^2}{x+2}dx$

$= \frac{1}{2}\log\frac{3}{2} - \frac{1}{2}\left[\frac{1}{2} - 2 + 4\log 3 - 4\log 2\right] = \frac{3}{4} + \frac{3}{2}\log\frac{2}{3}$

17. (a) Put $\log x = u$ in I_1, so that $dx = x\,du = e^u du$

Also as $x = e$ to $e^2, u = 1$ to 2

Thus, $I_1 = \int_1^2 \frac{e^u}{u}du = \int_1^2\frac{e^x}{x}dx$. Hence, $I_1 = I_2$.

18. (c) $\int_1^2 \log x dx = [x\log x - x]_1^2 = 2\log 2 - 2 + 1$

$= \log 4 - 1 = \log 4 - \log e = \log\frac{4}{e}$.

19. (b) $\int_0^1 \frac{e^x(x-1)}{(x+1)^3}dx = \int_0^1\frac{e^x(x+1-2)}{(x+1)^3}dx$

$\int_0^1\frac{e^x}{(x+1)^2}dx - 2\int_0^1\frac{e^x}{(x+1)^3}dx = \left[\frac{e^x}{(x+1)^2}\right]_0^1 = \frac{e}{4} - 1$.

20. (a) $\int_0^{\pi/4}\frac{1+\tan x}{1-\tan x}dx = \int_0^{\pi/4}\tan\left(\frac{\pi}{4}+x\right)dx$

$= \left[\log\left\{\sec\left(\frac{\pi}{4}+x\right)\right\}\right]_0^{-\pi/4} = -\frac{1}{2}\log 2$.

21. (a) Put $\sin x = t \Rightarrow \cos x\,dx = dt,$ so that reduced integral is $\int_0^1\left(\frac{1}{1+t} - \frac{1}{2+t}\right)dt = [\log(1+t) - \log(2+t)]_0^1$

$= \log\frac{2}{3} - \log\frac{1}{2} = \log\frac{4}{3}$.

22. (c) Let $I=\int_{\pi/4}^{\pi/2} e^x(\log\sin x+\cot x)dx$

$$I=\int_{\pi/4}^{\pi/2} e^x \log\sin x\, dx+\int_{\pi/4}^{\pi/2} e^x \cot x\, dx$$

$$=\int_{\pi/4}^{\pi/2} e^x \log\sin x dx+[e^x \log\sin x]_{\pi/4}^{\pi/2} -\int_{\pi/4}^{\pi/2} e^x \log\sin x\, dx$$

$$=e^{\pi/2}\log\sin\frac{\pi}{2}-e^{\pi/4}\log\sin\frac{\pi}{4}=\frac{1}{2}e^{\pi/4}\log 2 .$$

23. (a) Put $x=2\cos\theta \Rightarrow dx=-2\sin\theta\, d\theta$, then

$$\int_0^2\sqrt{\frac{2+x}{2-x}}dx=-2\int_{\pi/2}^0\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\sin\theta\, d\theta$$

$$=4\int_0^{\pi/2}\frac{\cos(\theta/2)}{\sin(\theta/2)}\sin\frac{\theta}{2}\cos\frac{\theta}{2}d\theta$$

$$=2\int_0^{\pi/2}(1+\cos\theta)d\theta$$

$$=2[\theta+\sin\theta]_0^{\pi/2}=2\left[\frac{\pi}{2}+1\right]=\pi+2 .$$

24. (c) $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=\left[\frac{1}{x}e^x\right]_1^2=\frac{e^2}{2}-e$.

25. (c) $\int_0^{\pi/2}\frac{(1+2\cos x)}{(2+\cos x)^2}dx=\int_0^{\pi/2}\frac{2(\cos x+2)-3}{(2+\cos x)^2}dx$

$$=2\int_0^{\pi/2}\frac{dx}{2+\cos x}-3\int_0^{\pi/2}\frac{dx}{(2+\cos x)^2}$$

$$=4\int_0^1\frac{dt}{3+t^2}-6\int_0^1\frac{1+t^2}{(3+t^2)^2}dt\ ,\quad \left[\text{Put }\tan\frac{x}{2}=t\right]$$

$$=-2\int_0^1\frac{dt}{3+t^2}+12\int_0^1\frac{dt}{(3+t^2)^2}$$

$$=-2\int_0^1\frac{dt}{3+t^2}+12\left[\frac{1}{6}.\frac{t}{t^2+3}\right]_0^1+\frac{1}{6}\int_0^1\frac{dt}{3+t^2}$$

$$=2\left[\frac{t}{t^2+3}\right]_0^1=\frac{1}{2} .$$

26. (b) Let $I=\int_0^{\pi/2}\frac{\sin x\cos x.dx}{\cos^2 x+3\cos x+2}$

We put $\cos x=t\Rightarrow -\sin x\, dx=dt$, then

$$I=\int_0^1\frac{t.dt}{t^2+3t+2}=\int_0^1\left[\frac{2}{t+2}-\frac{1}{t+1}\right]dt$$

$$=[2\log(t+2)-\log(t+1)]_0^1=[2\log 3-\log 2-2\log 2]$$

$$=[2\log 3-3\log 2]=[\log 9-\log 8]=\log\left(\frac{9}{8}\right) .$$

27. (d) Put $1+\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\therefore \int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$$

$$= \int_1^2 \frac{dt}{t(1+t)} = \int_1^2 \frac{dt}{t} - \int_1^2 \frac{dt}{1+t} = [\log t - \log(1+t)]_1^2$$

$$= \log_e 2 - \log_e 3 + \log_e 2 = \log_e \frac{4}{3}.$$

28. (c) We have $I = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt$

Putting $t = \sin^2 u$ in the first integral and $t = \cos^2 v$ in the second integral, we have

$$I = \int_0^x u \sin 2u\, du - \int_{\pi/2}^x v \sin 2v\, dv$$

$$= \int_0^{\pi/2} u \sin 2u du + \int_{\pi/2}^x u \sin 2u\, du - \int_{\pi/2}^x v \sin 2v\, dv$$

$$I = \int_0^{\pi/2} u \sin 2u du = \left(\frac{-u\cos 2u}{2}\right)_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2} \cos 2u\, du$$

$$= \left(\frac{-u\cos 2u}{2}\right)_0^{\pi/2} + \frac{1}{4}(\sin 2u)_0^{\pi/2} = \frac{\pi}{4}.$$

