APPLICATIONS OF DERIVATIVES

OBJECTIVES

1. **The approximate increase in the area of a square plane when each side expands from 3 c m to 3.01 cm is**

(1) 0.001 sq. cm (2) 0.006 sq. cm (3) 0.06 sq. cm (4) None

2. **If $y = \log x$ then 8y when $x = 3, 8x = 0.03$ is**

(1) 0.0l (2) 0.009 (3) 0.0091 (4) 0.0099

3. **The approximate percentage error in the volume of a sphere is equal to**

(1) Percentage error in *r* (2) Double the percentage error in r

(3) Treble the percentage error in *r* (4) None

4. **If $y = x''$, then ratio of relative errors in y and x is**

(1) 1:1 (2) 2 : 1 (3) 1: *n* (4) n : 1

5. **Let P be the pressure and V the volume of a gas such that PV = constant. If percentage error in P is *k* then percentage error in V is**

(1) *k* (2) *1/k* (3) -*k* (4) None

6. **If log 4 = 1.3868 then log 4.01 =**

(1) 1.3968 (2) 1.3898 (3) 1.3893 (4) None

7. $\frac{1}{\sqrt[3]{998}}$ **correct to 4 decimal places is**

(1) 0.3333 (2) 0.1667 (3) 0.1666 (4) None

8. **The circumference of a circle is measured as 28 cm with an error of 0.01 cm. The percentage error in the area is**

(1) 1/14 (2) 0.01 (3) 1/7 (4) None

9. **While measuring the side of an equilateral triangle an error of 0.5% is made. Percentage error in its area is**

(1) 0.5 (2) 1 (3) 10 (4) 1.5

10. **If there is an error of 0.02 cm in the measurement of the diameter of a sphere, then the percentage error in its volume when the radius = 10 cm, is**

(1) 0.1 (2) 0.2 (3) 0.3 (4) 3

11. **If the percentage error in the surface area of a sphere is α, then percentage error i n the volume is**

(1) (3/2) α (2) (2/3) α (3) α (4) None

12. **If there is an error of 2% in measuring the length of a simple pendulum then percentage error in its period is**

(1) 1 (2) 2 (3) 3 (4) 4

13. **The radius of a closed cylinder is half of its height. If an error of 0.5% is made in measuring the radius, the percentage error in the surface area is**

(1) 0.5 (2) 1 (3) 1.5 (4) none

14. **If** $T = 2\pi\sqrt{\frac{l}{g}}$ **then the ratio of the relative error in T to relative error in / is**

(1) ½ (2) 2 (3) 1/2 re (4) None

15. **The voltage E of a thermocouple as a function of temperature is given by $E = 6.2T + 0.0002\,T^3$. When T changes from 100° to 101° the approximate change in E is**

(1) 12 (2) 12.1 (3) 12.12 (4) 12.2

16. **In A ABC the sides *b, c* are given. If there is an error 8A i n measuring angle A then δa =**

1. $\frac{\triangle}{2a}\delta A$ 2. $\frac{2\triangle}{2a}\delta A$ 3. $bc\sin A.\delta A$ 4. None

17. **If the ratio of the base radius and height of a cone is 1 : 2 and percentage error in the radius is *k*, then percentage error in its volume is**

(1) *k* (2) *2k* (3) 3 *k* (4) None

18. **A circular hole of 4 mm in diameter and 12 mm deep in a metal block is rebored to increase the diameter to 4.12 mm, and then the amount of metal removed is approximately**

(1) 2.88 πmm^3 (2) 3.99 πmm^3 (3) 3.79 πmm^3 (4) 3.725 πmm^3

19. **The semi-vertical angle of a cone is 45°. If the height of the cone is 20.025, the approximate lateral surface area is**

1. $401\sqrt{2}\pi$ 2. $400\sqrt{2}\pi$ 3. $401\sqrt{2}\pi$ 4. None

20. **$\triangle$ABC is not right angled and is inscribed in a fixed circle. If *a*, A, *b*, B be slightly varied keeping, *c*, C fixed, then**

1)2R 2) π 3)0 4)None

21. **The focal length of a mirror is given by $\frac{1}{v}-\frac{1}{u}=\frac{2}{f}$. If equal errors α are made in measuring *u* and *v* then relative error is**

1) $\frac{2}{\alpha}$ 2) $\alpha\left(\frac{1}{u}+\frac{1}{v}\right)$ 3) $\alpha\left(\frac{1}{u}-\frac{1}{v}\right)$ 4)None

22. **Approximate value of cos61° given that sin60° = 0.86603 and 1° = 0.001745**

(1) 0.4849 (2) 0.4983 (3) 0.9969 (4) 0.5012

23. **A stone moving vertically upwards has its equation of motion $s = 490t - 4.9t^2$. The maximum height reached by the stone is**

(a) 12250 (b) 1225

(c) 36750 (d) None of these

24. **A particle moves in a straight line so that its velocity at any point is given by $v^2 = a + bx$, where $a, b \neq 0$ are constants. The acceleration is**

(a) Zero (b) Uniform

(c) Non-uniform (d) Indeterminate

25. **The maximum height is reached in 5 seconds by a stone thrown vertically upwards and moving under the equation 10*s* = 10*ut* – 49 t^2, where *s* is in *metre* and *t* is in *second*. The value of *u* is**

(a) $4.9 m/\sec$ (b) $49 m/\sec$

(c) $98 m/\sec$ (d) None of these

26. **A stone is falling freely and describes a distance *s* in *t* seconds given by equation $s = \frac{1}{2} g t^2$. The acceleration of the stone is**

(a) Uniform (b) Zero (c) Non-uniform (d) Indeterminate

27. A 10*cm* long rod *AB* moves with its ends on two mutually perpendicular straight lines *OX* and *OY*. If the end *A* be moving at the rate of $2 cm/\sec$, then when the distance of *A* from *O* is $8 cm$, the rate at which the end *B* is moving, is

(a) $\frac{8}{3} cm/\sec$ (b) $\frac{4}{3} cm/\sec$

(c) $\frac{2}{9} cm/\sec$ (d) None of these

28. If the radius of a circle increases from 3 *cm* to 3.2 *cm*, then the increase in the area of the circle is

(a) $1.2\pi\ cm^2$ (b) $12\pi\ cm^2$

(c) $6\pi\ cm^2$ (d) None of these

29. The equation of motion of a particle is given by $s = 2t^3 - 9t^2 + 12t + 1$, where *s* and *t* are measured in *cm* and *sec*. The time when the particle stops momentarily is

(a) 1 *sec* (b) 2 *sec*

(c) 1, 2 *sec* (d) None of these

30. A particle is moving in a straight line according as $s = 45t + 11t^2 - t^3$ then the time when it will come to rest, is

(a) – 9 *seconds* (b) $\frac{5}{3}$ *seconds*

(c) 9 *seconds* (d) $-\frac{5}{3}$ *seconds*

31. A ladder 5 *m* in length is resting against vertical wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of $1.5 m/\sec$. The length of the highest point of the ladder when the foot of the ladder $4.0 m$ away from the wall decreases at the rate of

(a) 2 *m/sec* (b) 3 *m/sec*

(c) 2.5 *m/sec* (d) 1.5 *m/sec*

32. The equation of motion of a particle moving along a straight line is $s = 2t^3 - 9t^2 + 12t$, where the units of *s* and *t* are *cm* and *sec*. The acceleration of the particle will be zero after

(a) $\frac{3}{2} sec$ (b) $\frac{2}{3} sec$

(c) $\frac{1}{2} sec$ (d) Never

33. A particle is moving on a straight line, where its position *s* (in *metre*) is a function of time *t* (in seconds) given by $s = at^2 + bt + 6, t \geq 0$. If it is known that the particle comes to rest after 4 seconds at a distance of 16 *metre* from the starting position $(t = 0)$, then the retardation in its motion is

(a) $-1 m/\sec^2$ (b) $\frac{5}{4} m/\sec^2$

(c) $-\frac{1}{2} m/\sec^2$ (d) $-\frac{5}{4} m/\sec^2$

34. If the law of motion in a straight line is $s = \frac{1}{2} v t$, then acceleration is

(a) Constant (b) Proportional to *t*

(c) Proportional to *v* (d) Proportional to *s*

35. If the distance travelled by a point in time *t* is $s = 180t - 16t^2$, then the rate of change in velocity is

(a) – 16 *unit* (b) 48 *unit*

(c) – 32 *unit* (d) None of these

36. The edge of a cube is increasing at the rate of $5 cm/\sec$. How fast is the volume of the cube increasing when the edge is 12*cm* long

(a) $432\ cm^3/\sec$ (b) $2160\ cm^3/\sec$

(c) $180\ cm^3/\sec$ (d) None of these

37. A body moves according to the formula $v = 1 + t^2$, where *v* is the velocity at time *t*. The acceleration after 3 *sec* will be (*v* in *cm/sec*)

