CONTINUITY

OBJECTIVES

1. If $f(x)=\begin{cases}(1+2x)^{1/x}, \text{for } x\neq 0\\ e^2, \text{for } x=0\end{cases}$ **, then**

(a) $\lim_{x\to 0+} f(x)=e$ (b) $\lim_{x\to 0-} f(x)=e^2$

(c) $f(x)$ **is discontinuous at** $x=0$ **(d)** None of these

2. Let $f(x)=\begin{cases}x^2+k, & \text{when } x\geq 0\\ -x^2-k, & \text{when } x<0\end{cases}$**. If the function** $f(x)$ **be continuous at** $x=0$**, then *k* =**

(a) 0 (b) 1

(c) 2 (d) –2

3. Which of the following statements is true for graph $f(x)=\log x$

(a) Graph shows that function is continuous

(b) Graph shows that function is discontinuous

(c) Graph finds for negative and positive values of *x*

(d) Graph is symmetric along *x*-axis

4. If the function $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, \text{when } x\neq\dfrac{\pi}{2}\\ 3, \quad \text{when } x=\dfrac{\pi}{2}\end{cases}$ **be continuous at** $x=\dfrac{\pi}{2}$**, then *k* =**

(a) 3 (b) 6

(c) 12 (d) None of these

5. If $f(x)=\begin{cases}x^2\sin\dfrac{1}{x}, & \text{when } x\neq 0\\ 0, & \text{when } x=0\end{cases}$**, then**

(a) $f(0+0)=1$ (b) $f(0-0)=1$

(c) f is continuous at $x=0$ (d) None of these

6. If $f(x)=\begin{cases}\dfrac{|x-a|}{x-a}, \text{when } x\neq a\\ 1, \text{when } x=a\end{cases}$ **,then**

(a) $f(x)$ is continuous at $x=a$ (b) $f(x)$ is discontinuous at $x=a$

(c) $\lim_{x\to a} f(x)=1$ (d) None of these

7. The points at which the function $f(x)=\frac{x+1}{x^2+x-12}$ **is discontinuous, are**

(a) –3, 4 (b) 3, –4

(c) –1,–3, 4 (d) –1, 3, 4

8. The function $f(x)=\frac{\log(1+ax)-\log(1-bx)}{x}$ **is not defined at** $x=0$**. The value which should be assigned to *f* at *x* =0 so that it is continuous at** $x=0$**, is**

(a) $a-b$ (b) $a+b$

(c) $\log a+\log b$ (d) $\log a-\log b$

9. At which points the function $f(x)=\frac{x}{[x]}$**, where** $[.]$ **is greatest integer function, is discontinuous**

(a) Only positive integers

(b) All positive and negative integers and (0, 1)

(c) All rational numbers

(d) None of these

10. Let $f(x)=\begin{cases}\frac{x^3+x^2-16x+20}{(x-2)^2}, \text{if } x\neq 2\\ k, \text{if } x=2\end{cases}$**. If** $f(x)$ **be continuous for all *x*, then *k* =**

(a) 7 (b) –7

(c) ± 7 (d) None of these

11. If $f(x)=\begin{cases}-x^2, \text{when } x\le 0\\ 5x-4, \text{when } 0<x\le 1\\ 4x^2-3x, \text{when } 1<x<2\\ 3x+4, \text{when } x\ge 2\end{cases}$**, then**

(a) $f(x)$ is continuous at $x=0$

(b) $f(x)$ is continuous $x=2$

(c) $f(x)$ is discontinuous at $x=1$

(d) None of these

12. If $f(x)=\begin{cases}\frac{5}{2}-x, \text{when } x<2\\ 1 \quad, \text{when } x=2\\ x-\frac{3}{2}, \text{when } x>2\end{cases}$**, then**

(a) $f(x)$ is continuous at $x=2$ (b) $f(x)$ is discontinuous at $x=2$

(c) $\lim_{x\to 2} f(x)=1$ (d) None of these

13. For the function $f(x)=\begin{cases}\dfrac{\sin^2 ax}{x^2}, \text{when } x\neq 0\\ 1, \text{when } x=0\end{cases}$ **which one is a true statement**

(a) $f(x)$ is continuous at $x=0$

(b) $f(x)$ is discontinuous at $x=0$, when $a\neq \pm 1$

(c) $f(x)$ is continuous at $x=a$

(d) None of these

14. If $f(x)=\begin{cases}\dfrac{\sin x}{x}+\cos x, \text{ when } x\neq 0\\ 2, \text{when } x=0\end{cases}$ **then**

(a) $\lim\limits_{x\to 0+} f(x)\neq 2$ (b) $\lim\limits_{x\to 0-} f(x)=0$

(c) $f(x)$ is continuous at $x=0$ (d) None of these

15. If function $f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ 1-x, & \text{if } x \text{ is irrational}\end{cases}$, **then** $f(x)$ **is continuous at ...... number of points**