29. (a) $I = \int_0^1 \frac{x^4+1}{x^2+1} dx = \int_0^1 \frac{x^4-1}{x^2+1} dx + 2\int_0^1 \frac{dx}{1+x^2}$

$$\Rightarrow I = \int_0^1 (x^2-1) dx + 2\int_0^1 \frac{dx}{1+x^2}$$

$$\Rightarrow I = \left[\frac{x^3}{3} - x\right]_0^1 + 2[\tan^{-1} x]_0^1 = -\frac{2}{3} + \frac{\pi}{2} = \frac{(3\pi-4)}{6}.$$

30. (d) $I = \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}.$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$I = \int_0^{\pi/2} \frac{\frac{1}{\cos^2 x} dx}{a^2 + b^2 \frac{\sin^2 x}{\cos^2 x}} = \int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2\tan^2 x} dx.$$

Substituting $b \tan x = t$ and $b \sec^2 x\, dx = dt$ limit when $x = 0$, then $t = 0$ and when $x = \frac{\pi}{2}$, then $t = \infty$,

therefore, $I = \int_0^\infty \frac{\frac{dt}{b}}{a^2+t^2} = \frac{1}{b}\left[\frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)\right]_0^\infty$

$$= \frac{1}{ab}\left[\tan^{-1}\infty - \tan^{-1} 0\right] = \frac{1}{ab}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{2ab}.$$

31. (c) $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}} dx$

$I = \int_0^{\pi/2} (\sin x + \cos x) dx = (-\cos x + \sin x)_0^{\pi/2}$

$I = 1 - (-1) = 2$.

32. (b) $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx$

Put $x = \cos\theta$, then $\sin\left[2\tan^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right]$

$= \sin\left[2\tan^{-1}\left(\cot\frac{\theta}{2}\right)\right]$

$= \sin\left[2\tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right]\right] = \sin\left[2\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right]$

$= \sin(\pi - \theta) = \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-x^2}$

Now, $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx = \int_0^1 \sqrt{1-x^2}\, dx$

$= \left[\frac{1}{2}x\sqrt{1-x^2}\right]_0^1 + \frac{1}{2}[\sin^{-1} x]_0^1 = \frac{\pi}{4}$.

33. (a) $I = \int_{-1}^3 \left[\tan^{-1}\left(\frac{x}{x^2+1}\right) + \cot^{-1}\left(\frac{x}{x^2+1}\right)\right] dx$

$= \int_{-1}^3 \left(\frac{\pi}{2}\right) dx = \left[\frac{\pi x}{2}\right]_{-1}^3 = 2\pi$, $\left(\because \tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}\right)$.

34. (b) We have, $(1+x^2) > x^2, \forall x$; $\sqrt{1+x^2} > x, \forall x \in (1, 2)$

$\Rightarrow \frac{1}{\sqrt{1+x^2}} < \frac{1}{x}, \forall x \in (1,2) \Rightarrow \int_1^2 \frac{dx}{\sqrt{1+x^2}} < \int_1^2 \frac{dx}{x}$

$\Rightarrow I_1 < I_2 \Rightarrow I_2 > I_1$.

35. (c) $I = \int_0^{\pi/4} [\sqrt{\tan x} + \sqrt{\cot x}] dx = \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$

$= \sqrt{2}\int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}} dx$

Put $\sin x - \cos x = t$; $(\cos x + \sin x)dx = dt$

$\therefore I = \sqrt{2}\int_{-1}^0 \frac{dt}{\sqrt{1-t^2}}$

$I = \sqrt{2}[\sin^{-1} t]_{-1}^0 = \sqrt{2}[0 - (-\pi/2)] = \frac{\pi}{\sqrt{2}}$.

36. (d) $I=\int_0^{\pi/4}(\cos x-\sin x)dx+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx+\int_{2\pi}^{\pi/4}(\cos x-\sin x)dx$

$=[\sin x+\cos x]_0^{\frac{\pi}{4}}-[\sin x+\cos x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}+[\sin x+\cos x]_{2\pi}^{\frac{\pi}{4}}$

$I=\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right]-\left[-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\right]+\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right]$

$I=[\sqrt{2}-1]-[-\sqrt{2}-\sqrt{2}]+[\sqrt{2}-1]$

37. (a) $I=\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}$

Put $x=\tan^2\theta\Rightarrow\theta=\tan^{-1}\sqrt{x}$

$dx=2\tan\theta\sec^2\theta\,d\theta$

$\therefore\ I=\int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}}\frac{2\tan\theta\sec^2\theta}{(\tan^2\theta-3)\sqrt{\tan^2\theta+1}}d\theta$

$=\int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}}\frac{2\tan\theta\sec^2\theta}{(\sec^2\theta-4)\sec\theta}d\theta$

$=\int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}}\frac{2\tan\theta\sec\theta}{(\sec^2\theta-4)}d\theta$

$=\int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}}\frac{2\tan\theta\sec\theta}{(\sec\theta-2)(\sec\theta+2)}d\theta$

$=\left[\frac{1}{2}\log\frac{(\sec\theta-2)}{(\sec\theta+2)}\right]_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}}$

38. (d) $I_n=\int_0^{\pi/4}(\sec^2\theta-1)\tan^{n-2}\theta\,d\theta$

$I_n=\int_0^{\pi/4}\sec^2\theta\tan^{n-2}\theta\,d\theta-\int_0^{\pi/2}\tan^{n-2}\theta\,d\theta$

$I_n=\left[\frac{\tan^{n-1}\theta}{n-1}\right]_0^{\pi/4}-I_{n-2}\Rightarrow I_n+I_{n-2}=\frac{1}{n-1}$

Hence $I_8+I_6=\frac{1}{8-1}=\frac{1}{7}$.

