AREAS

OBJECTIVES

1. Area bounded by $y = x\sin x$ **and** $x-$**axis between** $x=0$ **and** $x=2\pi,$ **is**

(a) 0 (b) 2π sq. unit (c) π sq. unit (d) 4π sq. unit

2. Area bounded by the parabola $y = 4x^2,$ $y-$**axis and the lines** $y=1, y=4$ **is**

(a) 3 sq. unit (b) $\frac{7}{5}$ sq. unit (c) $\frac{7}{3}$ sq. unit (d) None of these

3.Area bounded by the curve $y = xe^{x^2},$ $x-$axis and the ordinates $x=0, x=a$

(a) $\frac{e^{a^2}+1}{2}$ sq. unit (b) $\frac{e^{a^2}-1}{2}$ sq. unit (c) $e^{a^2}+1$ sq. unit (d) $e^{a^2}-1$ sq. unit

4. Area bounded by curve $y = x^3,$ $x-$**axis and ordinates** $x=1$ **and** $x=4,$ **is**

(a) 64 sq. unit (b) 27 sq. unit (c) $\frac{127}{4}$ sq. unit (d) $\frac{255}{4}$ sq. unit

5.The area of smaller part between the circle $x^2+y^2=4$ **and the line** $x=1$ **is**

(a) $\frac{4\pi}{3}-\sqrt{3}$ (b) $\frac{8\pi}{3}-\sqrt{3}$ (c) $\frac{4\pi}{3}+\sqrt{3}$ (d) $\frac{5\pi}{3}+\sqrt{3}$

6. Area under the curve $y = x^2-4x$ **within the x-axis and the line** $x=2$**, is**

(a) $\frac{16}{3} sq.unit$ (b) $-\frac{16}{3} sq.unit$ (c) $\frac{4}{7} sq.unit$ (d) Cannot be calculated

7. The ratio of the areas bounded by the curves $y=\cos x$ **and** $y = \cos 2x$ **between** $x=0,$ $x=\pi/3$ **and** $x-$**axis, is**

(a) $\sqrt{2}:1$ (b) $1:1$ (c) $1:2$ (d) None of these

8. The area bounded by the parabola $y^2 = 4ax,$ **and two ordinates** $x=4,$ $x=9$ **is**

(a) $4a^2$ (b) $4a^2.4$ (c) $4a^2(9-4)$ (d) $\frac{152\sqrt{a}}{3}$

9. If the ordinate $x=a$ **divides the area bounded by the curve** $y=\left(1+\frac{8}{x^2}\right),$ $x-$**axis and the ordinates** $x=2,$ $x=4$ **into two equal parts, then** $a=$

(a) 8 (b) $2\sqrt{2}$ (c) 2 (d) $\sqrt{2}$

10. Area bounded by the curve $y = \log x$, $x -$ axis and the ordinates $x = 1, x = 2$ is

(a) $\log 4$ sq. unit (b) $(\log 4 + 1)$ sq. unit (c) $(\log 4 - 1)$ sq. unit (d) None of these

11. Area bounded by the lines $y = x, x = -1, x = 2$ and $x -$ axis is

(a) $\frac{5}{2}$ sq. unit (b) $\frac{3}{2}$ sq. unit (c) $\frac{1}{2}$ sq. unit (d) None of these.

12. If the area above the x-axis, bounded by the curves $y = 2^{kx}$ and $x = 0$ and $x = 2$ is $\frac{3}{\ln 2}$, then the value of k is

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 2

13. Area bounded by parabola $y^2 = x$ and straight line $2y = x$ is

(a) $\frac{4}{3}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

14. The area of the region bounded by the $x -$ axis and the curves defined by $y = \tan x, (-\pi/3 \le x \le \pi/3)$ is

(a) $\log\sqrt{2}$ (b) $-\log\sqrt{2}$ (c) $2\log 2$ (d) 0

15. The area bounded by the circle $x^2 + y^2 = 4$, line $x = \sqrt{3}y$ and $x -$ axis lying in the first quadrant, is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) π

16. Area of the region bounded by the curve $y = \tan x$, tangent drawn to the curve at $x = \frac{\pi}{4}$ and the x-axis is

(a) $\frac{1}{4}$ (b) $\log\sqrt{2} + \frac{1}{4}$ (c) $\log\sqrt{2} - \frac{1}{4}$ (d) None of these

17. The area of figure bounded by $y = e^x, y = e^{-x}$ and the straight line $x = 1$ is

(a) $e + \frac{1}{e}$ (b) $e - \frac{1}{e}$ (c) $e + \frac{1}{e} - 2$ (d) $e + \frac{1}{e} + 2$

18. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

19.The area enclosed by the parabolas $y = x^2 - 1$ and $y = 1 - x^2$ is

(a) 1/3 (b) 2/3 (c) 4/3 (d) 8/3

20.The part of straight line $y = x + 1$ between $x = 2$ and $x = 3$ is revolved about x-axis, then the curved surface of the solid thus generated is

(a) $37\pi/3$ (b) $7\pi\sqrt{2}$ (c) 37π (d) $7\pi/\sqrt{2}$

21.The area bounded by $y = -x^2 + 2x + 3$ and $y = 0$ is

(a) 32 (b) $\frac{32}{3}$ (c) $\frac{1}{32}$ (d) $\frac{1}{3}$

22.The area bounded by the curves $y^2 = 8x$ and $y = x$ is

(a) $\frac{128}{3}$ sq. unit (b) $\frac{32}{3}$ sq. unit (c) $\frac{64}{3}$ sq. unit (d) 32 sq. unit