(a) $24\ cm/\sec^2$ (b) $12\ cm/\sec^2$

(c) $6\ cm/\sec^2$ (d) None of these

38. A point moves in a straight line during the time $t = 0$ to $t = 3$ according to the law $s = 15t - 2t^2$. The average velocity is

(a) 3 (b) 9

(c) 15 (d) 27

39. A man 2*metre* high walks at a uniform speed 5 *metre/hour* away from a lamp post 6 *metre* high. The rate at which the length of his shadow increases is

(a) 5 *m/h* (b) $\frac{5}{2}$ *m/h*

(c) $\frac{5}{3}$ *m/h* (d) $\frac{5}{4}$ *m/h*

40. If the path of a moving point is the curve $x = at$, $y = b \sin at$, then its acceleration at any instant

(a) Is constant

(b) Varies as the distance from the axis of *x*

(c) Varies as the distance from the axis of *y*

(d) Varies as the distance of the point from the origin

41. The rate of change of the surface area of a sphere of radius *r* when the radius is increasing at the rate of 2 *cm/sec* is proportional

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) r (d) r^2

42. A spherical iron ball 10 *cm* in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/*min*. When the thickness of ice is 5 *cm*, then the rate at which the thickness of ice decreases, is

(a) $\frac{1}{54\pi}$ *cm/min* (b) $\frac{5}{6\pi}$ *cm/min*

(c) $\frac{1}{36\pi}$ *cm/min* (d) $\frac{1}{18\pi}$ *cm/min*

43. A ladder is resting with the wall at an angle of 30^o. A man is ascending the ladder at the rate of 3 *ft/sec*. His rate of approaching the wall is

(a) 3 *ft/sec* (b) $\frac{3}{2}$ *ft/sec*

(c) $\frac{3}{4}$ *ft/sec* (d) $\frac{3}{\sqrt{2}}$ *ft/sec*

44. Gas is being pumped into a spherical balloon at the rate of 30 ft^3/*min*. Then the rate at which the radius increases when it reaches the value 15 *ft* is

(a) $\frac{1}{30\pi}$ ft/ min. (b) $\frac{1}{15\pi}$ ft/ min. (c) $\frac{1}{20}$ ft/ min. (d) $\frac{1}{25}$ ft/ min.

45. A ladder 10 *m* long rests against a vertical wall with the lower end on the horizontal ground. The lower end of the ladder is pulled along the ground away from the wall at the rate of 3 *cm/sec*. The height of the upper end while it is descending at the rate of 4 cm/*sec* is

(a) $4\sqrt{3}$ *m* (b) $5\sqrt{3}$ *m* (c) $5\sqrt{2}$ *m* (d) 8 *m*

46. The speed v of a particle moving along a straight line is given by $a + bv^2 = x^2$ (where *x* is its distance from the origin). The acceleration of the particle is

(a) bx (b) x/a (c) x/b (d) x/ab

47. If the volume of a spherical balloon is increasing at the rate of 900*cm*^{3}*persec*, then the rate of change of radius of balloon at instant when radius is 15*cm* [in *cm/sec*]

(a) $\frac{22}{7}$ (b)22 (c) $\frac{7}{22}$ (d) None of these

48. A spherical balloon is being inflated at the rate of 35 *cc/min*. The rate of increase of the surface area of the balloon when its diameter is 14 *cm* is

(a) 7 *sq. cm/min* (b) 10 *sq. cm/min* (c)17.5 *sq. cm/min* (d)28 *sq. cm/min*

49. The sides of an equilateral triangle are increasing at the rate of 2 *cm/sec*. The rate at which the area increases, when the side is 10 *cm* is

(a) $\sqrt{3}$ *sq. unit/sec* (b) 10 *sq. unit/sec* (c) $10\sqrt{3}$ *sq. unit/sec* (d) $\frac{10}{\sqrt{3}}$ *sq. unit/sec*

50. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) (2, – 4) (c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) (2, 4)

51. The position of a point in time '*t*' is given by $x = a + bt - ct^2$, $y = at + bt^2$. Its acceleration at time '*t*' is

(a) $b - c$ (b) $b + c$

(c) $2b - 2c$ (d) $2\sqrt{b^2 + c^2}$

52. A particle moves in a straight line so that $s = \sqrt{t}$, then its acceleration is proportional to

(a) Velocity (b) (Velocity)$^{3/2}$ (c)(Velocity)^3 (d) (Velocity)^2

53. If $x+y=10$, then the maximum value of xy is

(a) 5 (b) 20

(c) 25 (d) None of these

54. The necessary condition to be maximum or minimum for the function is

(a) $f'(x)=0$ and it is sufficient (b) $f''(x)=0$ and it is sufficient

(c) $f'(x)=0$ but it is not sufficient (d) $f'(x)=0$ and $f''(x)=-ve$

55. The value of a so that the sum of the squares of the roots of the equation $x^2-(a-2)x-a+1=0$ assume the least value, is

(a) 2 (b) 1

(c) 3 (d) 0

56. If $f(x)=2x^3-3x^2-12x+5$ and $x\in[-2,4]$, then the maximum value of function is at the following value of x

(a) 2 (b) –1

(c) – 2 (d) 4

57. If $x+y=16$ and x^2+y^2 is minimum, then the values of x and y are

(a) 3, 13 (b) 4, 12

(c) 6, 10 (d) 8, 8

58. A minimum value of $\int_0^x te^{-t^2}\,dt$ is

(a) 1 (b) 2

(c) 3 (d) 0

59. If two sides of a triangle be given, then the area of the triangle will be maximum if the angle between the given sides be

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

60. The sufficient conditions for the function $f : R \to R$ is to be maximum at $x = a$, will be

(a) $f'(a) > 0$ and $f''(a) > 0$ (b) $f'(a) = 0$ and $f''(a) = 0$

(c) $f'(a) = 0$ and $f''(a) < 0$ (d) $f'(a) > 0$ and $f''(a) < 0$

61. The maximum value of $2x^3 - 24x + 107$ in the interval [–3, 3] is

(a) 75 (b) 89

(c) 125 (d) 139

62. If for a function $f(x), f'(a) = 0, f''(a) = 0$, $f'''(a) > 0$, then at $x = a$, $f(x)$

(a) Minimum (b) Maximum

(c) Not an extreme point (d) Extreme point

63. The area of a rectangle will be maximum for the given perimeter, when rectangle is a

(a) Parallelogram (b) Trapezium

(c) Square (d) None of these

64. x and y be two variables such that $x > 0$ and $xy = 1$. Then the minimum value of $x + y$ is

(a) 2 (b) 3

(c) 4 (d) 0

65. If from a wire of length 36 *metre* a rectangle of greatest area is made, then its two adjacent sides in *metre* are

(a) 6, 12 (b) 9, 9

(c) 10, 8 (d) 13, 5

66. The minimum value of $\frac{\log x}{x}$ in the interval $[2, \infty)$ is

(a) $\frac{\log 2}{2}$ (b) Zero

(c) $\frac{1}{e}$ (d) Does not exist

67. The minimum value of $2x + 3y$, when $xy = 6$, is

(a) 12 (b) 9

(c) 8 (d) 6

68. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) $\sqrt{ab}$ (b) $\frac{a}{b}$

(c) $2ab$ (d) ab

69. If $y = a \log x + bx^2 + x$ has its extremum value at $x = 1$ and $x = 2$, then $(a, b) =$

(a) $\left(1, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, 2\right)$

(c) $\left(2, \frac{-1}{2}\right)$ (d) $\left(\frac{-2}{3}, \frac{-1}{6}\right)$

70. A cone of maximum volume is inscribed in a given sphere, then ratio of the height of the cone to diameter of the sphere is

(a) 2/3 (b) 3/4

(c) 1/3 (d) ¼

71. If $xy = c^2$, then minimum value of $ax + by$ is

(a) $c\sqrt{ab}$ (b) $2c\sqrt{ab}$

(c) $-c\sqrt{ab}$ (d) $-2c\sqrt{ab}$

72. If $A + B = \frac{\pi}{2}$, the maximum value of $\cos A \cos B$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) 1 (d) $\frac{4}{3}$

73. The minimum value of $4e^{2x} + 9e^{-2x}$ is

(a) 11 (b) 12

(c) 10 (d) 14

74. If *PQ* and *PR* are the two sides of a triangle, then the angle between them which gives maximum area of the triangle is

(a) π (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/2$

75. If $a^2x^4 + b^2y^4 = c^6$, then maximum value of *xy* is

(a) $\frac{c^2}{\sqrt{ab}}$ (b) $\frac{c^3}{ab}$

(c) $\frac{c^3}{\sqrt{2ab}}$ (d) $\frac{c^3}{2ab}$

76. The minimum value of $e^{(2x^2-2x+1)\sin^2 x}$ is

(a) e (b) $1/e$

(c) 1 (d) 0

77. If x is real, then greatest and least values of $\frac{x^2-x+1}{x^2+x+1}$ are

(a) $3, -\frac{1}{2}$ (b) $3, \frac{1}{3}$

(c) $-3, -\frac{1}{3}$ (d) None of these

78. The ratio of height of cone of maximum volume inscribed in a sphere to its radius is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