39. (a) $I=\int_1^{e^2}\frac{dx}{x(1+\ln x)^2}$

Let $(1+\ln x)=t\ \Rightarrow\ dt=\frac{1}{x}dx$

Now, when $x=1\rightarrow e^2$, then $t=1\rightarrow 3$

$\therefore\ I=\int_1^3\frac{dt}{t^2}=\left[\frac{-1}{t}\right]_1^3=-\left[\frac{1}{3}-1\right]=\frac{2}{3}$.

40. (c) $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ .....(i)

$= \int_0^{\pi/2} \frac{\sqrt{\cot\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cot\left(\frac{\pi}{2} - x\right)} + \sqrt{\tan\left(\frac{\pi}{2} - x\right)}} dx$

$= \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$ .....(ii)

Now adding (i) and (ii), we get

$2I = \int_0^{\pi/2} \frac{\sqrt{\cot x} + \sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx = [x]_0^{\pi/2} \Rightarrow I = \frac{\pi}{4}$.

41. (b) $\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$

Since $\int_0^a x f(x) dx = \frac{1}{2} a \int_0^a f(x) dx$, if $f(a - x) = f(x)$.

42. (c) $\int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx = I$ .....(i)

Now $I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right) - \sin\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx$

$= \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ .....(ii)

On adding, $2I = 0 \Rightarrow I = 0$.

43. (a) Since $f(-\theta) = \log\left(\frac{2 - \sin\theta}{2 + \sin\theta}\right)^{-1} = -\log\left(\frac{2 - \sin\theta}{2 + \sin\theta}\right) = -f(\theta)$

$\therefore$ $f(x)$ is an odd function of x.

Therefore, $2\int_0^{\pi/2} \log\left(\frac{2 - \sin\theta}{2 + \sin\theta}\right) d\theta = 0$.

44. (c) $I = \frac{1}{c} \int_{ac}^{bc} f(x/c) dx$

Put $\frac{x}{c} = t \Rightarrow dx = c\, dt$ and $x = bc \Rightarrow t = b$

$x = ac \Rightarrow t = a$ then, $I = \int_a^b f(t) dt = \int_a^b f(x) dx$.

45. (c) $x - [x]$ is a periodic function with period 1.

$\therefore \int_0^n \{x - [x]\} dx = n \int_0^1 (x - [x])\, dx$

$= n\left[\int_0^1 x\, dx - \int_0^1 [x] dx\right] = n\left[\left(\frac{x^2}{2}\right)_0^1 - 0\right] = \frac{n}{2}$.

46. (b) Let $f(x)=x|x|$. Then $f(-x)=-x|-x|=-x|x|=-f(x)$

Therefore $\int_{-1}^{1} x|x|\,dx = 0$,

47. (a) $\int_0^{\pi/2} \log \sin x\,dx = \int_0^{\pi/2} \log \cos x\,dx$

$$\Rightarrow 2I = \int_0^{\pi/2} \log \sin x \cos x\,dx = \int_0^{\pi/2} \log \sin 2x\,dx - \int_0^{\pi/2} \log 2dx$$

$$= \frac{1}{2}\int_0^{\pi} \log \sin t dt - \frac{\pi}{2}\log 2, \quad \text{(Putting } 2x = t\text{)}$$

$$= \frac{1}{2}.2\int_0^{\pi/2} \log \sin t\,dt - \frac{\pi}{2}\log 2$$

$$\Rightarrow 2I = I - \frac{\pi}{2}\log 2 \Rightarrow I = \frac{-\pi}{2}\log 2, \left\{\because \int_a^b f(x)dx = \int_a^b f(t)dt\right\}.$$

48. (c) $I = \int_0^{\pi/4} \log(1+\tan\theta)d\theta \Rightarrow I = \int_0^{\pi/4} \log\left\{1+\tan\left(\frac{\pi}{4}-\theta\right)\right\}d\theta$

$$\Rightarrow I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$$

$$\Rightarrow I = \int_0^{\pi/4} \log 2d\theta - \int_0^{\pi/4} \log(1+\tan\theta)d\theta$$

$$\Rightarrow I = \frac{1}{2}\int_0^{\pi/4} \log 2d\theta = \frac{\log 2}{2}|\,\theta|_0^{\pi/4} = \frac{\pi}{8}\log 2.$$

49. (b) $\int_0^{\pi/2} |\sin x - \cos x|\,dx$

$$= \int_0^{\pi/4} -(\sin x - \cos x)dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx = 2(\sqrt{2}-1).$$

50. (b) $\int_{-\pi/4}^{\pi/4} \sin^{-4} x\,dx = 2\int_0^{\pi/4} \frac{\cos^4 x}{\sin^4 x}\sec^4 x\,dx = 2\int_0^{\pi/4} \frac{\sec^4 x dx}{\tan^4 x}$

Put $\tan x = t$, we get $2\int_0^1 \frac{1+t^2}{t^4}dt$

$$= 2\left[\int_0^1 t^{-4}dt + \int_0^1 t^{-2}dt\right] = 2\left[\left|-\frac{1}{3t^3}\right|_0^1 + \left|-\frac{1}{t}\right|_0^1\right] = -\frac{8}{3}.$$