23. The area bounded by curves $y = \cos x$ and $y = \sin x$ and ordinates $x = 0$ and $x = \frac{\pi}{4}$ is

(a) $\sqrt{2}$ (b) $\sqrt{2} + 1$ (c) $\sqrt{2} - 1$ (d) $\sqrt{2}(\sqrt{2} - 1)$

24. Area bounded by the parabola $y^2 = 4ax$ and its latus rectum is

(a) $\frac{2}{3}a^2$ sq. unit (b) $\frac{4}{3}a^2$ sq. unit (c) $\frac{8}{3}a^2$ sq. unit (d) $\frac{3}{8}a^2$ sq. unit

25. The area bounded by the curve $y = 4x - x^2$ and the $x-$axis, is

(a) $\frac{30}{7}$ sq. unit (b) $\frac{31}{7}$ sq. unit (c) $\frac{32}{3}$ sq. unit (d) $\frac{34}{3}$ sq. unit

26. The area of the region bounded by the curves $y = x^2$ and $y = |x|$ is

(a) 1/6 (b) 1/3 (c) 5/6 (d) 5/3

27. The area enclosed between the parabolas $y^2 = 4x$ and $x^2 = 4y$ is

(a) $\frac{14}{3}$ sq. unit (b) $\frac{3}{4}$ sq. unit (c) $\frac{3}{16}$ sq. unit (d) $\frac{16}{3}$ sq. unit

28. The area between the parabola $y = x^2$ and the line $y = x$ is

(a) $\frac{1}{6}$ sq. unit (b) $\frac{1}{3}$ sq. unit (c) $\frac{1}{2}$ sq. unit (d) None of these

29. Area included between the two curves $y^2 = 4ax$ and $x^2 = 4ay$,

(a) $\frac{32}{3}a^2$ sq. unit (b) $\frac{16}{3}$ sq. unit (c) $\frac{32}{3}$ sq. unit (d) $\frac{16}{3}a^2$ sq. unit

30. **Area bounded by curves $y = x^2$ and $y = 2 - x^2$ is**

(a) 8/3 (b) 3/8 (c) 3/2 (d) None of these

31. **The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is**

(a) $2:1:2$ (b) $1:1:1$ (c) $1:2:1$ (d) $1:2:3$

32. **The part of circle $x^2 + y^2 = 9$ in between $y = 0$ and $y = 2$ is revolved about y-axis. The volume of generating solid will be**

(a) $\frac{46}{3}\pi$ (b) 12π (c) 16π (d) 28π

33. **The area bounded by the curves $y = \sqrt{x},\ 2y + 3 = x$ and $x-$axis in the 1st quadrant is**

(a) 9 (b) $\frac{27}{4}$ (c) 36 (d) 18

34. **The area of the smaller segment cut off from the circle $x^2 + y^2 = 9$ by $x = 1$ is**

(a) $\frac{1}{2}(9\sec^{-1}3 - \sqrt{8})$ (b) $9\sec^{-1}(3) - \sqrt{8}$ (c) $\sqrt{8} - 9\sec^{-1}(3)$ (d) None of these

35. **The area of region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$ is**

(a) $\frac{\pi^2}{5}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi}{4} - \frac{1}{2}$

36. **Area enclosed by the parabola $ay = 3(a^2 - x^2)$ and x-axis is**

(a) $4a^2$ sq. unit (b) $12a^2$ sq. unit (c) $4a^3$ sq. unit (d) None of these

37. **Area inside the parabola $y^2 = 4ax$, between the lines $x = a$ and $x = 4a$ is equal to**

(a) $4a^2$ (b) $8a^2$ (c) $28\frac{a^2}{3}$ (d) $35\frac{a^2}{3}$

38. **The area of the curve $xy^2 = a^2(a - x)$ bounded by y-axis is**

(a) πa^2 (b) $2\pi a^2$ (c) $3\pi a^2$ (d) $4\pi a^2$

39. **The area formed by triangular shaped region bounded by the curves $y = \sin x, y = \cos x$ and $x = 0$ is**

(a) $\sqrt{2} - 1$ (b) 1 (c) $\sqrt{2}$ (d) $1 + \sqrt{2}$

40. **The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1, x = b$ is equal to $\sqrt{b^2 + 1} - \sqrt{2}$ for all b > 1, then $f(x)$ is**

(a) $\sqrt{x-1}$ (b) $\sqrt{x+1}$ (c) $\sqrt{x^2+1}$ (d) $\frac{x}{\sqrt{1+x^2}}$

42.Area under the curve $y=\sqrt{3x+4}$ between $x=0$ and $x=4,$ is

(a) $\frac{56}{9}$ sq. unit (b) $\frac{64}{9}$ sq. unit (c) 8 sq. unit (d) None of these

43.The area bounded by curve $y^2=x,$ line $y=4$ and y-axis is

(a) $\frac{16}{3}$ (b) $\frac{64}{3}$ (c) $7\sqrt{2}$ (d) None of these

44.For $0 \le x \le \pi,$ the area bounded by $y=x$ and $y=x+\sin x,$ is

(a) 2 (b) 4 (c) 2π (d) 4π

45.The area bounded by the straight lines $x=0, x=2$ and the curves $y=2^x, y=2x-x^2$

(a) $\frac{4}{3}-\frac{1}{\log 2}$ (b) $\frac{3}{\log 2}+\frac{4}{3}$ (c) $\frac{4}{\log 2}-1$ (d) $\frac{3}{\log 2}-\frac{4}{3}$