79. The maximum value of $\sin x\ (1+\cos x)$ will be at the

(a) $x=\frac{\pi}{2}$ (b) $x=\frac{\pi}{6}$

(c) $x=\frac{\pi}{3}$ (d) $x=\pi$

80. The perimeter of a sector is *p*. The area of the sector is maximum when its radius is

(a) $\sqrt{p}$ (b) $\frac{1}{\sqrt{p}}$

(c) $\frac{p}{2}$ (d) $\frac{p}{4}$

81. The function $y=a(1-\cos x)$ is maximum when $x=$

(a) π (b) $\pi/2$

(c) $-\pi/2$ (d) $-\pi/6$

82. If $P=(1,1)$, $Q=(3,2)$ and *R* is *a* point on *x*-axis then the value of $PR+RQ$ will be minimum at

(a) $\left(\frac{5}{3},0\right)$ (b) $\left(\frac{1}{3},0\right)$

(c) (3, 0) (d) (1, 0)

83. The function $\sin x - bx + c$ will be increasing in the interval $(-\infty, \infty)$, if

(a) $b \le 1$ (b) $b \le 0$

(c) $b < -1$ (d) $b \ge 0$

84. The function *f* defined by $f(x) = (x+2)e^{-x}$ is

(a) Decreasing for all x

(b) Decreasing in $(-\infty, -1)$ and increasing in $(-1, \infty)$

(c) Increasing for all x

(d) Decreasing in $(-1, \infty)$ and increasing in $(-\infty, -1)$

85. If the function $f(x) = \frac{K \sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all values of *x*, then

(a) $K < 1$ (b) $K > 1$

(c) $K < 2$ (d) $K > 2$

86. The interval for which the given function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is decreasing, is

(a) (– 2, 3) (b) (2, 3)

(c) (2,– 3) (d) None of these

87. $f(x) = x^3 - 27x + 5$ is an increasing function, when

(a) $x < -3$ (b) $|x| > 3$

(c) $x \le -3$ (d) $|x| < 3$

88. If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in each interval,

(a) $k < 3$ (b) $k \le 3$

(c) $k > 3$ (d) None of these

89. Function $f(x)=\frac{\lambda\sin x+6\cos x}{2\sin x+3\cos x}$ **is monotonic increasing, if**

(a) $\lambda>1$ (b) $\lambda<1$

(c) $\lambda<4$ (d) $\lambda>4$

90. The values of '*a*' for which the function $(a+2)x^3-3ax^2+9ax-1$ **decreases monotonically throughout for all real *x*, are**

(a) $a<-2$ (b) $a>-2$

(c) $-3<a<0$ (d) $-\infty<a\le-3$

91. The function $\frac{a\sin x+b\cos x}{c\sin x+d\cos x}$ **is decreasing, if**

(a) $ad-bc>0$ (b) $ad-bc<0$

(c) $ab-cd>0$ (d) $ab-cd<0$

92. The function $\frac{1}{1+x^2}$ **is decreasing in the interval**

(a) $(-\infty,-1]$ (b) $(-\infty,0]$

(c) $[1,\infty)$ (d) $(0,\infty)$

93. The least value of *k* for which the function x^2+kx+1 **is an increasing function in the interval** $1<x<2$ **is**

(a) – 4 (b) – 3

(c) – 1 (d) – 2

94. If $f(x)=x^3-6x^2+9x+3$ **be a decreasing function, then *x* lies in**

(a) $(-\infty,-1)\cap(3,\infty)$ (b) $(1, 3)$

(c) $(3,\infty)$ (d) None of these

95. **The function** $f(x) = \tan^{-1}(\sin x + \cos x)$**,** $x > 0$ **is always an increasing function on the interval**

(a) $(0, \pi)$ (b) $(0, \pi/2)$

(c) $(0, \pi/4)$ (d) $(0, 3\pi/4)$

96. **Let** $f(x) = x^3 + bx^2 + cx + d, 0 < b^2 < c$ **. Then *f***

(a) Is bounded (b) Has a local maxima

(c) Has a local minima (d) Is strictly increasing

97. **The function** $f(x) = x^{1/x}$ **is**

(a) Increasing in $(1, \infty)$ (b) Decreasing in $(1, \infty)$

(c) Increasing in $(1, e)$, decreasing in (e, ∞) (d) Decreasing in $(1, e)$, increasing in (e, ∞)

98. **For all** $x \in (0, 1)$

(a) $e^x < 1 + x$ (b) $\log_e(1 + x) < x$

(c) $\sin x > x$ (d) $\log_e x > x$

99. **Given function** $f(x) = \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)$ **is**

(a) Increasing (b) Decreasing

(c) Even (d) None of these

100. **If** $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1 + x^2} - x)$**, then** $f(x)$

(a) Increases in $[0, \infty)$ (b) Decreases in $[0, \infty)$

(c) Neither increases nor decreases in $(0, \infty)$ (d) Increases in $(-\infty, \infty)$

101. Rolle's theorem is not applicable to the function $f(x) = |x|$ defined on [–1, 1] because

(a) f is not continuous on [–1, 1] (b) f is not differentiable on (–1,1)

(c) $f(-1) \neq f(1)$ (d) $f(-1) = f(1) \neq 0$

102. From mean value theorem $f(b) - f(a) = (b-a)f'(x_1); a < x_1 < b$ if $f(x) = \frac{1}{x}$, then $x_1 =$

(a) $\sqrt{ab}$ (b) $\frac{a+b}{2}$

(c) $\frac{2ab}{a+b}$ (d) $\frac{b-a}{b+a}$

103. The function $f(x) = x(x+3)e^{-(1/2)x}$ satisfies all the conditions of Rolle's theorem in [–3, 0]. The value of *c* is

(a) 0 (b) –1

(c) – 2 (d) – 3

104. The function $f(x) = x^3 - 6x^2 + ax + b$ satisfy the conditions of Rolle's theorem in [1, 3]. The values of *a* and *b* are

(a) 11, – 6 (b) – 6, 11

(c) –11, 6 (d) 6, –11

105. In the Mean-Value theorem $\frac{f(b) - f(a)}{b-a} = f'(c)$, if $a = 0, b = \frac{1}{2}$ and $f(x) = x(x-1)(x-2)$, the value of *c* is

(a) $1 - \frac{\sqrt{15}}{6}$ (b) $1 + \sqrt{15}$ (c) $1 - \frac{\sqrt{21}}{6}$ (d) $1 + \sqrt{21}$

106. If from mean value theorem, $f'(x_1) = \frac{f(b) - f(a)}{b-a}$, then

(a) $a < x_1 \leq b$ (b) $a \leq x_1 < b$

(c) $a < x_1 < b$ (d) $a \leq x_1 \leq b$

107. If $f(x)$ satisfies the conditions of Rolle's theorem in $[1,2]$ and $f(x)$ is continuous in $[1,2]$ then $\int_1^2 f'(x)dx$ is equal to

(a) 3 (b) 0

(c) 1 (d) 2

108. Let $f(x)$ satisfy all the conditions of mean value theorem in [0, 2]. If *f* (0) = 0 and $|f'(x)| \le \frac{1}{2}$ for all *x*, in [0, 2] then

(a) $f(x) \le 2$ (b) $|f(x)| \le 1$

(c) $f(x) = 2x$ (d) $f(x) = 3$ for at least one *x* in [0, 2]

109. If the function $f(x) = x^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval $[1,3]$ and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$, then

(a) $a = -11$ (b) $a = -6$

(c) $a = 6$ (d) $a = 11$

110. The radius of the cylinder of maximum volume, which can be inscribed in a sphere of radius *R* is

(a) $\frac{2}{3}R$ (b) $\sqrt{\frac{2}{3}}R$

(c) $\frac{3}{4}R$ (d) $\sqrt{\frac{3}{4}}R$

111. Let $f(x) = \begin{cases} x^{\alpha}\ln x, & x > 0 \\ 0, & x = 0 \end{cases}$, Rolle's theorem is applicable to *f* for $x \in [0,1]$, if $\alpha =$

(a) – 2 (b) – 1

(c) 0 (d) $\frac{1}{2}$

112. The function $f(x)=\int_{-1}^{x} t(e^{t}-1)(t-1)(t-2)^{3}(t-3)^{5}dt$ **has a local minimum at** x **=**

(a) 0 (b) 1

(c) 2 (d) 3

113. If the function $f(x)=x^{3}-6ax^{2}+5x$ **satisfies the conditions of Lagrange's mean value theorem for the interval [1, 2] and the tangent to the curve** $y=f(x)$ **at** $x=\frac{7}{4}$ **is parallel to the chord that joins the points of intersection of the curve with the ordinates** $x=1$ **and** $x=2$**. Then the value of** a **is**