51. (b) $\int_{-2}^{2} |1-x^2|\,dx = \int_{-2}^{-1} |1-x^2|\,dx + \int_{-1}^{1} |1-x^2|dx + \int_1^2 |1-x^2|\,dx$

$$= -\int_{-2}^{-1}(1-x^2)dx + \int_{-1}^{1}(1-x^2)dx - \int_1^2 (1-x^2)dx$$

$$= \frac{4}{3}+\frac{4}{3}+\frac{4}{3} = 4.$$

52. (b) Let $f(x) = \int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x.dx$

Since $\cos(2n+1)(\pi - x) = \cos[(2n+1)\pi - (2n+1)x]$

$= -\cos(2n+1)x$ and $\sin^2(\pi - x) = \sin^2 x$

Hence by the property of definite integral,

$\int_0^{\pi} e^{\sin^2 x} \cos^3(2n+1)x\,dx = 0$,

53. (c) $I = \int_0^1 x(1-x)^n\,dx$

$-I = \int_0^1 -x(1-x)^n\,dx = \int_0^1 (1-x-1)(1-x)^n\,dx$

$= \int_0^1 (1-x)^{n+1}\,dx - \int_0^1 (1-x)^n\,dx$

$= \left[\frac{(1-x)^{n+2}}{-(n+2)}\right]_0^1 - \left[\frac{(1-x)^{n+1}}{-(n+1)}\right]_0^1 = \frac{1}{n+2} - \frac{1}{n+1}$

$\Rightarrow I = \frac{1}{n+1} - \frac{1}{n+2}$.

54. (a) Let $I = \int_0^{\pi} \frac{x\tan x}{\sec x + \cos x}\,dx = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \cos(\pi - x)}\,dx$

It gives $I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$

Now put $\cos x = t$ and solve, we get $I = \frac{\pi}{2} \times \frac{\pi}{2} = \frac{\pi^2}{4}$.

55. (b) Since $I = \int_a^b x f(x)dx = \int_a^b (a+b-x) f(a+b-x)dx$

$\Rightarrow \quad I = \int_a^b (a+b) f(x)dx - \int_a^b x f(x)dx$

$\{\because f(a+b-x) = f(x) \text{given}\}$

$\Rightarrow 2I = (a+b)\int_a^b f(x)dx \Rightarrow I = \int_a^b x\, f(x)dx = \frac{a+b}{2}\int_a^b f(x)dx$.

56. (c) Consider the function $g(a) = \int_a^{a+T} f(x)dx$

$= \int_a^0 f(x)dx + \int_0^T f(x)dx + \int_T^{a+T} f(x)dx$

Putting $x - T = y$ in last integral, we get $\int_T^{a+T} f(x)dx = \int_0^a f(y+T)dy = \int_0^a f(y)dy$

$\Rightarrow g(a) = \int_a^0 f(x)dx + \int_0^1 f(x)dx + \int_0^a f(x)dx = \int_0^T f(x)dx$

Hence $g(a)$ is independent of a.

57. (a) Let $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$.

Then $f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2} = -f(x)$

Hence $f(x)$ is an odd function and so $\int_{-1}^1 f(x)dx = 0$.

58. (b) $\int_{1/e}^{e} |\log x|\,dx = \int_{1/e}^1 -\log x\,dx + \int_1^e \log x\,dx$

$= [x - x\log x]_{1/e}^1 + [x\log x - x]_1^e$

$$= (1-0) - \left\{\frac{1}{e} - \frac{1}{e}(-1)\right\} + e - e + 1 = 2 - \frac{2}{e} = 2\left(1 - \frac{1}{e}\right).$$

59. (a) $I = \int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi} d\phi = \int_{\pi/4}^{3\pi/4} \frac{\pi - \phi}{1 + \sin(\pi - \phi)} d\phi$

$$\left\{\because \frac{\pi}{4} + \frac{3\pi}{4} = \pi\right\}$$

$\Rightarrow \; 2I = \int_{\pi/4}^{3\pi/4} \frac{\pi}{1+\sin\phi} d\phi$

On simplification, we get $I = \pi(\sqrt{2} - 1) = \pi \tan\frac{\pi}{8}$.

60. (d) $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$ …..(i)

Using the property $I = \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

i.e., change in $x = (2+3-x) = 5-x$ or $dx = -dx$

$\therefore I = \int_3^2 \frac{\sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}}(-dx) = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx$ …..(ii)

Adding (i) and (ii), $2I = \int_2^3 \frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx = \int_2^3 1 dx$

$$= [x]_2^3 = 3 - 2 = 1 \Rightarrow I = \frac{1}{2}.$$

61. (c) Let $I = \int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$.

Then, $I = \int_0^{\pi/2} \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right) dx$,

$\left[\because \int_0^{\pi/2} f(x)dx = \int_0^{\pi/2} f\left(\frac{\pi}{2} - x\right)dx\right]$

$\Rightarrow \; I = -\int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx = -I$

$\Rightarrow \; 2I = 0 \Rightarrow I = 0$.

62. (b) $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx = \int_0^1 \tan^{-1}\left(\frac{x + (x-1)}{1 - x(x-1)}\right) dx$

$I = \int_0^1 (\tan^{-1} x + \tan^{-1}(x-1)) dx$

$I = \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(x-1) dx$

$I = \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(1 - x - 1) dx$,

{Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ in second integral}

$$I=\int_0^1 \tan^{-1} x\,dx+\int_0^1 \tan^{-1}(-x)dx$$

$$I=\int_0^1 \tan^{-1} x\,dx-\int_0^1 \tan^{-1} x\,dx=0 .$$

63. (c) $I=\int_{-1}^1 \frac{\sin x-x^2}{3-|x|}dx=\int_{-1}^1 \frac{\sin x}{3-|x|}dx-\int_{-1}^1 \frac{x^2}{3-|x|}dx$

Here, $f(x)=\frac{\sin x}{3-|x|}$ is an odd function but $f(x)=\frac{x^2}{3-|x|}$ is an even function

$$\therefore I=-\int_{-1}^1 \frac{x^2}{3-|x|}dx=-2\int_0^1 \frac{x^2}{3-|x|}dx=2\int_0^1 \frac{-x^2}{3-|x|}dx .$$

64. (b) It is a fundamental property.