46.The area between the curve $y=4+3x-x^2$ and x-axis is

(a) 125/6 (b) 125/3 (c) 125/2 (d) None of these

47.The area between the curve $y=\sin^2 x,$ $x-$axis and the ordinates $x=0$ and $x=\frac{\pi}{2}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) π

48.Area bounded by the curve $xy-3x-2y-10=0,$ x-axis and the lines $x=3, x=4$ is

(a) $16\log 2-13$ (b) $16\log 2-3$ (c) $16\log 2+3$ (d) None of these

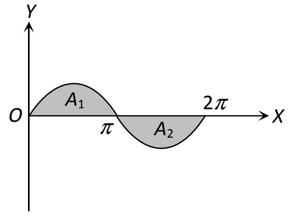
49.The area of the triangle formed by the tangent to the hyperbola $xy=a^2$ and co-ordinate axes is

(a) a^2 (b) $2a^2$ (c) $3a^2$ (d) $4a^2$

AREAS

HINTS AND SOLUTIONS

1. (d) Required area is $A_1 + A_2 = \int_0^{\pi} y\ dx + \left|\int_{\pi}^{2\pi} y\,dx\right| = 4\pi\,sq.unit$



2. (c) Required area = $\int_1^4 x\,dy = \int_1^4 \frac{\sqrt{y}}{2}dy$

$= \frac{1}{2}.\frac{2}{3}|\,y^{3/2}\,|_1^4 = \frac{7}{3}$ *sq. unit.*

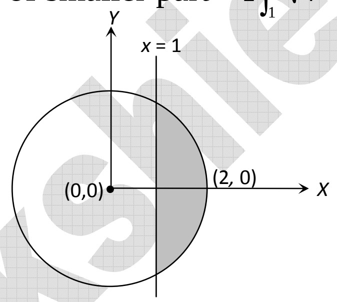
3. (b) Required area is $\int_0^a y\ dx = \int_0^a xe^{x^2}dx$

We put $x^2 = t \Rightarrow dx = \frac{dt}{2x}$ as $x = 0 \Rightarrow t = 0$ and $x = a \Rightarrow t = a^2$, then it reduces to

$\frac{1}{2}\int_0^{a^2} e^t dt = \frac{1}{2}[e^t]_0^{a^2} = \frac{e^{a^2} - 1}{2}$ *sq. unit.*

4. (d) Required area $= \int_1^4 x^3 dx = \left[\frac{x^4}{4}\right]_1^4 = \frac{255}{4}$ *sq. unit.*

5. (b) Area of smaller part $= 2\int_1^2 \sqrt{4 - x^2}\ dx$

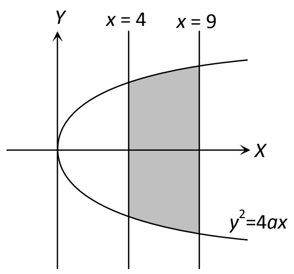


$= \frac{8\pi}{3} - \sqrt{3}$.

6. (a) $\int_0^2 (x^2 - 4x)dx = \left[\frac{x^3}{3} - \frac{4x^2}{2}\right]_0^2 = \frac{16}{3}$ *sq. unit.*

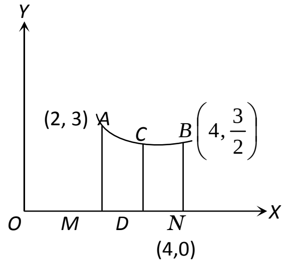
7. (d) $A_1 = \int_0^{\pi/3} \cos x\,dx$, $A_2 = \int_0^{\pi/4} \cos 2x\,dx - \int_{\pi/4}^{\pi/3} \cos 2x\,dx$.

8. (d) Shaded area $A = 2\int_4^9 \sqrt{4ax}\,dx$



$A = 4\sqrt{a} \times \frac{2}{3}[x^{3/2}]_4^9 = \frac{152\sqrt{a}}{3}$.

9. (b) Let the ordinate at $x = a$ divide the area into two equal parts



Area of $AMNB = \int_2^4 \left(1 + \frac{8}{x^2}\right) dx = \left[x - \frac{8}{x}\right]_2^4 = 4$

Area of $ACDM = \int_2^a \left(1 + \frac{8}{x^2}\right) dx = 2$

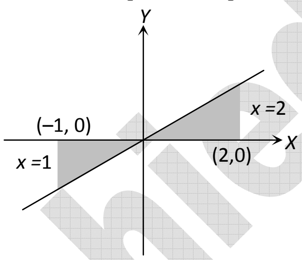
On solving, we get $a = \pm 2\sqrt{2}$;Since $a > 0 \Rightarrow a = 2\sqrt{2}$.

10. (c) Given curve $y = \log x$ and $x = 1$, $x = 2$.

Hence required area = $\int_1^2 \log x \, dx = (x \log x - x)_1^2$

$= 2\log 2 - 1 = (\log 4 - 1)$ *sq. unit.*

11. (a) Required area $\int_{-1}^{2} y\, dx = \int_{-1}^{0} y \,.dx + \int_0^2 y\,.dx = \frac{5}{2}$ *sq. unit.*



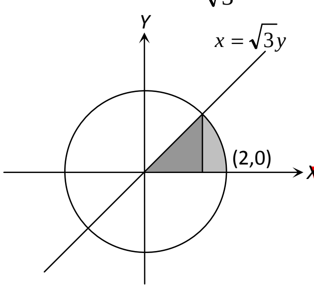
12. (b) $\int_0^2 2^{kx}\, dx = \frac{3}{\log 2} \Rightarrow 2^{2k} - 1 = 3k$. Now check from options, only (b) satisfies the above condition.