(a) $\frac{35}{16}$ (b) $\frac{35}{48}$ (c) $\frac{7}{16}$ (d) $\frac{5}{16}$

114. If $f(x)=x^{2}+2bx+2c^{2}$ **and** $g(x)=-x^{2}-2cx+b^{2}$ **such that** ***min*** $f(x)>$ ***max*** $g(x)$**, then the relation between** ***b*** **and** ***c*** **is**

(a) No real value of b and c (b) $0<c<b\sqrt{2}$

(c) $|c|<|b|\sqrt{2}$ (d) $|c|>|b|\sqrt{2}$

115. Let $h(x)=f(x)-(f(x))^{2}+(f(x))^{3}$ **for every real number** ***x*****. Then**

(a) h is increasing whenever f is increasing

(b) h is increasing whenever f is decreasing

(c) h is decreasing whenever f is decreasing

(d) Nothing can be said in general

116. The function $f(x)=\frac{\ln(\pi+x)}{\ln(e+x)}$ **is**

(a) Increasing on $[0,\infty)$

(b) Decreasing on $[0,\infty)$

(c) Decreasing on $\left[0,\frac{\pi}{e}\right)$ and increasing on $\left[\frac{\pi}{e},\infty\right)$

(d) Increasing on $\left[0,\frac{\pi}{e}\right)$ and decreasing on $\left[\frac{\pi}{e},\infty\right)$

117. In [0, 1] Lagrange's mean value theorem is NOT applicable to

(a) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$

(b) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(c) $f(x) = x|x|$

(d) $f(x) = |x|$

118. On the interval [0, 1], the function $x^{25}(1-x)^{75}$ takes its maximum value at the point

(a) 0

(b) 1/2

(c) 1/3

(d) 1/4

APPLICATIONS OF DERIVATIVES

HINTS AND SOLUTIONS

1. (3)

Let side = x and area = A then A = x^2.

$$\therefore \delta A = \frac{dA}{dx}\times dx = 2\times 3\times 0.01 = 0.06$$

2. (1)

$$\delta y = \frac{dy}{dx}\times \delta x = \frac{1}{3}\times 0.03 = 0.01$$

3. (3)

For a sphere of radius = r and volume = V, we have V = $\frac{4}{3}\pi r^3$

$$\Rightarrow \log V = \log\frac{4\pi}{3} + 3\log r$$

$$\Rightarrow \frac{\delta V}{V} = 0 + \frac{3\delta r}{r}$$

$$\Rightarrow \frac{\delta V}{V}\times 100 = \frac{3\delta r}{r}\times 100$$

4. (4)

$$y = x^n \Rightarrow \log y = n\log x$$

$$\Rightarrow \frac{\delta y}{y} = n\frac{\delta x}{x} \Rightarrow n:1$$

5. (4)

PV = Constant

$\Rightarrow \log P + \log V = \log C$

$\Rightarrow \frac{\delta P}{P} \times 100 + \frac{\delta V}{V} \times 100 = 0.$

6. (3)

Take f(x) = log x so that f′(x) = 1/x.

We have x = 4 and δx = 0.01

$f(x + \delta x) = f(x) + f'(x) \cdot \delta x$

$= 1.3868 + \frac{1}{4}(0.01) = 1.3893$

7. (1)

Consider $f(x) = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$ so that

$f'(x) = (-1/3)x^{-4/3} = -(1/3)(x^{-1/3})^4$

Take x = 1000, δx = –2

Then f(x) = 0.1 and $f'(x) = -\frac{1}{3} \times \frac{1}{10^4}$

$f(x + \delta x) \Omega 0.1 - \frac{1}{3 \times 10^4}(-2)$

$= 0.1 - (0.6666)10^{-4}$

8. (1)

If r is radius then C = 2πr and A = πr^2

$\therefore A = \frac{C^2}{4\pi}$

By log differentiation

$\frac{\delta A}{A} \times 100 = 2\frac{\delta C}{C} \times 100 = 2\frac{(0.01)}{28} \times 100 = \frac{1}{14}$

9. (2)

Let x = side and A = area for the equilateral triangle.

Then $A = \frac{\sqrt{3}}{4}x^2$

$\Rightarrow \log A = \log\frac{\sqrt{3}}{4} + 2\log x$

$\Rightarrow \frac{\delta A}{A}\times 100 = 2(0.5)$

10. (3)

Let d = diameter and V = volume for a sphere.

Then, $V = \frac{1}{6}\pi d^3$

$\Rightarrow \log V = \log\frac{\pi}{6} + 3\log d$

$\Rightarrow \frac{\delta V}{V}\times 100 = 3\frac{\delta d}{d}\times 100$

$= \frac{3(0.02)}{20}\times 100 = 0.3$

11. (1)

Let radius = r, surface area = S and volume = V for a sphere.

Then $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$

$\Rightarrow V = \frac{4\pi}{3}\left(\frac{S}{4\pi}\right)^{3/2}$

$\Rightarrow \log V = \log(\text{constant}) + \frac{3}{2}\log S$

$\Rightarrow \frac{\delta V}{V}\times 100 = \frac{3}{2}\frac{\delta S}{S}\times 100$

12. (1)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \log T = \log 2\pi + \frac{1}{2}\log \ell - \frac{1}{2}\log g$$

$$\Rightarrow \frac{\delta T}{T}\times 100 = \frac{1}{2}\frac{\delta \ell}{\ell}\times 100 = \frac{1}{2}(2) = 1.$$

13. (2)

Let r = radius and S = surface area.

Given height = 2r

$$\therefore S = 2\pi r(2r) + 2\pi r^2 = 6\pi r^2$$

$$\Rightarrow \frac{\delta S}{S}\times 100 = 2\frac{\delta r}{r}\times 100 = 2(0.5) = 1.$$

14. (1)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow \log T = \log 2\pi + \frac{1}{2}\log \ell - \frac{1}{2}\log g$$

$$\Rightarrow \frac{\delta T}{T} = \frac{1}{2}\frac{\delta \ell}{\ell}$$

15. (4)

$$\frac{dE}{dT} = 6.2 + 0.0006T^2$$

Take T = 100°, δT = 1°

$$\therefore \delta E = [6.2 + (0.0006)(100)^2]1 = 12.2$$

16. (2)

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Rightarrow 2a \cdot \delta a = 0 + 0 - 2bc(-\sin A)\delta A$$
$$\Rightarrow \delta a = \frac{bc\sin A}{a} \cdot \delta A = \frac{2\Delta}{a} \cdot \delta A$$

17. (3)

Let r = radius, h = height and V = volume of a cone.

Given that r : h = 1 : 2 $\Rightarrow$ h = 2r

$$V = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$$
$$\Rightarrow \frac{\delta V}{V} \times 100 = 3\frac{\delta r}{r} \times 100 = 3k$$

18. (1)

Let r = radius and h = depth.

Then r = 2, δr = 0.06 and h = 12.

Volume V = $\pi r^2 h = 12\pi r^2$

$$\therefore \delta V \underline{\Omega} \frac{dV}{dr} \cdot \delta r = 24\pi r \times \delta r = 24\pi \times 2 \times 0.06$$

19. (1)

Semi vertical angle = 45°

$\Rightarrow$ r = h and $\ell = h\sqrt{2}$

Take h = 20 and δh = 0.025

Let S = lateral surface area.

Then, $S = \pi h(h\sqrt{2}) = \pi\sqrt{2}h^2$

$$S + \delta S = \pi\sqrt{2}(20)^2 + 2\sqrt{2}\pi(20)(0.025)$$
$$= 401\sqrt{2}\pi$$

20. (3)

We have

a = 2R sin A, b = 2R sin B, c = 2R sin C.

δa = 2R cos AδA, δb = 2R cos B δB, δc = 0

$$\therefore \frac{\delta a}{\cos A} + \frac{\delta b}{\cos B} = 2R(\delta A + \delta B)$$
$$= 2R(\delta A + \delta B + \delta C) = 2R\delta(\pi) = 0$$

21. (2)

$$\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$$
$$\Rightarrow -\frac{1}{v^2} \cdot \delta v + \frac{1}{u^2} \cdot \delta u = -\frac{2}{f^2} \cdot \delta f$$
$$\Rightarrow \frac{\delta f}{f} = \frac{f}{2}\left[\frac{1}{v^2} - \frac{1}{u^2}\right]\alpha = \left(\frac{1}{v} + \frac{1}{u}\right)\alpha$$

22. (1)

Let f(x) = cos x so that f′(x) = –sin x.

Take x = 60° and δx = 1°

Then cos 60° = 0.5, sin 60° = 0.86603

$$\cos 61° \,\underline{\Omega}\, \cos 60° + (-\sin 60°) \times 1(0.001745)$$
$$= 0.5 - 0.0151 = 0.4849$$

23. (a) Here $u = 490$, $g = 9.8$ (downward)

Therefore, $S = \frac{u^2}{2g} = 12250$.