65. (c) $I=\int_0^{\pi/2}\frac{\sin x.dx}{\sin x+\cos x}=\int_0^{\pi/2}\frac{\cos x.dx}{\cos x+\sin x}$,

$\left(\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$$2I=\int_0^{\pi/2}dx \Rightarrow I=\frac{\pi}{4} .$$

66. (a) $I=\int_0^{\pi/2}\frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx$ .....(i)

$$I=\int_0^{\pi/2}\frac{2^{\sin\left(\frac{\pi}{2}-x\right)}}{2^{\sin\left(\frac{\pi}{2}-x\right)}+2^{\cos\left(\frac{\pi}{2}-x\right)}}dx=\int_0^{\pi/2}\frac{2^{\cos x}}{2^{\cos x}+2^{\sin x}}dx \quad \text{.....(ii)}$$

Adding equations (i) and (ii), we get

$$2I=\int_0^{\pi/2}\left(\frac{2^{\sin x}+2^{\cos x}}{2^{\sin x}+2^{\cos x}}\right)dx=\int_0^{\pi/2}1\,dx=[x]_0^{\pi/2}=\frac{\pi}{2}$$

Therefore, $I=\frac{\pi}{4}$.

67. (b) $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx=\int_{e^{-1}}^{1}\left|\frac{\log_e x}{x}\right|dx+\int_1^{e^2}\left|\frac{\log_e x}{x}\right|dx$

$$=\int_{e^{-1}}^1-\frac{\log x}{x}dx+\int_1^{e^2}\frac{\log x}{x}dx=\int_{-1}^0-zdz+\int_0^2 zdz ,$$

(Putting $\log_e x=z \Longrightarrow (1/x)dx=dz$)

$$=\left[-\frac{z^2}{2}\right]_{-1}^0+\left[\frac{z^2}{2}\right]_0^2=\frac{1}{2}+2=\frac{5}{2} .$$

68. (c) $I=\int_0^{\sqrt{2}}[x^2]\,dx=\int_0^1[x^2]dx+\int_1^{\sqrt{2}}[x^2]\,dx$

$$=\int_0^1 0\,dx+\int_1^{\sqrt{2}}dx=[x]_1^{\sqrt{2}}=\sqrt{2}-1 .$$

69. (b) $\int_0^{\pi} 2\sin x\, dx + \int_{\pi}^{2\pi} 0.\, dx = 2[-\cos x]_0^{\pi} + 0$

$= -2(\cos\pi - \cos 0) = -2(-1-1) = 4$.

70. (a) Let $f(x) = \log(x + \sqrt{1+x^2})$

Now, $f(-x) = \log\left(\sqrt{1+x^2} - x\right) = \log(\sqrt{1+x^2} - x).\frac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}$

$= \log\frac{[(1+x^2)-x^2]}{(\sqrt{1+x^2}+x)} = \log 1 - \log(\sqrt{1+x^2}+x)$

$= -\log(\sqrt{1+x^2}+x) = -f(x)$

Hence, $\int_{-1}^{1} \log(x + \sqrt{1+x^2}) = 0$,

$\left[\because \int_{-a}^{a} f(x) = 0, \text{if } f(-x) = -f(x)\right]$.

71. (b) $I = \int_1^5 [|x-3|]\, dx \Rightarrow I = \int_1^3 [-(x-3)]dx + \int_3^5 [(x-3)]dx$

$\Rightarrow I = \int_1^2 [-(x-3)]dx + \int_2^3 [-(x-3)]dx + \int_3^4 [x-3]\,dx + \int_4^5 [x-3]\,dx$

$\Rightarrow I = \int_1^2 dx + \int_2^3 0\,dx + \int_3^4 0\,dx + \int_4^5 dx = [x]_1^2 + [x]_4^5$

$\Rightarrow I = (2-1) + (5-4) = 2$.

72. (b) $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx = \int_{e^{-1}}^{1} \left|\frac{\log_e x}{x}\right| dx + \int_1^{e^2} \left|\frac{\log_e x}{x}\right| dx$

$= \int_{e^{-1}}^{1} -\frac{\log x}{x}\,dx + \int_1^{e^2} \frac{\log x}{x}\,dx = \int_{-1}^{0} -zdz + \int_0^2 zdz$,

(Putting $\log_e x = z \Rightarrow (1/x)dx = dz$)

$= \left[-\frac{z^2}{2}\right]_{-1}^{0} + \left[\frac{z^2}{2}\right]_0^2 = \frac{1}{2} + 2 = \frac{5}{2}$.

73. (d) Given $f(-x) = -f(x)$

We know that, $\int_{-a}^{a} f(x)dx = 0 = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$

$\Rightarrow \int_{-1}^{0} f(x)\,dx + \int_0^1 f(x)\,dx = 0 \Rightarrow \int_{-1}^{0} f(x)dx = -5$

$\Rightarrow \int_{-1}^{0} f(t)\,dt = -5$.

74. (c) It is a fundamental property.

75. (d) $\int_{-2}^{2}|[x]|\,dx = \int_{-2}^{-1}|[x]|\,dx + \int_{-1}^{0}|[x]|\,dx + \int_{0}^{1}|[x]|\,dx| + \int_{1}^{2}|[x]|\,dx$

$$= \int_{-2}^{-1} 2dx + \int_{-1}^{0} 1dx + \int_{0}^{1} 0\,dx + \int_{1}^{2} 1dx$$

$$= 2[x]_{-2}^{-1} + [x]_{-1}^{0} + 0 + [x]_{1}^{2}$$

$$= 2(-1+2)+(0+1)+(2-1) = 2+1+1 = 4.$$

76. (c) $e^{x-[x]}$ is a periodic function with period 1.