13. (a) $y^2 = x$ and $2y = x \Rightarrow y^2 = 2y \Rightarrow y = 0, 2$

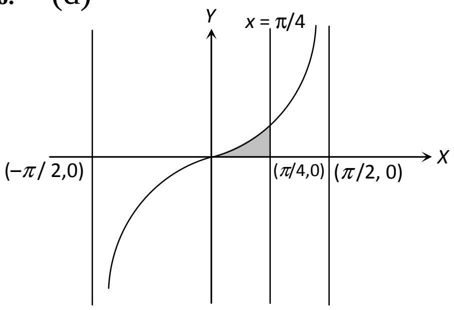
$\therefore$ Required area $= \int_0^2 (y^2 - 2y) dy$ *sq. unit.*

14. (c) Required area $= 2\int_0^{\pi/3} \tan x\, dx = 2[\log \sec x]_0^{\pi/3} = 2\log(2)$.

15. (c) Required area $= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$



16. (d)



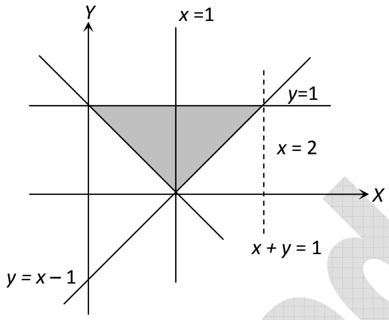
Shaded area $= \int_0^{\pi/2} \tan x\, dx = [\log \sec x]_0^{\pi/4}$

$= \log \sec(\pi/4) - \log \sec 0 = \log\sqrt{2} - \log 1 = \log\sqrt{2}$.

17. (c) Given equations of curves $y = e^x; y = e^{-x}$ and straight line $x = 1$ We know that area of the figure bounded by the curves and straight line

$= \int_0^1 (e^x - e^{-x})dx = [e^x + e^{-x}]_0^1 = e + \frac{1}{e} - 2.$

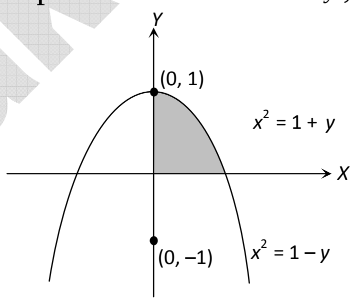
18. (b) $y = x - 1$, if $x > 1$ and $y = -(x - 1)$, if $x < 1$



Area $= \int_0^1 (1 - x)dx + \int_1^2 (x - 1)dx = \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$

$= \left[1 - \frac{1}{2}\right] + \left[-\left(\frac{1}{2} - 1\right)\right] = \frac{1}{2} + \frac{1}{2} = 1$.

19. (d) Given parabolas are $x^2 = 1 + y$, $x^2 = 1 - y$



Required area $= 4\int_0^1 (1-x^2)dx = 4\left[x - \frac{x^3}{3}\right]_0^1 = \frac{8}{3}$.

20. (b) Curved surface $= \int_a^b 2\pi y\sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}dx$

Given that $a=2,\ b=3$ and $y = x+1$.

On differentiating with respect to x,

$$\frac{dy}{dx} = 1+0 \text{ or } \frac{dy}{dx} = 1$$

Therefore, curved surface

$$= \int_2^3 2\pi(x+1)\sqrt{[1+(1)^2]}\,dx = \int_2^3 2\pi(x+1)\sqrt{2}\,dx$$

$$= 2\sqrt{2}\,\pi\int_2^3 (x+1)\,dx = 2\sqrt{2}\,\pi\left[\frac{(x+1)^2}{2}\right]_2^3$$

$$= \frac{2\sqrt{2}}{2}\pi[(3+1)^2 - (2+1)^2] = \sqrt{2}\pi(16-9) = 7\sqrt{2}\pi = 7\pi\sqrt{2}.$$

21. (b) Given, $y = -x^2 + 2x + 3$ and $y = 0$

Therefore, $x = -1$ and $x = 3$

$\therefore$ Required area $= \int_{-1}^3 (-x^2+2x+3)dx$

$$= \left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^3 = \frac{32}{3}.$$

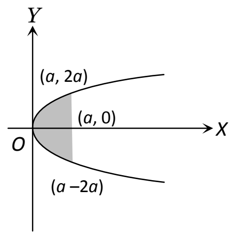
22. (b) $y^2 = 8x$ and $y = x \Rightarrow x^2 = 8x \Rightarrow x = 0, 8$

$\therefore$ Required area $= \int_0^8 (2\sqrt{2}\sqrt{x} - x)dx$

$$= \left[\frac{4\sqrt{2}}{3}x^{3/2} - \frac{x^2}{2}\right]_0^8 = \frac{128}{3} - \frac{64}{2} = \frac{32}{3}\ sq.\ unit.$$

23. (c) Given equations of curves $y = \cos x$ and $y = \sin x$ and ordinates $x = 0$ to $x = \frac{\pi}{4}$. We know that area bounded by the curves $= \int_{x_1}^{x_2} y dx = \int_0^{\pi/4} \cos x dx - \int_0^{\pi/4} \sin x\, dx$

