Theory of Equations Part 2

Long Answer Questions:

1. **If α, β, γ are the roots of $x^3 - 6x^2 + 11x - 6 = 0$, then find the equation whose roots are $\alpha^2 + \beta^2, \beta^2 + \gamma^2, \gamma^2 + \alpha^2$**

Sol: **1st Method**

Let α, β, γ are the roots of the equation

$x^3 - 6x^2 + 11x - 6 = 0$

$\therefore \alpha + \beta + \gamma = 6, \alpha\beta + \beta\gamma + \gamma\alpha = 11$

Let $y = \alpha^2 + \beta^2 = \alpha^2 + \beta^2 + \gamma^2 - \gamma^2$

$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) - x^2$

$= 36 - 22 - x^2$

$\Rightarrow x^2 = 14 - y$

$\Rightarrow x = \sqrt{14 - y}$

Substitute $x = \sqrt{14 - y}$ in $x^3 - 6x^2 + 11x - 6 = 0$

$\Rightarrow \left(\sqrt{14 - y}\right)^3 - 6\left(\sqrt{14 - y}\right)^2 + 11\left(\sqrt{14 - y}\right) - 6 = 0$

$\Rightarrow (14 - y)\sqrt{14 - y} - 6(14 - y) + 11\sqrt{14 - y} - 6 = 0$

$\Rightarrow -6(14 - y + 1) = \sqrt{14 - y}\,[-11 - 14 + y]$

$\Rightarrow -6(15 - y) = \left(\sqrt{14 - y}\right)(y - 25)$

Squaring on both sides

i.e., $\left[-6(15 - y)\right]^2 = \left[\sqrt{14 - y}\,(y - 25)\right]^2$

$\Rightarrow 36\left(225 - 30y + y^2\right)$

$= (14 - y)(y^2 - 50y + 625)$

$\Rightarrow 1800 - 1080y + 36y^2$

$= 14y^2 - 700y + 8750 - y^3 + 50y^2 - 625y$

$\Rightarrow y^3 - 28y^2 + 245y - 650 = 0$

Sol: 2nd Method

Let α, β, γ are the roots of $x^3 - 6x^2 + 11x - 6 = 0$

It is an odd degree reciprocal equation of class two.

$\therefore$ $x-1$ is a factor of $x^3 - 6x^2 + 11x - 6$

	1	-6	11	-6
x = 1	0	1	-5	6
	1	-5	6	0

$\therefore x^3 - 6x^2 + 11x - 6 = (x-1)(x^2 - 5x + 6)$

$= (x-1)(x-2)(x-3)$

$\therefore$ The roots of $x^3 - 6x^2 + 11x - 6 = 0$ are

$\alpha = 1, \beta = 2, \gamma = 3$

Now $\alpha^2 + \beta^2 = 1 + 2^2 = 5$

$\beta^2 + \gamma^2 = 2^2 + 3^2 = 13$

$\gamma^2 + \alpha^2 = 3^2 + 1^2 = 10$

Therefore the cubic equation with roots

$\alpha^2 + \beta^2, \beta^2 + \gamma^2, \gamma^2 + \alpha^2$ is

$(x-5)(x-13)(x-10) = 0$

$x^3 - (5+13+10)x^2 + (65+130+50)x - 650 = 0$

(i.e.,) $x^3 - 28x^2 + 245x - 650 = 0$

2. If α,β,γ are the roots of $x^3-7x+6=0$, then find the equation whose roots are $(\alpha-\beta)^2,(\beta-\gamma)^2,(\gamma-\alpha)^2$

Sol: 1st Method

Let α,β,γ are the roots of the equation

$x^3-7x+6=0 \quad \text{........(1)}$

$\alpha+\beta+\gamma=0,\ \alpha\beta\gamma=-6$

Let $y=(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$=(-\gamma)^2-4\left(\frac{6}{\gamma}\right)$

$=\gamma^2+\frac{24}{\gamma}=x^2+\frac{24}{x}$

$\Rightarrow xy=x^3+24$

$\Rightarrow xy=7x-6+24$ [From (1)]

$\Rightarrow x(y-7)=18$

$\Rightarrow x=\frac{18}{y-7}$

Substitution $x=\frac{18}{y-7}$ in $x^3-7x+6=0$

$\left(\frac{18}{y-7}\right)^3-7\left(\frac{18}{y-7}\right)+6=0$

$\Rightarrow (18)^3-7(18)(y-7)^2+6(y-7)^3=0$

$\Rightarrow 5832-126(y^2-14y+49)+6(y^3-21y^2+147y-343)=0$

$\Rightarrow y^3-42y^2+441y-400=0$

$\therefore$ The equation with roots

$\therefore (\alpha-\beta)^2,(\beta-\gamma)^2,(\gamma-\alpha)^2$ is $x^3-42x^2+441x-400=0$

Sol: 2nd Method

α, β, γ are the roots of $x^3 - 7x + 6 = 0$

By trial and error method x = 1 satisfy this equation.