24. (b) $v^2 = a + bx \Rightarrow 2v\frac{dv}{dt} = b\frac{dx}{dt} \Rightarrow 2v\frac{dv}{dt} = bv \Rightarrow \frac{dv}{dt} = \frac{b}{2}$

Hence acceleration is constant or uniform.

25. (b) Given equation is $10s = 10ut - 49t^2$ or $s = ut - 4.9t^2$

$$\Rightarrow \frac{ds}{dt} = u - 9.8t = v$$

When stone reached the maximum height, then $v = 0$

$\Rightarrow u - 9.8t = 0 \Rightarrow u = 9.8t$

But time $t = 5$ *sec*

So the value of $u = 9.8 \times 5 = 49.0$ *m/sec*

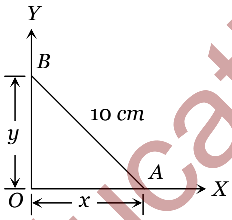
Hence initial velocity $= 49$ *m/sec*.

26. (a) Given $s = \frac{1}{2}gt^2 \Rightarrow \frac{ds}{dt} = gt$; Again $\frac{d^2s}{dt^2} = g$

27. (a) By figure, $x^2 + y^2 = 100$ .....(i)

$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ .....(ii)

$x = 8$



Therefore by (i) and (ii),

$\frac{dy}{dt} = -\frac{16}{6} = -\frac{8}{3} cm/\sec$.

$= \frac{8}{3} cm/\sec$.

28. (a) We know that area of a circle is $A = \pi R^2$

$\therefore \frac{dA}{dt} = 2\pi R\frac{dR}{dt} = 1.2\pi cm^2$.

29. (c) $\frac{ds}{dt} = 6t^2 - 18t + 12$ = velocity = 0

(when particle stopped)

$\Rightarrow 6t^2 - 18t + 12 = 0 \Rightarrow (t-1)(t-2) = 0$

Hence time 1, 2 *sec*.

30. (c) $\frac{ds}{dt}$ = velocity $= 45 + 22t - 3t^2$

When particle will come to rest, then $v = 0$

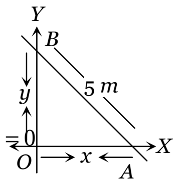
$\Rightarrow 3t^2 - 22t - 45 = 0 \Rightarrow t = 9, \left(\text{since } t \neq -\frac{5}{3}\right)$.

31. (a) According to fig. $x^2 + y^2 = 25$ .....(i)

Differentiate (i) *w.r.t .t*, we get

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$ .....(ii)

Here $x=4$ and $\frac{dx}{dt}=1.5$



From (i), $4^2+y^2=25 \Rightarrow y=3$

$\therefore$ From (ii), $2(4)(1.5)+2(3)\frac{dy}{dt}=0$

So, $\frac{dy}{dt}=-2m/\sec$

Hence, length of the highest point decreases at the rate of 2*m*/*sec*.

32. (a) $\frac{ds}{dt}=6t^2-18t+12$

Again $\frac{d^2s}{dt^2}=12t-18$ = acceleration

If acceleration becomes zero, then $0=12t-18$

$\Rightarrow t=\frac{3}{2}\sec$. Hence acceleration will be zero after $\frac{3}{2}$ *sec*.

33. (b) Given equation $s=at^2+bt+6$ ......(i)

Differentiating *w.r.t.* time, we get

Velocity (v) $=2at+b$ ......(ii)

After 4*sec*, $v=0$ and distance $s=16\ metres$

$\therefore 0=2a\times 4+b \Rightarrow 8a+b=0$ .....(iii)

and $16=16a+4b+6 \Rightarrow 16=16a+4(-8a)+6$

$\therefore a=-\frac{5}{8}$

But retardation in its motion is, $2a=\frac{-5}{4}m/\sec^2$

$\therefore$ Retardation $=\frac{5}{4}m/s^2$ (Retardation itself means –*ve*).

34. (a) $s=\frac{1}{2}vt \Rightarrow 2s=vt \Rightarrow 2\frac{ds}{dt}=v+t.\frac{dv}{dt}$

$\Rightarrow 2\frac{d^2s}{dt^2}=\frac{dv}{dt}+t.\frac{d^2v}{dt^2}+\frac{dv}{dt}$

But $\frac{dv}{dt}$ = acceleration (a)

$\Rightarrow 2a = a + t.\frac{da}{dt} + a \Rightarrow \frac{da}{dt} = 0$ or $t = 0$

But for whole notation $t = 0$ is impossible so that $\frac{da}{dt} = 0$ *i.e.*,a is constant.

35. (c) $\frac{d^2s}{dt^2} = -32\, unit$.

36. (b) Let velocity $v = 5\, cm/\sec$

(Increasing the rate/sec is called the velocity)

$\frac{da}{dt} = 5$ .....(i)

Where a is distance and t is time.

But if a is edge of a cube, then $V = a^3$

Differentiating *w.r.t.* time t, so

$\frac{dV}{dt} = 3a^2\frac{da}{dt} = 3a^2.5 = 15a^2 = 15 \times (12)^2$

$= 2160\ cm^3/\sec$ ($\because$ edge $a = 12\, cm$).

37. (c) Acceleration $f = \frac{dv}{dt} = 2t,$ then acceleration after 3 second $= 2 \times 3 = 6 cm/\sec^2$.

38. (b) Motion of a particle $s = 15t - 2t^2$

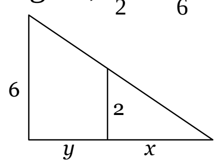
Therefore, velocity $\frac{ds}{dt} = 15 - 4t$

$\Rightarrow \left(\frac{ds}{dt}\right)_{t=0} = 15$ and $\left(\frac{ds}{dt}\right)_{t=3} = 3$

Therefore, average $= \frac{15+3}{2} = 9$.

39. (b) $\frac{dy}{dt} = 5, \frac{dx}{dt} = ?$

From figure, $\frac{x}{2} = \frac{x+y}{6} \Rightarrow 4x = 2y \Rightarrow x = \frac{1}{2}y$



Hence $\frac{dx}{dt} = \frac{1}{2}\frac{dy}{dt} = \frac{5}{2} metre\ /\ hour$.

40. (c) $\frac{dx}{dt} = v_x = a \Rightarrow \frac{d^2x}{dt^2} = 0 = a_x$

a_x is acceleration in x-axis

$\frac{d^2y}{dt^2} = -ba^2 \sin at \Rightarrow a_y = -a^2 y$

Hence, a_y changes as y changes.

41. (c) $\because$ Surface area $s = 4\pi r^2$ and $\frac{dr}{dt} = 2$

$\therefore \frac{ds}{dt} = 4\pi \times 2r\frac{dr}{dt} = 8\pi r \times 2 = 16\pi r \Rightarrow \frac{ds}{dt} \propto r$.

42. (d) $V = \frac{4}{3}\pi (x+10)^3$ where$x$ is thickness of ice.

$\therefore \frac{dV}{dt} = 4\pi(10+x)^2 \frac{dx}{dt}$

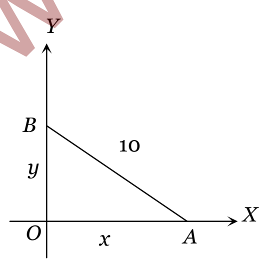
43. (b)

44. (a) Given that $dV/dt = 30\ ft^3/min$ and $r = 15\ ft$

$V = \frac{4}{3}\pi r^3;\ \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2} = \frac{30}{4 \times \pi \times 15 \times 15} = \frac{1}{30\pi}$ *ft/min*

45. (b) We have $x^2 + y^2 = 10^2$



$\Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=0 \Rightarrow x.3+y.(-4)=0$

$x=\frac{4}{3}y$. Thus, $\left(\frac{4}{3}y\right)^2+y^2=10^2 \Rightarrow y=6\,m$.

46. (c) $a+bv^2=x^2 \Rightarrow 0+b\left(2v.\frac{dv}{dt}\right)=2x.\frac{dx}{dt}$

$\Rightarrow v.b\frac{dv}{dt}=x.\frac{dx}{dt} \Rightarrow \frac{dv}{dt}=\frac{x}{b}$, $\left(\because \frac{dx}{dt}=v\right)$.

47. (c) $V=\frac{4}{3}\pi r^3$

Differentiate with respect to *t*,

$\frac{dV}{dt}=\frac{4}{3}\pi 3r^2.\frac{dr}{dt} \Rightarrow \frac{dr}{dt}=\frac{1}{4\pi r^2}.\frac{dV}{dt}$

$\frac{dr}{dt}=\frac{1}{4\times\pi\times15\times15}\times900 \Rightarrow \frac{dr}{dt}=\frac{1}{\pi}=\frac{7}{22}$.

48. (b) Volume $=V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt}=4\pi r^2.\frac{dr}{dt}$, at $r=7\,cm$

35 cc/min $=4\pi(7)^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt}=\frac{5}{28\pi}$

Surface area, $S=4\pi r^2$

$\frac{dS}{dt}=8\pi r\frac{dr}{dt}=\frac{8\pi.7.5}{28\pi}=10\ cm^2/min$.