$\therefore \int_{0}^{1000} e^{x-[x]}dx = 1000\int_{0}^{1} e^{x-[x]}dx$,

$[\because [x]=0, \text{ if } 0<x<1]$

$$= 1000\ [e^{x}]_{0}^{1} = 1000\,(e-1).$$

77. (d) $\int_{0}^{1}\tan^{-1}\left(\frac{1}{x^2-x+1}\right)dx = \int_{0}^{1}\tan^{-1}x\,dx - \int_{0}^{1}\tan^{-1}(x-1)\,dx$

$$= 2\int_{0}^{1}\tan^{-1}x\,dx = 2[\tan^{-1}x - \frac{1}{2}\log(1+x^2)]_{0}^{1} = \frac{\pi}{2} - \log 2.$$

78. (c) Since $\log\left(\frac{1+x}{1-x}\right)$ is an odd function

$\therefore \int_{-2}^{2}\left\{p\log\left(\frac{1+x}{1-x}\right) + q\log\left(\frac{1-x}{1+x}\right)^{-2} + r\right\}dx$

$= r\int_{-2}^{2} dx = 4r.$ Hence depends on the value of r.

79. (d) Since, f is continuous function. Let $x=t-1$

$\therefore dx = dt$. When $x=-3\to 5$, then $t=-2\to 6$

Therefore, $\int_{-3}^{5} f(x)dx = \int_{-2}^{6} f(t-1)dt = \int_{-2}^{6} f(x-1)dx$.

80. (b) Let $x+1=t$ when $x=-2\to 0$, then $t=-1\to 1$

$I = \int_{-1}^{1}(t^3+2+t\cos t)\,dt$

Since t^3 and $t\cos t$ are odd functions

$\therefore I = \int_{-1}^{1} 2dt = [2t]_{-1}^{1} = 4$.

81. (b) Let $I = \int_{0}^{\pi} xf(\sin x)dx = A\int_{0}^{\pi/2} f(\sin x)dx$

Now, $2I = \int_{0}^{\pi} xf(\sin x)dx + \int_{0}^{\pi}(\pi - x)f[\sin(\pi - x)]dx$

$$= \int_0^{\pi} \pi f(\sin x)dx = \pi \int_0^{\pi} f(\sin x)dx$$

$$\Rightarrow 2I = 2\pi \int_0^{\pi/2} f(\sin x)dx$$

$\therefore I = \pi \int_0^{\pi/2} f(\sin x)dx = A\int_0^{\pi} f(\sin x)dx$. Hence $A = \pi$.

82. (a) $g(x+\pi) = \int_0^{x+\pi} \cos^4 t\,dt = \int_0^{\pi} \cos^4 t\,dt + \int_{\pi}^{x+\pi} \cos^4 t\,dt$

$$= g(\pi) + f(x)$$

$f(x) = \int_0^{x} \cos^4 u\,du = g(x)$, $(\because t = \pi + u)$

$\therefore g(x+\pi) = g(x) + g(\pi)$.

83. (a) Let $I = \int_0^{\pi/2} \left(\frac{\theta}{\sin\theta}\right)^2 d\theta = [-\theta^2 \cot\theta]_0^{\pi/2} + \int_0^{\pi/2} 2\theta.\cot\theta.d\theta$

$$= 2[\theta.\log\sin\theta]_0^{\pi/2} - 2\int_0^{\pi/2} \log\sin\theta\,d\theta$$

$$\Rightarrow \frac{I}{2} = 0 - \lim_{\theta\to 0} \theta\log.\sin\theta - \int_0^{\pi/2} \log\sin\theta\,d\theta$$

$\Rightarrow \frac{\pi}{2}\log 2$. Hence $I = \pi\log 2$.

84. (c) $\int_0^3 (3ax^2 + 2bx + c)dx = \int_1^3 (3ax^2 + 2bx + c)dx$

$$\Rightarrow \int_0^1 (3ax^2 + 2bx + c)dx + \int_1^3 (3ax^2 + 2bx + c)dx$$

$$= \int_1^3 (3ax^2 + 2bx + c)dx$$

$$\Rightarrow \int_0^1 (3ax^2 + 2bx + c)dx = 0$$

$$\Rightarrow \left[\frac{3ax^3}{3} + \frac{2bx^2}{2} + cx\right]_0^1 = 0 \Rightarrow a + b + c = 0.$$

85. (c) Given function $L(x) = \int_1^x \frac{1}{t}dt = [\log t]_1^x = \log x - \log 1$

$\Rightarrow L(x) = \log x$, Hence $L(xy) = L(x) + L(y)$.

86. (d) $\int_{-2}^{3} |1 - x^2|\,dx = \int_{-2}^{-1} (x^2 - 1)dx + \int_{-1}^{1} (1 - x^2)dx + \int_1^3 (x^2 - 1)dx$

$$= \left[\frac{x^2}{3} - x\right]_{-2}^{-1} + \left[x - \frac{x^2}{3}\right]_{-1}^{1} + \left[\frac{x^2}{3} - x\right]_1^2$$

$$= \frac{2}{3} + \frac{2}{3} + 2\left(\frac{2}{3}\right) + (9-3) - \left(\frac{1}{3} - 1\right) = \frac{10}{3} + 6 = \frac{28}{3}.$$

87. (a) On differentiating both sides, we get

$$-\sin^2 x\ f(\sin x)\cos x = -\cos x$$

$\Rightarrow f(\sin x) = \frac{1}{\sin^2 x} \Rightarrow f(x) = \frac{1}{x^2} \Rightarrow f\left(\frac{1}{\sqrt{3}}\right) = 3$.