(a) ∞ (b) 1

(c) 0 (d) None of these

16. Let $f(x)=\begin{cases}\dfrac{x^4-5x^2+4}{|(x-1)(x-2)|}, & x\neq 1,2\\ 6, & x=1\\ 12, & x=2\end{cases}$

Then $f(x)$ is continuous on the set

(a) R (b) $R-\{1\}$

(c) $R-\{2\}$ (d) $R-\{1,2\}$

17. If $f(x)=\begin{cases}x\sin x, \text{when } 0<x\leq\dfrac{\pi}{2}\\ \dfrac{\pi}{2}\sin(\pi+x), \text{when } \dfrac{\pi}{2}<x<\pi\end{cases}$**, then**

(a) $f(x)$ is discontinuous at $x=\pi/2$

(b) $f(x)$ is continuous at $x=\pi/2$

(c) $f(x)$ is continuous at $x=0$

(d) None of these

18. If $f(x)=\begin{cases}\sin x,\ x\neq n\pi,\ n\in Z\\ 2,\text{otherwise}\end{cases}$ **and** $g(x)=\begin{cases}x^2+1,\ x\neq 0,2\\ 4,\ x=0\\ 5,\ x=2\end{cases}$, **then** $\lim_{x\to 0} g\{f(x)\}$ **is**

(a) 5 (b) 6

(c) 7 (d) 1

19. If $f(x)=\begin{cases}\dfrac{x^4-16}{x-2},\ \text{when } x\neq 2\\ 16,\ \text{when } x=2\end{cases}$**, then**

(a) $f(x)$ is continuous at $x=2$

(b) $f(x)$ is discontinuous at $x=2$

(c) $\lim_{x\to 2} f(x)=16$

(d) None of these

20. If $f(x)=\dfrac{x^2-10x+25}{x^2-7x+10}$ **for** $x\neq$ **5 and *f* is continuous at** $x=5$, **then** $f(5)=$

(a) 0 (b) 5

(c) 10 (d) 25

21. If $f(x)=|x|$**, then** $f(x)$ **is**

(a) Continuous for all x

(b) Differentiable at $x=0$

(c) Neither continuous nor differentiable at $x=0$

(d) None of these

22. If $f(x)=\begin{cases}\dfrac{x-|x|}{x},\ \text{when } x\neq 0\\ 2,\ \text{when } x=0\end{cases}$**, then**

(a) $f(x)$ is continuous at $x=0$

(b) $f(x)$ is discontinuous at $x=0$

(c) $\lim_{x\to 0} f(x)=2$

(d) None of these

23. **If** $f(x)=\begin{cases}\dfrac{\sin[x]}{[x]+1}, & \text{for } x>0\\ \dfrac{\cos\frac{\pi}{2}[x]}{[x]}, & \text{for } x<0\\ k, & \text{at } x=0\end{cases}$ **; where [*x*] denotes the greatest integer less than or equal to *x*, then in order that *f* be continuous at** $x=0$**, the value of *k* is**

(a) Equal to 0 (b) Equal to 1

(c) Equal to –1 (d) Indeterminate

24. **If the function** $f(x)=\begin{cases}(\cos x)^{1/x}, & x\neq 0\\ k, & x=0\end{cases}$ **is continuous at** $x=0$**, then the value of *k* is**

(a) 1 (b) –1

(c) 0 (d) *e*

25. **If the function** $f(x)=\begin{cases}1+\sin\dfrac{\pi x}{2}, & \text{for } -\infty<x\leq 1\\ ax+b, & \text{for } 1<x<3\\ 6\tan\dfrac{x\pi}{12}, & \text{for } 3\leq x<6\end{cases}$ **is continuous in the interval** $(-\infty, 6)$**, then the values of *a* and *b* are respectively**

(a) 0, 2 (b) 1, 1

(c) 2, 0 (d) 2, 1

26. **The values of *A* and *B* such that the function** $f(x)=\begin{cases}-2\sin x, & x\leq -\dfrac{\pi}{2}\\ A\sin x+B, & -\dfrac{\pi}{2}<x<\dfrac{\pi}{2}\\ \cos x, & x\geq \dfrac{\pi}{2}\end{cases}$**, is continuous everywhere are**