49. (c) If x is the length of each side of an equilateral triangle and A is its area, then $A=\frac{\sqrt{3}}{4}x^2 \Rightarrow \frac{dA}{dt}=\frac{\sqrt{3}}{4}2x\frac{dx}{dt}$

Here, $x=10\ cm$ and $\frac{dx}{dt}=2\ cm/sec$

50. (a) $y^2=18x$

Differentiate both sides *w.r.t.t*

$2y\left(\frac{dy}{dt}\right)=18\left(\frac{dx}{dt}\right)$

$\Rightarrow 2y\left(2\frac{dx}{dt}\right)=18\left(\frac{dx}{dt}\right),\left(\because \frac{dy}{dt}=2\frac{dx}{dt}\right)$

$\therefore 4y = 18$ or $y = \frac{9}{2}$ and $x = \frac{y^2}{18} = \frac{9}{8}$

Hence the required point is $\left(\frac{9}{8}, \frac{9}{2}\right)$.

51. (d) Acceleration in direction of x-axis $= \frac{d^2x}{dt^2} = -2c$ and acceleration in direction of y-axis $= \frac{d^2y}{dt^2} = 2b$

Resultant acceleration is

$= \sqrt{(-2c)^2 + (2b)^2} = 2\sqrt{b^2 + c^2}$

52. (a) Given $s = \sqrt{t}$. Now $v = \frac{ds}{dt} = \frac{1}{2\sqrt{t}}$

Also $a = \frac{dv}{dt} = \frac{-1}{2 \times 2(t)^{3/2}} \Rightarrow a \propto \frac{1}{t\sqrt{t}}$ or $a \propto v^3$.

53. (c) $x + y = 10$; $\therefore y = 10 - x$ .....(i)

Now $f(x) = xy = x(10 - x) = 10x - x^2$

$\therefore f'(x) = 10 - 2x$

For maximum value of $f(x)$, $f'(x) = 0$

$\therefore x = 5$ and $y = 5$

So maximum value of $xy = 5 \times 5 = 25$.

54. (c) The necessary condition to be maximum or minimum for function $f'(x) = 0$ and for maximum $f''(x) = -ve$ and for minimum $f''(x) = +ve$.

Hence $f'(x) = 0$, but it is not sufficient.

55. (b) Let α, β be the roots of the equation

56. (d) $f'(x) = 6x^2 - 6x - 12$

$f'(x) = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$

Here $f(4) = 128 - 48 - 48 + 5 = 37$

$f(-1) = -2 - 3 + 12 + 5 = 12$

$f(2) = 16 - 12 - 24 + 5 = -15$

$f(-2) = -16 - 12 + 24 + 5 = 1$

Therefore the maximum value of function is 37 at $x = 4$.

57. (d) $x + y = 16 \Rightarrow y = 16 - x \Rightarrow x^2 + y^2 = x^2 + (16 - x)^2$

Let $z = x^2 + (16 - x)^2 \Rightarrow z' = 4x - 32$

To be minimum of $z, z'' > 0$, and it is.

Therefore $4x - 32 = 0 \Rightarrow x = 8 \Rightarrow y = 8$

58. (d) $f(x) = \int_0^x te^{-t^2} dt \Rightarrow f'(x) = xe^{-x^2} = 0 \Rightarrow x = 0$

$f''(x) = e^{-x^2}(1 - 2x^2); \quad f''(0) = 1 > 0$

$\therefore$ Minimum value $f(0) = 0$.

59. (d) Let a and b are given, then area $A = \frac{1}{2} ab \sin C \Rightarrow \frac{dA}{dC} = \frac{1}{2} ab \cos C$

Hence A is maximum, when $\frac{dA}{dC} = 0 \Rightarrow C = 90°$.

60. (c) Given function $f : R \to R$ is to be maximum, if $f'(a) = 0$ and $f''(a) < 0$.

61. (d) Let $f(x) = 2x^3 - 24x + 107$

At $x = -3, \ f(-3) = 2(-3)^3 - 24(-3) + 107 = 125$

At $x = 3, \ f(3) = 2(3)^3 - 24(3) + 107 = 89$

For maxima or minima, $f'(x) = 6x^2 - 24 = 0$

$\Rightarrow x = 2, -2$

So at $x = 2, \ f(2) = 2(2)^3 - 24(2) + 107 = 75$

At $x = -2, \ f(-2) = 2(-2)^3 - 24(-2) + 107 = 139$

Thus the maximum value of the given function in $[-3, 3]$ is 139.

62. (c) It is a fundamental property.

63. (c) We know that perimeter of a rectangle $s = 2(x + y)$, where x and y are adjacent sides

$\Rightarrow y = \frac{S-2x}{2}$. Now area of rectangle,

$A = xy = \frac{x}{2}(S-2x) = \frac{1}{2}(Sx - 2x^2)$

64. (a) $xy = 1 \Rightarrow y = \frac{1}{x}$ and let $z = x + y$

$z = x + \frac{1}{x} \Rightarrow \frac{dz}{dx} = 1 - \frac{1}{x^2}$

65. (b) Given $2(a+b) = 36$, $a + b = 18$

Area of rectangle $= ab = a(18-a)$

$A = 18a - a^2$, $\therefore \frac{dA}{da} = 18 - 2a$

66. (d) Let $y = \frac{\log x}{x} \Rightarrow \frac{dy}{dx} = \frac{x.\frac{1}{x} - \log x}{x^2} = \frac{1-\log x}{x^2}$

Put $\frac{dy}{dx} = 0 \Rightarrow \frac{1-\log x}{x^2} = 0$

$\Rightarrow 1 - \log x = 0 \Rightarrow x = e$ and $\frac{d^2y}{dx^2} = \frac{-3x + 2x\log x}{x^4}$

At $x = e$, $\frac{d^2y}{dx^2} = \frac{1}{-e^3} < 0$

$\therefore$ In $[2, \infty)$ the function $\frac{\log x}{x}$ will be maximum and minimum value does not exist.

67. (a) $f(x) = 2x + 3y$ when $xy = 6$

$f(x) = 2x + 3y = 2x + \frac{18}{x}$

$f'(x) = 2 - \frac{18}{x^2} = 0$

$\Rightarrow x = \pm 3$ and $f''(x) = \frac{36}{x^3} \Rightarrow f''(3) > 0$

Putting $x = +3$, we get the minimum value to be 12.

68. (c) Concept

69. (d) $\frac{dy}{dx}=\frac{a}{x}+2bx+1 \Rightarrow \left(\frac{dy}{dx}\right)_{x=1}=a+2b+1=0 \Rightarrow a=-2b-1$

and $\left(\frac{dy}{dx}\right)_{x=2}=\frac{a}{2}+4b+1=0$

$\Rightarrow \frac{-2b-1}{2}+4b+1=0 \Rightarrow \qquad -b+4b+\frac{1}{2}=0$

$\Rightarrow 3b=\frac{-1}{2} \Rightarrow b=\frac{-1}{6}$ and $a=\frac{1}{3}-1=\frac{-2}{3}$.

70. (a) Standard Problem

71. (b) $xy=c^2 \Rightarrow y=\frac{c^2}{x} \Rightarrow f(x)=ax+by=ax+\frac{bc^2}{x}$

Differentiate with respect to x $f'(x)=a-\frac{bc^2}{x^2}$

Put $f'(x)=0 \Rightarrow ax^2-bc^2=0$

$\Rightarrow x^2=\frac{bc^2}{a} \Rightarrow x=\pm c\sqrt{b/a}$

At $x=+c\sqrt{b/a},\ ax+by$ will be minimum.

The minimum value $f\left(c\sqrt{\frac{a}{b}}\right)=a.c\sqrt{\frac{a}{b}}+\frac{bc^2}{c}.\sqrt{\frac{b}{a}}$

$= 2c\sqrt{ab}$.

72. (a) Let $f(A)=\cos A\cos B=\cos A\cos\left(\frac{\pi}{2}-A\right)=\cos A\sin A$

$\therefore f'(A)=\cos^2 A-\sin^2 A=\cos 2A$

Now, $f'(A)=0 \Rightarrow \cos 2A=0 \Rightarrow 2A=\frac{\pi}{2} \Rightarrow A=\frac{\pi}{4}$

Now $f''(A)=-2\sin 2A=-2\sin\frac{\pi}{2}=-2 \quad (-ve)$

73. (b) Let $f(x)=4e^{2x}+9e^{-2x}$

$\therefore f'(x)=8e^{2x}-18e^{-2x}$

Put $f'(x)=0 \Rightarrow 8e^{2x}-18e^{-2x}=0$

$e^{2x}=3/2 \Rightarrow x=\log(3/2)^{1/2}$

Again $f''(x) = 16e^{2x} + 36e^{-2x} > 0$

Now $f(\log(3/2)^{1/2}) = 4e^{2.(\log(3/2)^{1/2})} + 9e^{-2(\log(3/2)^{1/2})}$

$= 4 \times \frac{3}{2} + 9 \times \frac{2}{3} = 6 + 6 = 12$

Hence minimum value = 12.