24. (c) Area = $2\int_0^a y\,dx = 2\int_0^a \sqrt{4ax}\,dx$

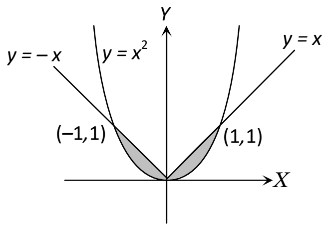


$2\times 2\sqrt{a}\times\frac{2}{3}\left|x^{3/2}\right|_0^a=\frac{8}{3}a^2$ *sq. unit.*

25. (c) We have $y=4x-x^2$ and $y=0$; $\therefore$ $x=0$, 4

Required area $=\int_0^4(4x-x^2)dx=\left[\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^4$

26. (b)



Required area = 2 (shaded area in first quadrant)

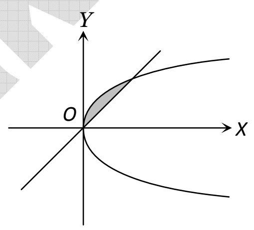
$=2\int_0^1(x-x^2)dx=2\times\frac{1}{6}=\frac{1}{3}$.

27. (d) Equations of curves $y^2=4x$ and $x^2=4y$. The given equations may be written as $y=2\sqrt{x}$ and $y=\frac{x^2}{4}$.

We know that area enclosed by the parabolas $=\int_0^4 2\sqrt{x}\,dx-\int_0^4\frac{x^2}{4}dx=\frac{32}{3}-\frac{16}{3}=\frac{16}{3}$ *sq. unit.*

28. (a) Given curves are $y=x^2$ and $y=x$

On solving, we get $x=0$, $x=1$

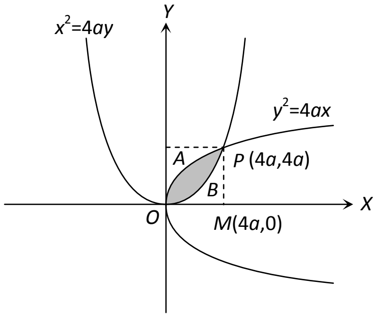


Therefore, required area $A=\int_0^1(x^2-x)dx$

$=\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_0^1=\frac{1}{3}-\frac{1}{2}=\frac{1}{6}$ *sq. unit.*

29. (d) Solving the two equations, we have $x^4 = 64a^3x$

$\Rightarrow x = 0, 4a$



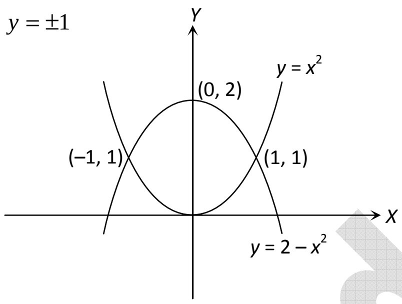
Required area $= \int_0^{4a} 2a^{1/2}x^{1/2}dx - \int_0^{4a} \frac{x^2}{4a}dx$

30. (a) $y = x^2$ .....(i)

$y = 2 - x^2$ .....(ii)

$\therefore$ By equation (i) and (ii) , we get, $x = \pm 1$

$\therefore \; y = \pm 1$

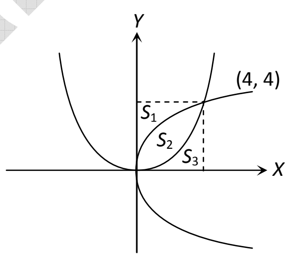


$\therefore$ Required area $= 2\left[\int_0^1 (2 - x^2)dx - \int_0^1 x^2 dx\right]$

$= 2\left[2x - \frac{2x^3}{3}\right]_0^1 = 4\left[x - \frac{x^3}{3}\right]_0^1 = 4\left(\frac{2}{3}\right) = \frac{8}{3}$.

31. (b) $y^2 = 4x$ and $x^2 = 4y$ are symmetric about line $y = x$

$\Rightarrow$ Area bounded between $y^2 = 4x$ and $y = x$ is $\int_0^4 (2\sqrt{x} - x)dx = \frac{8}{3}$



$\Rightarrow \; A_{s_2} = \frac{16}{3}$ and $A_{s_1} = A_{S_3} = \frac{16}{3}$

$\Rightarrow \; A_{S_1} : A_{S_2} : A_{S_2} :: 1:1:1$.

32. (a) The part of circle $x^2 + y^2 = 9$ in between $y = 0$ and $y = 2$ is revolved about *y*-axis. Then a frustum of sphere will be formed.

The volume of this frustum

$= \pi \int_0^2 x^2 dy \; = \pi \int_0^2 (9 - y^2) dy$

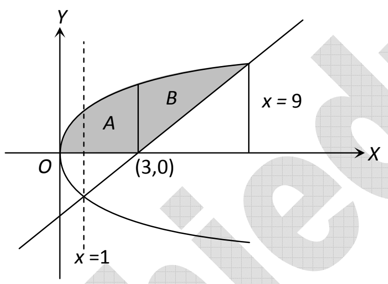
$= \pi \left[9y - \frac{1}{3} y^3 \right]_0^2 = \pi \left[9 \times 2 - \frac{1}{3}(2)^3 - (9.0 - \frac{1}{3}.0) \right]$

$= \pi \left[18 - \frac{8}{3} \right] = \frac{46}{3} \pi$ *cubic unit.*

33. (a) Solving $y^2 = x$ and $x = 2y + 3$

$4y^2 = (x-3)^2$, $4x = x^2 - 6x + 9$

$\Rightarrow \; x^2 - 10x + 9 = 0 \Rightarrow (x-1)(x-9) = 0 \Rightarrow x = 1, 9$

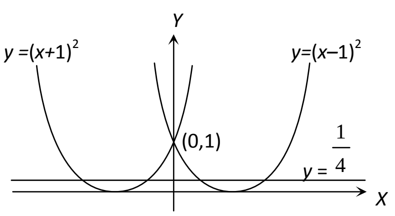


$= -4\,[x \log x - x]_0^1 = -4(-1) = 4$ *sq. unit,*

$(\because \lim_{x \to 0} x \log x = 0)$.