(a) $A=0, B=1$ (b) $A=1, B=1$

(c) $A=-1, B=1$ (d) $A=-1, B=0$

27. **The function** $f(x)=\sin|x|$ **is**

(a) Continuous for all *x*

(b) Continuous only at certain points

(c) Differentiable at all points

(d) None of these

28. If $f(x)=\begin{cases} e^x; & x \le 0 \\ |1-x|; & x>0 \end{cases}$, then

(a) $f(x)$ is not differentiable at $x=0$

(b) $f(x)$ is continuous at $x=0$

(c) $f(x)$ is differentiable at $x=1$

(d) $f(x)$ is continuous at $x=1$

29. For the function $f(x)=\begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1}, & x \ne 0 \\ 0 & , x=0 \end{cases}$, which of the following is correct

(a) $\lim_{x \to 0} f(x)$ does not exist

(b) $f(x)$ is continuous at $x=0$

(c) $\lim_{x \to 0} f(x)=1$

(d) $\lim_{x \to 0} f(x)$ exists but $f(x)$ is not continuous at $x=0$

30. The function $f(x)=|x|+\dfrac{|x|}{x}$ is

(a) Continuous at the origin

(b) Discontinuous at the origin because $|x|$ is discontinuous there

(c) Discontinuous at the origin because $\dfrac{|x|}{x}$ is discontinuous there

(d) Discontinuous at the origin because both $|x|$ and $\dfrac{|x|}{x}$ are discontinuous there

31. If $f(x)=\begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & ,\text{for } -1 \le x < 0 \\ 2x^2+3x-2 & ,\text{for } 0 \le x \le 1 \end{cases}$, is continuous at $x=0$, then $k=$

(a) – 4 (b) – 3

(c) – 2 (d) – 1

32. If $f(x)=\begin{cases} \dfrac{1-(x)}{1+x}, & x \ne -1 \\ 1 & , x=-1 \end{cases}$, then the value of $f(|2k|)$ will be (where [ ] shows the greatest integer function)

(a) Continuous at $x=-1$ (b) Continuous at $x=0$

(c) Discontinuous at $x=\frac{1}{2}$ (d) All of these

33. For the function $f(x)=\dfrac{\log_e(1+x)-\log_e(1-x)}{x}$ to be continuous at $x=0$, the value of $f(0)$, should be

(a) –1 (b) 0 (c) –2 (d) 2

34. In the function $f(x)=\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x},(x\neq 0)$ is continuous at each point of its domain, then the value of $f(0)$ is

(a) 2 (b) $1/3$

(c) $2/3$ (d) $-1/3$

CONTINUITY

HINTS AND SOLUTIONS

1. (b) $\lim_{x\to 0-} f(x)=\lim_{x\to 0}\left[(1+2x)^{1/2x}\right]^2=e^2$.

2. (a) $\lim_{x\to 0+} f(x)=k,\ \lim_{x\to 0-} f(x)=-k$ and $f(0)=k$

$f(0)=0=k$

3. (a) concept

4. (b) $f(\pi/2)=3$. Since $f(x)$ is continuous at $x=\pi/2$

$$\Rightarrow \lim_{x\to\pi/2}\left(\frac{k\cos x}{\pi-2x}\right)=f\left(\frac{\pi}{2}\right)\Rightarrow\frac{k}{2}=3\Rightarrow k=6.$$

5. (c) $\lim_{x\to 0^+} f(x)=x^2\sin\frac{1}{x}$, but $-1\leq\sin\frac{1}{x}\leq 1$ and $x\to 0$

Therefore, $\lim_{x\to 0^+} f(x)=0=\lim_{x\to 0^-} f(x)=f(0)$

Hence $f(x)$ is continuous at $x=0$.

6. (b) $\lim_{x\to a-} f(x)=-1,\ \lim_{x\to a+} f(x)=1,\ f(a)=1.$

7. (b) $f(x)=\frac{x+1}{(x-3)(x+4)}$. Hence the points are 3, – 4.

8. (b) Since limit of a function is $a+b$ as $x\to 0$, therefore to be continuous at a function, its value must be

$a+b$ at $x=0$ $\Rightarrow f(0)=a+b.$

9. (b) Standard problem

10. (a) For continuous $\lim_{x\to 2} f(x)=f(2)=k$

$$\Rightarrow k=\lim_{x\to 2}\frac{x^3+x^2-16x+20}{(x-2)^2}$$

11. (b) Standard problem

12. (b) $\lim_{x\to 2-} f(x)=\frac{1}{2}$ and $\lim_{x\to 2+} f(x)=\frac{1}{2}$ and $f(2)=1$.

13. (b) $\lim_{x\to 0} f(x)=\frac{\sin^2 ax}{(ax)^2}a^2=a^2$ and $f(0)=1$.

Hence $f(x)$ is discontinuous at $x=0$, when $a\neq 0$.