74. (d) Let $PQ = a$ and $PR = b$, then $\Delta = \frac{1}{2} ab \sin\theta$

$\because -1 \le \sin\theta \le 1$

Since, area is maximum when $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$.

75. (c) $a^2x^4 + b^2y^4 = c^6 \Rightarrow y = \left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4}$

Hence $f(x) = xy = x\left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4}$

$\Rightarrow f(x) = \left(\frac{c^6x^4 - a^2x^8}{b^2}\right)^{1/4}$

Differentiate $f(x)$ with respect to *x*, then

$f'(x) = \frac{1}{4}\left(\frac{c^6x^4 - a^2x^8}{b^2}\right)^{-3/4}\left(\frac{4x^3c^6}{b^2} - \frac{8x^7a^2}{b^2}\right)$

Put $f'(x) = 0$, $\frac{4x^3c^6}{b^2} - \frac{8x^7a^2}{b^2} = 0$

$\Rightarrow x^4 = \frac{c^6}{2a^2} \Rightarrow x = \pm\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$

At $x = \frac{c^{3/2}}{2^{1/4}\sqrt{a}}$ the $f(x)$ will be maximum

76. (c) Given $y = e^{(2x^2 - 2x + 1)\sin^2 x}$

For minima or maxima, $\frac{dy}{dx} = 0$

$\therefore e^{(2x^2-2x+1)\sin^2 x}[(4x-2)\sin^2 x + 2(2x^2 - 2x + 1)\sin x \cos x] = 0$

$\Rightarrow [(4x-2)\sin^2 x + 2(2x^2 - 2x + 1)\sin x \cos x] = 0$

$\Rightarrow 2\sin x[(2x-1)\sin x+(2x^2-2x+1)\cos x]=0$

$\Rightarrow \sin x = 0$

$\therefore y$ is minimum for $\sin x = 0$

77. (b) Let $y=\frac{x^2-x+1}{x^2+x+1}$

$$\Rightarrow \frac{dy}{dx}=\frac{(x^2+x+1)(2x-1)-(x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x^2-2}{(x^2+x+1)^2}=0 \Rightarrow 2x^2-2=0 \Rightarrow x=-1,+1$$

$$\frac{d^2y}{dx^2}=\frac{4(-x^3+3x+1)}{x^2+x+1}$$

78. (b) Standard Problem

79. (c) $y=\sin x(1+\cos x)=\sin x+\frac{1}{2}\sin 2x$

$\therefore \frac{dy}{dx}=\cos x+\cos 2x$ and $\frac{d^2y}{dx^2}=-\sin x-2\sin 2x$

On putting $\frac{dy}{dx}=0$, $\cos x+\cos 2x=0$

$\Rightarrow \cos x=-\cos 2x=\cos(\pi-2x) \Rightarrow x=\pi-2x$

$$\therefore x=\frac{\pi}{3};\ \therefore \left(\frac{d^2y}{dx^2}\right)_{x=\pi/3}=-\sin\left(\frac{1}{3}\pi\right)-2\sin\left(\frac{2}{3}\pi\right)$$

$$=\frac{-\sqrt{3}}{2}-2.\frac{\sqrt{3}}{2}=\frac{-3\sqrt{3}}{2}$$, which is negative.

80. (d) Standard Problem

81. (a) Standard Problem

82. (a) Standard Problem

83. (c) Let $f(x)=\sin x-bx+c$

$\therefore f'(x)=\cos x-b>0$ or $\cos x>b$ or $b<-1$.

84. (d) $f(x) = (x+2)e^{-x}$

$f'(x) = e^{-x} - e^{-x}(x+2)$

85. (d) Since $f(x) = \frac{K\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all x, therefore $f'(x) > 0$ for all x

$\Rightarrow \frac{K-2}{(\sin x + \cos x)^2} > 0$ for all x

$\Rightarrow K - 2 > 0 \Rightarrow K > 2$.

86. (a) $f(x) = 2x^3 - 3x^2 - 36x + 7$

$\Rightarrow f'(x) = 6x^2 - 6x - 36$ but for decreasing $f'(x) < 0$

$\Rightarrow x^2 - x - 6 < 0 \Rightarrow (x-3)(x+2) < 0 \Rightarrow -2 < x < 3$

Hence the required interval is (–2, 3).

87. (b) To be increasing $f'(x) = 3x^2 - 27 > 0$

$\Rightarrow x^2 > 9 \Rightarrow |x| > 3$.

88. (c) $f'(x) = 3kx^2 - 18x + 9 = 3[kx^2 - 6x + 3] > 0, \forall x \in R$

$\therefore \Delta = b^2 - 4ac < 0$, $k > 0$ *i.e.*, $36 - 12k < 0$ or $k > 3$.

89. (d) The function is monotonic increasing, if $f'(x) > 0$

$\Rightarrow \frac{(2\sin x + 3\cos x)(\lambda\cos x - 6\sin x)}{(2\sin x + 3\cos x)^2} - \frac{(\lambda\sin x + 6\cos x)(2\cos x - 3\sin x)}{(2\sin x + 3\cos x)^2} > 0$

$\Rightarrow 3\lambda(\sin^2 x + \cos^2 x) - 12(\sin^2 x + \cos^2 x) > 0$

$\Rightarrow 3\lambda - 12 > 0 \Rightarrow \lambda > 4$.

90. (d) If $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically for all $x \in R$, then $f'(x) \le 0$ for all $x \in R$

$\Rightarrow 3(a+2)x^2 - 6ax + 9a \le 0$ for all $x \in R$

$\Rightarrow (a+2)x^2 - 2ax + 3a \le 0$ for all $x \in R$

$\Rightarrow a + 2 < 0$ and Discriminant ≤ 0

$\Rightarrow a < -2, -8a^2 - 24a \le 0 \Rightarrow a < -2$ and $a(a+3) \ge 0$

$\Rightarrow a < -2$, $a \le -3$ or $a \ge 0 \Rightarrow a \le -3 \Rightarrow -\infty < a \le -3$

91. (b) Let $y = \frac{a\sin x + b\cos x}{c\sin x + d\cos x}$

The function will be decreasing, when $\frac{dy}{dx} < 0$.

$\frac{(c\sin x + d\cos x)(a\cos x - b\sin x) - (a\sin x + b\cos x)(c\cos x - d\sin x)}{(c\sin x + d\cos x)^2} < 0$

$\Rightarrow ac\sin x\cos x - bc\sin^2 x + ad\cos^2 x$

$- bd\sin x\cos x - ac\sin x\cos x + ad\sin^2 x \ - bc\cos^2 x + bd\sin x\cos x < 0$

$\Rightarrow ad(\sin^2 x + \cos^2 x) - bc(\sin^2 x + \cos^2 x) < 0$

$\Rightarrow (ad - bc) < 0$.

92. (d) $y = \frac{1}{1+x^2} \Rightarrow \frac{dy}{dx} = -\frac{2x}{(1+x^2)^2}$

To be decreasing, $-\frac{2x}{(1+x^2)^2} < 0 \Rightarrow x > 0 \Rightarrow x \in (0,\infty)$.

93. (d) To be increasing, $\frac{d}{dx}(x^2 + kx + 1) > 0 \Rightarrow 2x + k > 0$

For $x \in (1,2)$, the least value of k is –2.

94. (b) $f(x) = x^3 - 6x^2 + 9x + 3$, For decreasing $f'(x) < 0$

$\Rightarrow 3x^2 - 12x + 9 < 0 \Rightarrow x^2 - 4x + 3 < 0$

$\Rightarrow (x-3)(x-1) < 0$, $\therefore x \in (1,3)$.

95. (c) $f(x) = y = \tan^{-1}\left(\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right)$

$\Rightarrow \tan y = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) \Rightarrow \sec^2 y\frac{dy}{dx} = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$

$\frac{dy}{dx} > 0 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) > 0$. $\therefore x \in \left(0, \frac{\pi}{4}\right)$.

96. (d) Given $f(x) = x^3 + bx^2 + cx + d$

$\therefore f'(x) = 3x^2 + 2bx + c$

Now its discriminant $= 4(b^2 - 3c)$

$\Rightarrow 4(b^2 - c) - 8c < 0$, as $b^2 < c$ and $c > 0$

Therefore, $f'(x) > 0$ for all $x \in R$

Hence f is strictly increasing.

97. (c) Let $y = x^{1/x} \Rightarrow \log y = \frac{1}{x}\log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2} = \frac{1-\log x}{x^2} \Rightarrow \frac{dy}{dx} = x^{1/x}\left(\frac{1-\log x}{x^2}\right)$

Now, $x^{1/x} > 0$ for all x and $\frac{1-\log x}{x^2} > 0$ in (1, e) and $\frac{1-\log x}{x^2} < 0$ in (e, ∞)

$\therefore f(x)$ is increasing in (1, e) and decreasing in (e, ∞).