88. (c) Let $I = \int_0^1 f(k-1+x)dx$

$\Rightarrow I = \int_{k-1}^{k} f(t)\,dt$, Where $t = k-1+x \Rightarrow I = \int_{k-1}^{k} f(x)dx$

$\therefore \sum_{k=1}^{n} \int_{k-1}^{k} f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx + ... + \int_{n-1}^{n} f(x)dx$

$= \int_0^n f(x)\,dx$.

89. (c) $\int_0^\infty \frac{xdx}{(1+x)(1+x^2)} = \int_0^\infty \frac{-\frac{1}{2}dx}{(1+x)} + \int_0^\infty \frac{\left(\frac{1}{2}x + \frac{1}{2}\right)}{1+x^2}dx$

$= \left[\frac{-1}{2}\log(1+x)\right]_0^\infty + \frac{1}{2}\times\frac{1}{2}[\log(1+x^2)]_0^\infty + \frac{1}{2}[\tan^{-1} x]_0^\infty$

$= 0 + 0 + \frac{1}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{4}$.

90. (b) $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x(\sin x + \cos x)dx$

$= \int_{-\pi/2}^{\pi/2} \sin^3 x \cos^2 xdx + \int_{-\pi/2}^{\pi/2} \sin^2 x \cos^3 x\,dx$

$= 0 + 2\int_0^{\pi/2} \sin^2 x \cos^3 xdx = 0 + 2\times\frac{2}{15} = \frac{4}{15}$

91. (c) $F'(x) = |x| > 0 \forall x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Hence the function is increasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and therefore $F(x)$ has maxima at the right end point of $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

$\Rightarrow$ Max $F(x) = F\left(\frac{1}{2}\right) = \int_1^{1/2} |t|\,dt = -\frac{3}{8}$.

92. (c) $\int_{-1}^{1} \frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)dx = -2[\tan^{-1}(x)]_0^1 = -\frac{\pi}{2}$.

93. (a) $I = \int_0^\infty \frac{xdx}{(1+x)(1+x^2)}$

Put $x = \tan\theta$, we get

$I = \int_0^{\pi/2} \frac{\tan\theta}{1+\tan\theta}d\theta = \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta + \sin\theta}d\theta = \frac{\pi}{4}$.

94. (a) Let $F_1(x) = y_1 = \int_2^x (2t-5)dt$ and $F_2(x) = y_2 = \int_0^x 2t\,dt$

Now point of intersection means those point at which $y_1 = y_2 = y \Rightarrow y_1 = x^2 - 5x + 6$ and $y_2 = x^2$.

On solving, we get $x^2 = x^2 - 5x + 6 \Rightarrow x = \frac{6}{5}$ and $y = x^2 = \frac{36}{25}$. Thus point of intersection is $\left(\frac{6}{5}, \frac{36}{25}\right)$.

95. (a) $I = \int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx$

Put $x = \tan\theta \implies dx = \sec^2\theta \, d\theta$

$$\therefore I = \int_0^{\pi/2} \frac{\tan\theta \log(\tan\theta)}{\sec^4\theta} \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/2} \sin\theta \cos\theta \log(\tan\theta) d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \sin 2\theta \log(\tan\theta) d\theta = 0,$$

$$\left\{\because \int_0^{\pi/2} \sin 2\theta \ \log \ \tan\theta \ d\theta = 0\right\}.$$

96. (c) $I = \left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2}$.

97. (b) We have $f(x) = \int_{x^2}^{x^4} \sin\sqrt{t} \, dt$

$$\therefore f'(x) = \frac{d}{dx}(x^4)(\sin\sqrt{x^4}) - \frac{d}{dx}(x^2)(\sin\sqrt{x^2})$$

$$= 4x^3 \sin x^2 - 2x \sin x.$$

98. (a) Putting $x = \tan\theta$, we get $\int_0^{\infty} \frac{dx}{\left(x + \sqrt{x^2+1}\right)^3}$

$$= \int_0^{\pi/2} \frac{\sec^2\theta \, d\theta}{(\tan\theta + \sec\theta)^3} = \int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)^3} d\theta$$

$$= \left[-\frac{1}{2(1+\sin\theta)^2}\right]_0^{\pi/2} = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}.$$

99. (b) Put $x = a\sin\theta \Rightarrow dx = a\cos\theta \, d\theta$

Now $\int_0^a x^4\sqrt{a^2 - x^2} dx = a^6 \int_0^{\pi/2} \sin^4\theta \cos\theta \cos\theta \, d\theta$

$$= a^6 \int_0^{\pi/2} \sin^4\theta \cos^2\theta \, d\theta$$

100. (d) $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x} \Rightarrow \int_1^4 \frac{3}{x} e^{\sin x^3} dx = \int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$$F(t) = \int_1^{64} \frac{e^{\sin t}}{t} dt = \int_1^{64} F(t) dt = F(64) - F(1),$$

On comparing, $k = 64$.

101. (a) $I=\int_0^a x^2(a^2-x^2)^{3/2}dx$

Put $x=a\sin\theta \Rightarrow dx=a\cos\theta\,d\theta$

$$I=\int_0^{\pi/2} a^2\sin^2\theta . a^3\cos^3\theta . a\cos\theta\,d\theta$$

$$=a^6\int_0^{\pi/2}\sin^2\theta\cos^4\theta\,d\theta=a^6\frac{\Gamma\frac{3}{2}.\Gamma\frac{5}{2}}{2.\Gamma\frac{8}{2}}$$

$$=a^6\frac{\frac{1}{2}.\sqrt{\pi}.\frac{3}{2}.\frac{1}{2}.\sqrt{\pi}}{2.3.2.1}=\frac{\pi a^6}{32}.$$

102. (b) We have, $\lim_{n\to\infty}\left[\frac{n}{1+n^2}+\frac{n}{4+n^2}+......+\frac{1}{2n}\right]$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{r^2+n^2}=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2\left(1+\frac{r^2}{n^2}\right)}$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)}=\int_0^1\frac{dx}{1+x^2},$$