14. (c) $f(0+)=f(0-)=2$ and $f(0)=2$

Hence $f(x)$ is continuous at $x=0$.

15. (c) At no point, function is continuous.

16. (d) Check continuity at $x=1, 2$.

17. (a) $\lim_{x\to\infty}\frac{4x}{(\sqrt{x^2+8x+3}+\sqrt{x^2+4x+3}}$ and $f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$.

18. (d) As we are given $f(x)=\sin x$, if $x\neq n\pi$

i.e., $x\neq 0,\pi, 2\pi,....$ = 2 otherwise

$\therefore \lim_{x\to 0^+} g\{f(x)\}=\lim_{x\to 0^+} g\{\sin x\}=\lim_{x\to 0^+}(\sin^2 x+1)=1$

Similarly, $\lim_{x\to 0^-} g\{f(x)\}=1$.

19. (b) $\lim_{x\to 2} f(x)=\lim_{x\to 2}(x+2)(x^2+4)=32,\ f(2)=16$.

20. (a) $f(5)=\lim_{x\to 5} f(x)=\lim_{x\to 5}\frac{x^2-10x+25}{x^2-7x+10}$

$$=\lim_{x\to 5}\frac{(x-5)^2}{(x-2)(x-5)}=\frac{5-5}{5-2}=0.$$

21. (a) $|x|$ is continuous for all x.

22. (b) $\lim_{x\to 0-} f(x)=1+1=2,\ \lim_{x\to 0+} f(x)=0,\ f(0)=2$.

23. (a) $k=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\frac{\cos\frac{\pi}{2}[0-h]}{[0-h]}$

$$k=\lim_{h\to 0}\frac{\cos\frac{\pi}{2}[-h]}{[-h]}=\lim_{h\to 0}\frac{\cos\frac{\pi}{2}[-h-1]}{[-h-1]}$$

$$k=\lim_{h\to 0}\frac{\cos\left(-\frac{\pi}{2}\right)}{-1};\ k=0.$$

24. (a) $\lim_{x\to 0}(\cos x)^{1/x}=k \Rightarrow \lim_{x\to 0}\frac{1}{x}\log(\cos x)=\log k$

$\Rightarrow \lim_{x\to 0}\frac{1}{x}\lim_{x\to 0}\log\cos x=\log k$

$\Rightarrow \lim_{x\to 0}\frac{1}{x}\times 0=\log_e k \Rightarrow k=1$

25. (c) check continuity at $x=1,\ x=3$

26. (c) For continuity at all $x\in R$, we must have $f\left(-\frac{\pi}{2}\right)=\lim_{x\to(-\pi/2)^-}(-2\sin x)=\lim_{x\to(-\pi/2)^+}(A\sin x+B)$

$\Rightarrow 2=-A+B$ .....(i)

and $f\left(\frac{\pi}{2}\right)=\lim_{x\to(\pi/2)^-}(A\sin x+B)=\lim_{x\to(\pi/2)^+}(\cos x)$

$\Rightarrow 0=A+B$ .....(ii)

From (i) and (ii), $A=-1$ and $B=1$.

27. (a) It is obvious.

28. (d) $f(x)=\begin{cases} e^x\ ;\ x\le 0\\ 1-x;\ 0<x\le 1\\ x-1;\ x>1\end{cases}$

Continuous at $x=0$, 1.

29. (d) $f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x\ne 0\\ 0, & x=0\end{cases}$

30. (c) $|x|$ is continuous at $x=0$ and $\frac{|x|}{x}$ is also discontinuous at $x=0$

$\therefore\ f(x)=|x|+\frac{|x|}{x}$ is discontinuous at $x=0$.

31. (c) L.H.L. $=\lim_{x\to 0^-}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}=k$

R.H.L. $=\lim_{x\to 0^+}(2x^2+3x-2)=-2 \Rightarrow k=-2$.

32. (d) $f(x)=\begin{cases}\dfrac{1-|x|}{1+x}, & x\ne -1\\ 1, & x=-1\end{cases}$ and $f(x)=\begin{cases}1, & x<0\\ \dfrac{1-x}{1+x}, & x\ge 0\end{cases}$

$$f(2x)=\begin{cases}1, & x<0\\ \dfrac{1-[2x]}{1+[2x]}, & x>0\end{cases}\Rightarrow f(2x)=\begin{cases}1, & x<0\\ 1, & 0\le x<\frac{1}{2}\\ 0, & \frac{1}{2}\le x\le 1\\ -\frac{1}{3}, & 1\le x<\frac{3}{2}\end{cases}$$

33. (d)

34. (b) $f(0)=\lim_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}=\frac{2-1}{2+1}=\frac{1}{3}$.