Required area = *A*+*B* $= \int_0^3 \sqrt{x}\, dx + \int_3^9 \left[\sqrt{x} - \left(\frac{x-3}{2} \right) \right] dx$

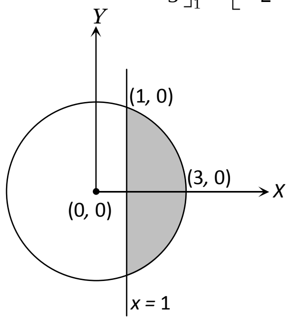
34. (d) Required area $= 2 \left| \int_{1/4}^{1} (\sqrt{y} - 1)\, dy \right|$,(From the symmetry)



On solving, we get required area $= \frac{1}{3}$ *sq. unit*.

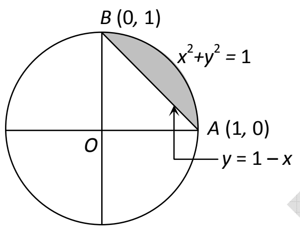
35. (b) Area of smaller part $I = 2\int_1^3 \sqrt{9 - x^2}\, dx$

$$= 2.\frac{1}{2}\left[x\sqrt{9 - x^2} + 9\sin^{-1}\frac{x}{3}\right]_1^3 = \left[9\frac{\pi}{2} - \sqrt{8} - 9\sin^{-1}\left(\frac{1}{3}\right)\right]$$



36. (d) $x^2 + y^2 = 1, x + y = 1$ meet when

$$x^2 + (1 - x)^2 = 1 \Rightarrow x^2 + 1 + x^2 - 2x = 1$$



$$\Rightarrow 2x^2 - 2x = 0 \Rightarrow 2x(x - 1) = 0$$

$\Rightarrow x = 0, x = 1 \quad \Rightarrow \quad y = 1, y = 0$, *i.e.*, $A(1,0); B(0,1)$

Required area $= \int_0^1 [\sqrt{1 - x^2} - (1 - x)]\, dx$

37. (a) The parabola meets x-axis at the points, where $\frac{3}{a}(a^2 - x^2) = 0 \Rightarrow x = \pm a.$ So the required area

$$= \int_{-a}^{a} \frac{3}{a}(a^2 - x^2)dx = \frac{6}{a}\int_0^a (a^2 - x^2)dx = 4a^2 \text{ sq. unit.}$$

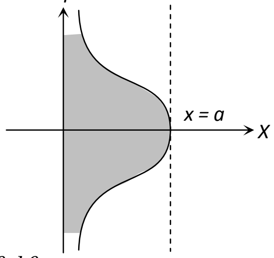
38. (c) We have $y^2 = 4ax \Rightarrow y = 2\sqrt{ax}$

We know the equations of lines $x = a$ and $x = 4a$

$\therefore$ The area inside the parabola between the lines

$$A = \int_a^{4a} y\,dx = \int_a^{4a} 2\sqrt{ax}\,dx = 2\sqrt{a}\int_a^{4a} x^{\frac{1}{2}}dx = 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^{4a}$$

39. (a) Since the curve is symmetrical about x-axis, therefore Required area $A = 2\int_0^a a\sqrt{\frac{a-x}{x}}\,dx$



Put $x = a\sin^2\theta$

$\Rightarrow dx = 2a\sin\theta.\cos\theta\,d\theta$

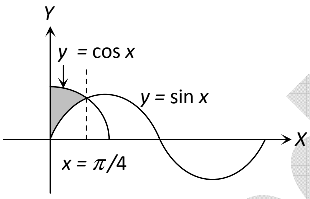
$$A = 2\int_0^{\pi/2} a\sqrt{\frac{a\cos^2\theta}{a\sin^2\theta}}\,a\sin 2\theta\,d\theta$$

$$= 2a^2\int_0^{\pi/.2}\frac{\cos\theta}{\sin\theta}2\sin\theta\cos\theta\,d\theta$$

$$A = 4a^2\int_0^{\pi/2}\cos^2\theta\,d\theta \Longrightarrow A = 4a^2.\frac{1}{2}.\frac{\pi}{2} = \pi a^2$$

40. (a) Given required area has been shown in the figure.

$x = \frac{\pi}{4}$ is the point of intersection of both curve



$\therefore$ Required area = $\int_0^{\pi/4}(\cos x - \sin x)\,dx$

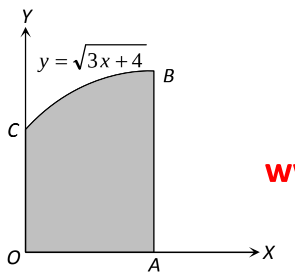
$$= [\sin x + \cos x]_0^{\pi/4} = \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$$

$$= \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1.$$

41. (d) $\int_1^b f(x)dx = \sqrt{b^2+1} - \sqrt{2} = \sqrt{b^2+1} - \sqrt{1+1} = [\sqrt{x^2+1}]_1^b$

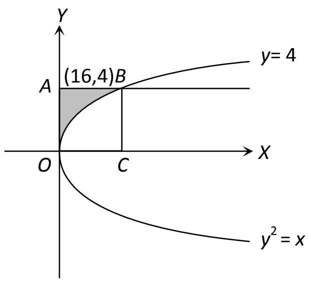
$\therefore\ f(x) = \frac{d}{dx}\sqrt{x^2+1} = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$.