98. (b) Both e^x and $1+x$ are increasing and $\sqrt{e} \ge 1 + \frac{1}{2}$, because $\sqrt{e} = 1.65$ nearly. so the answer (a) is not correct. Since $\sin\frac{\pi}{6} < \frac{\pi}{6}$ because $\frac{1}{2} < \frac{22}{42}$. So, (c) is not correct. $\log\frac{1}{2} < \frac{1}{2}$ because $\log\frac{1}{2}$ is negative.

99. (a) $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$

$\Rightarrow$(i) $f(-x) = \frac{e^{-2x}-1}{e^{-2x}+1} = \frac{1-e^{2x}}{1+e^{2x}} \Rightarrow f(x) = -\frac{e^{2x}-1}{e^{2x}+1} = -f(x)$

$f(x)$ is an odd function.

Again $f(x) = \frac{e^{2x}-1}{e^{2x}+1} \Rightarrow f'(x) = \frac{4e^{2x}}{(1+e^{2x})^2} > 0 \; \forall n \in R$

$\Rightarrow f(x)$ is an increasing function

100. (d) We have $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1+x^2} - x)$

$\therefore f'(x) = 2 - \frac{1}{1-x^2} + \frac{1}{\sqrt{1+x^2} - x}\left(\frac{x}{\sqrt{1-x^2}} - 1\right)$

$= \frac{1+2x^2}{1+x^2} - \frac{1}{\sqrt{1+x^2}} = \frac{1+2x^2}{1+x^2} - \frac{\sqrt{(1+x^2)}}{1+x^2}$

$= \frac{x^2 + \sqrt{1+x^2}(\sqrt{1+x^2} - 1)}{1+x^2} \ge 0$ for all x

Hence $f(x)$ is an increasing function on $(-\infty, \infty)$ and in particularon $[0, \infty)$.

101. (b) $f(x)=\begin{cases}-x, \text{when } -1\le x<0\\ x, \text{when } 0\le x\le 1\end{cases}$

Clearly $f(-1)=|-1|=1=f(1)$

But $Rf'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{|h|}{h}=\lim_{h\to 0}\frac{h}{h}=1$

$Lf'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{h}=\lim_{h\to 0}\frac{|-h|}{-h}=\lim_{h\to 0}\frac{h}{-h}=-1$

$\therefore Rf'(0)\ne Lf'(0)$

Hence it is not differentiable on $(-1, 1)$.

102. (a) $f'(x_1)=\frac{-1}{x_1^2}$

$\therefore \frac{-1}{x_1^2}=\frac{\frac{1}{b}-\frac{1}{a}}{b-a}=-\frac{1}{ab}\Rightarrow x_1=\sqrt{ab}$.

103. (c) To determine 'c' in Rolle's theorem, $f'(c)=0$.

Here $f'(x)=(x^2+3x)e^{-(1/2)x}.\left(-\frac{1}{2}\right)+(2x+3)e^{-(1/2)x}$

$=e^{-(1/2)x}\left\{-\frac{1}{2}(x^2+3x)+2x+3\right\}$

$=-\frac{1}{2}e^{-(x/2)}\{x^2-x-6\}$

$\therefore f'(c)=0\Rightarrow c^2-c-6=0\Rightarrow c=3,-2,$

But $c=3\notin[-3,0]$.

104. (a) $f(1)=f(3)\Rightarrow a+b-5=3a+b-27\Rightarrow a=11$, which is given in option (a) only.

105. (c) From mean value theorem $f'(c)=\frac{f(b)-f(a)}{b-a}$

$a=0, f(a)=0\Longrightarrow b=\frac{1}{2}, f(b)=\frac{3}{8}$

$f'(x)=(x-1)(x-2)+x(x-2)+x(x-1)$

$f'(c)=(c-1)(c-2)+c(c-2)+c(c-1)$

$=c^2-3c+2+c^2-2c+c^2-c$

$f'(c) = 3c^2 - 6c + 2$

According to mean value theorem, $f'(c) = \frac{f(b) - f(a)}{b - a}$

$\Rightarrow 3c^2 - 6c + 2 = \frac{(3/8) - 0}{(1/2) - 0} = \frac{3}{4} \Rightarrow 3c^2 - 6c + \frac{5}{4} = 0$

$c = \frac{6 \pm \sqrt{36 - 15}}{2 \times 3} = \frac{6 \pm \sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}$.

106. (c) According to mean value theorem,

In interval $[a, b]$ for $f(x)$

$\frac{f(b) - f(a)}{b - a} = f'(c)$, where $a < c < b$

$\therefore a < x_1 < b$.

107. (b) $\int_1^2 f'(x)dx = [f(x)]_1^2 = f(2) - f(1) = 0$

$f(2) = f(1)]$)

108. (b)

109. (d) $f(x) = x^3 - 6x^2 + ax + b$

$\Rightarrow f'(x) = 3x^2 - 12x + a$

$\Rightarrow f'(c) = 0 \Rightarrow f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$

$\Rightarrow 3\left(2 + \frac{1}{\sqrt{3}}\right)^2 - 12\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$

$\Rightarrow 3\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) - 12\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$

$12 + 1 + 4\sqrt{3} - 24 - 4\sqrt{3} + a = 0 \Rightarrow a = 11$.

110. (b) Standard Problem

111. (d) For Rolle's theorem to be applicable to *f*, for $x \in [0,1]$, we should have (i) $f(1) = f(0)$,

(ii) *f* is continuous for $x \in [0,1]$ and *f* is differentiable for $x \in (0,1)$

From (i), $f(1)=0$, which is true.

From (ii), $0=f(0)=f(0_+)=\lim_{x\to 0_+} x^{\alpha}\ln x$

Which is true only for positive values of α, thus (d) is correct

112. (b,d) $f(x)=\int_{-1}^{x} t(e^t-1)(t-1)(t-2)^3(t-3)^5 dt$

$\therefore f'(x)=x(e^x-1)(x-1)(x-2)^3(x-3)^5$

113. (b) $f(b)=f(2)=8-24a+10=18-24a$

$f(a)=f(1)=1-6a+5=6-6a$

$f'(x)=3x^2-12ax+5$

From Lagrange's mean value theorem,

$f'(x)=\dfrac{f(b)-f(a)}{b-a}=\dfrac{18-24a-6+6a}{2-1}$

$\therefore f'(x)=12-18a$

At $x=\dfrac{7}{4},\ 3\times\dfrac{49}{16}-12a\times\dfrac{7}{4}+5=12-18a$

$\Rightarrow 3a=\dfrac{147}{16}-7 \Rightarrow 3a=\dfrac{35}{16}\Rightarrow a=\dfrac{35}{48}$.

114. (d) $f(x)=(x+b)^2+2c^2-b^2$ is minimum at $x=-b$ and $g(x)=b^2+c^2-(x+c)^2$ is maximum at $x=-c$

$\Rightarrow 2c^2-b^2>b^2+c^2 \Rightarrow |c|>\sqrt{2}|b|$.

115. (a,c) $h(x)=f(x)-[f(x)]^2+[f(x)]^3$

$h'(x)=f'(x)-2f(x)f'(x)+3[f(x)]^2f'(x)$

$=f'(x)[1-2f(x)+3[f(x)]^2]$

$=3f'(x)\left\{\left(f(x)-\dfrac{1}{3}\right)^2+\dfrac{2}{9}\right\}$

$\therefore h'(x)$ and $f'(x)$ have same sign.

116. (b) Let $f(x)=\dfrac{\ln(\pi+x)}{\ln(e+x)}$

$$\therefore f'(x)=\frac{\ln(e+x)\times\frac{1}{\pi+x}-\ln(\pi+x)\frac{1}{e+x}}{\ln^2(e+x)}$$

$$=\frac{(e+x)\ln(e+x)-(\pi+x)\ln(\pi+x)}{\ln^2(e+x)\times(e+x)(\pi+x)}$$

$\Rightarrow f'(x)<0$ for all $x\geq 0$, $\{\because \pi>e\}$

Hence $f(x)$ is decreasing in $[0,\infty)$.

117. (a) The function defined in option (a) is not differentiable at $x=\frac{1}{2}$.

118. (d) $f(x)=x^{25}(1-x)^{75}$

$f'(x)=x^{25}(75)(1-x)^{74}(-1)+25x^{24}(1-x)^{75}$

For maxima and minima,

$-75x^{25}(1-x)^{74}+25x^{24}(1-x)^{75}=0$

$\Rightarrow 25x^{24}(1-x)^{74}[(1-x)-3x]=0$

$\Rightarrow$ Either $x=0$ or $x=1$ or $x=\frac{1}{4}$

At $x=\frac{1}{4}$, $f'\left(\frac{1}{4}-h\right)>0$ and $f'\left(\frac{1}{4}+h\right)<0$

$\therefore f(x)$ is maximum at $x=\frac{1}{4}$.