103. (b) $\lim_{n\to\infty}\frac{1^{99}+2^{99}+.....+n^{99}}{n^{100}}=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r^{99}}{n^{100}}\right)$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{99}=\int_0^1 x^{99}dx=\left[\frac{x^{100}}{100}\right]_0^1=\frac{1}{100}.$$

104. (a) $\lim_{n\to\infty}\frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}=\lim_{n\to\infty}\sum_{r=1}^{n}\left[\frac{r^p}{n^{p+1}}\right]$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^p=\int_0^1 x^p dx=\left[\frac{x^{p+1}}{p+1}\right]_0^1=\frac{1}{p+1}.$$

105. (b) Let $P=\lim_{x\to\infty}\left(\frac{n!}{n^n}\right)^{1/n}=\lim_{n\to\infty}\left(\frac{1}{n}.\frac{2}{n}.\frac{3}{n}.\frac{4}{n}..........\frac{n}{n}\right)^{1/n}$

$$\therefore \log P=\frac{1}{n}\lim_{n\to\infty}\left(\log\frac{1}{n}+\log\frac{2}{n}+......+\log\frac{n}{n}\right)$$

$$\log P=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\log\frac{r}{n}$$

$$\log P=\int_0^1\log x\,dx=(x\log x-x)_0^1=(-1)\Rightarrow P=\frac{1}{e}$$

106. (b) Let $I=\int_0^\infty\frac{\log(1+x^2)}{1+x^2}dx$

Put $x=\tan\theta\Rightarrow dx=\sec^2\theta\,d\theta,$

$\therefore \quad I = \int_0^{\pi/2} \log(\sec\theta)^2 d\theta = 2\int_0^{\pi/2} \log\sec\theta \; d\theta$

$= -2\int_0^{\pi/2} \log\cos\theta \;\; d\theta = -2.\,\frac{\pi}{2}\log\frac{1}{2} = -\pi\log\frac{1}{2} = \pi\log 2$.

107. (b) $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \frac{3}{n^2}\sec^2\frac{9}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ is equal to $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{n^2}\sec^2\frac{r^2}{n^2} = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{n}\sec^2\frac{r^2}{n^2}$

Given limit is equal to the value of integral $\int_0^1 x\sec^2 x^2 dx$

$= \frac{1}{2}\int_0^1 2x\sec^2 x^2 dx = \frac{1}{2}\int_0^1 \sec^2 t\, dt$, [Put $x^2 = t$]

$= \frac{1}{2}[\tan t]_0^1 = \frac{1}{2}\tan 1$.

108. (b) $y = \lim_{n\to\infty}\left[\frac{1}{n} + \frac{1}{\sqrt{n^2+n}} + ... + \frac{1}{\sqrt{n^2+(n-1)n}}\right]$

$\Rightarrow \quad y = \lim_{n\to\infty}\left[\frac{1}{n} + \frac{1}{n\sqrt{1+\frac{1}{n}}} + ... + \frac{1}{n\sqrt{1+\frac{(n-1)}{n}}}\right]$

$\Rightarrow \quad y = \frac{1}{n}\lim_{n\to\infty}\left[1 + \frac{1}{\sqrt{1+\frac{1}{n}}} + ... + \frac{1}{\sqrt{1+\frac{(n-1)}{n}}}\right]$

$y = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\sqrt{1+\frac{(k-1)}{n}}}$, Put $\frac{k-1}{n} = x$ and $\frac{1}{n} = dx$

$\Rightarrow \quad y = \lim_{n\to\infty}\int_0^{\frac{n-1}{n}}\frac{dx}{\sqrt{1+x}} = \lim_{n\to\infty} 2\left[\sqrt{1+x}\right]_0^{\left(\frac{n-1}{n}\right)}$

$\Rightarrow \quad y = 2\lim_{n\to\infty}\left[\sqrt{\frac{2n-1}{n}} - 1\right] = 2\lim_{n\to\infty}\sqrt{\frac{2n-1}{n}} - 2$

$\Rightarrow \quad y = 2\lim_{n\to\infty}\sqrt{2-\frac{1}{n}} - 2 = 2\sqrt{2} - 2$.

109. (b) $L = \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{n}.\frac{r/n}{\sqrt{1+(r/n)^2}} = \int_0^2\frac{x}{\sqrt{1+x^2}}dx = \sqrt{5} - 1$.

110. (d) Let $I(b) = \int_0^1\frac{x^b - 1}{\log x}dx \Rightarrow I'(b) = \int_0^1\frac{x^b\log x}{\log x}dx$

(If $I(\alpha) = \int_0^b f(x,\alpha)dx$, then $I'(\alpha) = \int_0^b f'(x,\alpha)dx$, where $f'(x,\alpha)$ is derivative of $f(x,\alpha)$ *w.r.t.* α keeping x constant)

$I'(b) = \int_0^1 x^b dx = \frac{1}{b+1}$

$\Rightarrow I(b) = \int \frac{db}{b+1} + c = \log(b+1) + c$

If $b = 0$, then $I(b) = 0$, so $c = 0 \Rightarrow I(b) = \log(b+1)$.

111. (d) $\lim_{n\to\infty}\left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n}\right]$

$= \lim_{n\to\infty}\left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}\right]$

$= \frac{1}{n}\lim_{n\to\infty}\left[1 + \frac{1}{1+\frac{1}{n}} + \frac{1}{1+\frac{2}{n}} + ... + \frac{1}{1+\frac{n}{n}}\right]$

$= \frac{1}{n}\lim_{n\to\infty}\sum_{r=0}^{n}\left[\frac{1}{1+\frac{r}{n}}\right] = \int_0^1 \frac{1}{1+x}\,dx$