42. (d) Area $= \int_0^4\sqrt{3x+4}\,dx = \left|\frac{(3x+4)^{3/2}}{3.(3/2)}\right|_0^4$



$= \frac{2}{9} \times 56 = \frac{112}{9}$ *sq. unit.*

43. (b) Required area = area of *OABC* – area of *OBC*



$$= 16 \times 4 - \int_0^{16} \sqrt{x}dx = 64 - \left[\frac{x^{3/2}}{3/2}\right]_0^{16} = \frac{64}{3}.$$

44. (a) The curves $y = x$ and $y = x + \sin x$ intersect at (0, 0) and (π, π). Hence area bounded by the two curves

$$= \int_0^{\pi} (x + \sin x)dx - \int_0^{\pi} x\, dx = \int_0^{\pi} \sin x\, dx$$

$$= [-\cos x]_0^{\pi} = -\cos \pi + \cos 0 = -(-1) + (1) = 2\,.$$

45. (d) Required area $= \int_0^2 [2^x - (2x - x^2)]dx$

$$= \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$$

46. (a) Solving $y = 0$ and $y = 4 + 3x - x^2$, we get $x = -1, 4$. Curve does not intersect *x*-axis between $x = -1$ and $x = 4$.

$\therefore$ Area $= \int_{-1}^{4} (4 + 3x - x^2)dx = \frac{125}{6}$.

47. (b) Required area $A = \int_0^{\pi/2} \sin^2 x.dx = \int_0^{\pi/2} \left(\frac{1 - \cos 2x}{2}\right) dxa$

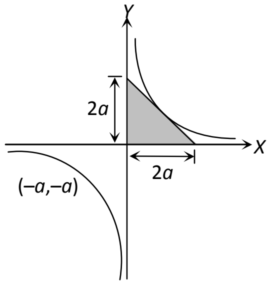
48. (c) Given curve is $y(x-2) = 3x + 10 \Rightarrow y = \frac{3x+10}{x-2}$

Required area is $\int_3^4 y\, dx = \int_3^4 \frac{3x+10}{x-2}\, dx$

$= [3x + 16 \log(x-2)]_3^4 = 3 + 16 \log 2$ *sq. unit.*

49. (b) Given $xy = a^2$ or $y = \frac{a^2}{x}$ .....(i)

There are two points on the curve $(a, a), (-a, -a)$



The equation of the line at (a, a) is,

$$y - a = \left(\frac{dy}{dx}\right)_{(a,a)}(x - a) = \left(\frac{-a^2}{x^2}\right)_{(a,a)}(x - a)$$

$y - a = -(x - a)$ therefore, equation of the tangent at (a, a) is $x + y = 2a$.The interception of line $x + y = 2a$ with x-axis is $2a$ and with y-axis is $2a$.

$\therefore$ Required area = $\frac{1}{2} \times 2a \times 2a = 2a^2$.

AREAS

PRACTICE EXERCISE

1. **The area bounded by $y = 5x - x^2 - 4$ and the x -axis**

   1) $\frac{9}{4}$ sq.units 2) $\frac{9}{8}$ sq.units 3) $\frac{3}{2}$ sq.units 4) $\frac{9}{2}$ sq.units

2. **The area bounded by the curve $y = (x - 1)^2 - 25$ and the x-axis is**

   1) $\frac{200}{3}$ sq.units 2) $\frac{300}{4}$ sq.units 3) $\frac{400}{3}$ sq.units 4) $\frac{500}{3}$ sq.units

3. **The area bounded by $x^2 = 4y$, $x = 4y - 2$**

   1) 9/8 sq.units 2) 9/4 sq.units 3) 9/16 sq.units 4) 3/2 sq.units

4. **The area bounded by $y^2 = 4x$ and the line $y = 2x-4$**

   1) 18 sq.units 2) 9/2 sq.units 3) 9 sq.units 4) 3/2 sq.units

5. **The area between the curves $y = 8 - x^2$ and $y = x^2$ in sq.units is**

   1) 32/3 2) 64/3 3) 16/3 4) 8/3

6. **The area enclosed within the curve $|x| + |y| = 1$ is**

   1) 4 sq.units 2) 1 sq.unit 3) 2 sq.unit 4) 8 sq.unit

7. **Area of the region bounded by $y = 1 - |x|$ and the x-axis**

   1) ½ 2) 1 3) ¼ 4) 2

8. **Area of the region bounded by $y = [x]$, the x-axis and the coordinates $x = 1$,** $x = 2$ is

   1) 2 2) 1 3) ½ 4) 1/3

9. **The area bounded by $y = x^3 - 6x^2 + 8x$ and the x - axis**

   1) 8 sq.units 2) 4 sq.units 3) 16 sq.units 4) 4 sq.units

10. **The whole area bounded by** $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ **in sq. units**

    1) 24π 2) 48π 3) 12π 4) 36π

11. **The area of the region bounded by the curve y=sinx and the x-axis between $-\pi$ and π is**

    1) 8 sq.units 2) 4 sq.units 3) 2 sq.units 4) 1 sq.unit

12. **The area bounded by one of the ac of y = cosax and the x-axis is**

1) $\frac{1}{|a|}$ 2) $\frac{1}{a}$ 3) $\frac{2}{a}$ 4) $\frac{2}{|a|}$

13. **The area between the curves $y^2 = x/2$ and $3y^2 = x + 1$ in sq.units is**

1) 4/3 2) 2/3 3) 8/3 4) 16/3

14. **The area between the curves $y = \frac{x^2}{4}$ and $y = 3 - \frac{x^2}{2}$ in sq.units is**

1) 8 2) 16/3 3) 8/3 4) 12

15. **The area of the region between the x-axis and the curve $f(x) = \frac{1}{4}x^2 + \frac{1}{4}x - \frac{1}{2}$ in [0, 2] is**

1) $\frac{3}{4}$ sq.units 2) $\frac{3}{2}$ sq.units 3) $\frac{3}{8}$ sq.units 4) $\frac{3}{5}$sq.units

16. **The area bounded by the x - axis, part of the curve $y = 1 + 8/x^2$ and the ordinates at x=2 and x=4 is divided by the ordinate x = a into two equal parts. Then a =**

1) $2\sqrt{2}$ 2) 2 3) 4 4) $\sqrt{3}$

17. **The area bounded by the curve $ay^2 = x^3$, the x-axis and the ordinate x = a**

1) $\frac{8a^2}{3}$ sq.units 2) $\frac{2a^2}{5}$sq.units 3) $\frac{4a^2}{5}$ sq.units 4) $\frac{3a^2}{5}$sq.units

18. **The whole area bounded by $a^2y^2 = a^2x^2 - x^4$ is**

1) $\frac{2}{3}a^2$ sq.units 2) $\frac{8}{3}a^2$ sq.units 3) $\frac{4}{3}a^2$ sq.units 4) $\frac{5a^2}{3}$sq.units

19. **The area bounded by $a^2y^2 = x^3(2a-x)$**

1) πa^2 sq.units 2) $\frac{\pi a^2}{2}$sq.units 3) $2\pi a^2$ sq.units 4) $\frac{\pi a^2}{4}$ sq.units

20. **The area bounded by the line y = x curve and $y = x^3$ is**

1) 1 sq. units 2) ½ sq. units 3) 1/3 sq. units 4) ¼ sq. units

21. **Area bounded by y = (x - 1)(x - 2)(x - 3) between x = 0, x = 3 in sq. units is**

1) $\frac{9}{4}$ 2) $\frac{11}{4}$ 3) $\frac{7}{4}$ 4) $\frac{3}{4}$

22. **Area of the region bounded by $y = e^x$ and $y = e^{-x}$ and the line x = 1 in sq. units is**

1) $e + \frac{1}{e}$ 2) $e - \frac{1}{e}$ 3) $e + \frac{1}{e} + 2$ 4) $e + \frac{1}{e} - 2$

23. The area bounded by $y = x^2$, $y = [x + 1]$, $x \leq 1$, and the y-axis in sq. units is

1) 1/3 2) 2/3 3) 1 4) 7/3

24. The area bounded by the curve xy=4 and x-axis the ordinates x=2, x=4 in sq. units is

1) 4 log 2 2) 2 log 2 3) 8 log 2 4) log 2

25. The area of the curve $x = a\cos^3 t$, $y = b\sin^3 t$ in sq. units is

1) $\frac{3\pi}{4}ab$ 2) $\frac{3\pi}{8}ab$ 3) $\frac{\pi}{4}ab$ 4) $\frac{\pi}{8}ab$

26. The area bounded by one arc of y = sin2x and x-axis in sq. units is

1) 1 2) 2 3) 3 4) 4

27. The area bounded by the curve y = sinx-cosx. X-axis and x = 0, x = $\pi/2$ in sq.units is

1) $\sqrt{3} - 1$ 2) $2(\sqrt{3} - 1)$ 3) $2(\sqrt{2} - 1)$ 4) $2(\sqrt{2} + 1)$

28. Area of the region bounded by y = tanx, and tangent at $x = \frac{\pi}{4}$ and the x-axis in sq. units

1) $\log\sqrt{2} - \frac{1}{4}$ 2) $\log\sqrt{2} + \frac{\pi^2}{16}$ 3) $\log\sqrt{2} - \frac{\pi}{4}$ 4) $\log\sqrt{2}$

29. The area bounded by y = cosx, y = x +1 and y = 0 in the second quadrant in sq. units is

1) 1/2 2) 3/2 3) 1/4 4) 5/4

30. The area between $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$ in sq. units is

1) $\frac{1}{2}ab$ 2) $\frac{\pi ab}{2}$ 3) $\frac{ab}{4}$ 4) $\frac{\pi ab}{4} - \frac{ab}{2}$

31. The area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ in sq. units is

1) $\sqrt{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) $2\sqrt{3}$ 4) $2/\sqrt{3}$

32. The area of the region bounded by the curves y = |x-2|, x = 1, x = 3 and the x-axis is

1) 1 sq. units 2) 4 sq. units 3) 3 sq. units 4) 2 sq. units

AREAS

Key for Practice Exercise

1	2	3	4	5	6	7	8	9	10
4	4	1	3	2	3	2	2	2	1
11	12	13	14	15	16	17	18	19	20
1	2	1	1	1	1	3	3	1	2
21	22	23	24	25	26	27	28	29	30
4	4	2	1	2	1	1	1	1	4
31	32								
3	1								