$\therefore x-1$ is a factor of $x^3 - 7x + 6$

x = 1	1	0	-7	6
	0	1	1	-6
	1	1	-6	0

$\therefore x^3 - 7x + 6 = (x-1)(x^2 + x - 6)$

$= (x-1)(x+3)(x-2)$

$\because \alpha, \beta, \gamma$ are the roots of $x^3 - 7x + 6 = 0$

$\alpha = 1, \beta = -3, \gamma = 2$

Now $(\alpha - \beta)^2 = [1-(-3)]^2 = (4)^2 - 16$

$(\beta - \gamma)^2 = [-3-2]^2 = 25$

$(\gamma - \alpha)^2 = [2-1]^2 = 1$

$\therefore$ The cubic equation whose roots are

$(\alpha-\beta)^2, (\beta-\gamma)^2 (\gamma-\alpha)^2$, is

$(x-16)(x-25)(x-1) = 0$

$\Rightarrow x^3 - (16+25+1)x^2 + (400+25+16)x - 400 = 0$

$x^3 - 42x^2 + 441x - 400 = 0$

3. If α, β, γ are the roots of the equation $x^3 - 3ax + b = 0$, then prove that $\Sigma(\alpha-\beta)(\alpha-\gamma) = 9a$.

Sol: Given α, β, γ are the roots of

$x^3 - 3ax + b = 0$

$\therefore \alpha+\beta+\gamma = 0, \alpha\beta+\beta\gamma+\gamma\alpha = -3a, \alpha\beta\gamma = -b$ Now $\Sigma(\alpha-\beta)(\alpha-\gamma) =$

$= \Sigma\left[\alpha^2 - \alpha\beta - \alpha\gamma + \beta\gamma\right]$

$= (\alpha^2+\beta^2+\gamma^2) - (\alpha\beta+\beta\gamma+\gamma\alpha)$

$= (\alpha+\beta+\gamma)^2 - 3(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= 0 - 3(-3a)$

$= 9a$

$\therefore \Sigma(\alpha-\beta)(\alpha-\gamma) = 9a$

4. Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, given that the product of two of the roots is 6

Sol: Suppose $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + x^3 - 16x^2 - 4x + 48 = 0$ -------------- (1)

Sum of the roots $\Rightarrow \alpha+\beta+\gamma+\delta = -1$ and product of the roots $= \alpha\beta\gamma\delta = 48$

$\therefore$ Product of two roots is 6

Let $\alpha\beta = 6$

From (1) $\gamma\delta = \frac{48}{\alpha\beta} = \frac{48}{6} = 8$

Let $\alpha + \beta = p$ *and* $\gamma + \delta = q$

The equation having roots α, β is

$x^2 + (\alpha+\beta)x + \alpha\beta = 0$

i.e., $x^2 - px + 6 = 0$ --------- (2)

The equation having the roots γ, δ is

$x^2 - (\gamma+\delta)x + \gamma\delta = 0$

i.e., $x^2 - qx + 8 = 0$ ------------- (3)

$\therefore$ From (1), (2) and (3)

$x^4 + x^3 - 16x^2 - 4x + 48$

$= (x^2 - px + 6)(x^2 - qx + 8)$

$= x^4 - (p+q)x^3 + (pq+14)x^2 - (8p+86q)x + 48$

Comparing the like terms

$p + q = -q, 8p + 6q = 4$

i.e., $4p + 3q = 2$

Solving $4p + 4 = -4$

$4p + 3q = 2$

$- \quad - \quad -$

$q = -6$

$\therefore p = -1 + 6 = 5$

Substitute the value of p is eq (2)

$x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$

Substitute, the value of q in eq(3)

$x^2 + 6x + 8 = 0 \Rightarrow x = -2, = -4$

$\therefore$ The roots of the given

Equation are $-4, -2, 2, 3$

5. Solve $8x^4 - 2x^3 - 27x^2 + 6x + 9 = 0$ given that two roots have the same absolute value, but are opposite in signs

Sol: Suppose $\alpha, \beta, \gamma, \delta$ are the roots of the equation

$8x^4 - 2x^3 - 27x^2 + 6x + 9 = 0$

i.e., $x^4 - \frac{1}{4}x^3 - \frac{27}{8}x^2 + \frac{3}{4}x + \frac{9}{8} = 0$ --------(1)

Sum of the root $\alpha + \beta + \gamma + \delta = \frac{1}{4}$ *and*

Product of the roots $\alpha\beta\gamma\delta = \frac{9}{8}$

Given $\beta = -\alpha \Rightarrow \alpha + \beta = 0$

$\therefore 0 + \gamma + \delta = \frac{1}{4} \Rightarrow \gamma + \delta = \frac{1}{4}$

Let $\alpha\beta = p, \gamma\delta = q$, so that $pq = \frac{9}{8}$

The equation having the roots

α, β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e., $x^2 + p = 0$

The equation having the roots

γ, δ is $x^2 - (\gamma + \delta)x + \gamma\delta = 0$

i.e., $x^2 - \frac{1}{4}x + q = 0$

From (1), (2) and (3)

$$x^4 - \frac{1}{4}x^3 - \frac{27}{8}x^2 + \frac{3}{4} + \frac{9}{8} = (x^2 + p)$$

$$\left(x^2 - \frac{1}{4}x + q\right)$$

$$\Rightarrow x^4 - \frac{1}{4}x^3 + (p+q)x^2 - \frac{p}{4}x + pq$$

Comparing the coefficients of x and constants

$\frac{-p}{4} = \frac{3}{4} \Rightarrow p = -3$

$pq = \frac{9}{8} \Rightarrow q = \frac{9}{8} \times \frac{-1}{3} = \frac{-3}{8}$

Substitute the value of p in eq--------- (2)

$x^2 - 3 = 0$

$\Rightarrow x = \pm\sqrt{3}$

Substitute the value of q in eq----------- (3)

$x^2 - \frac{1}{4}\times - \frac{3}{8} = 0$

$\Rightarrow 8x^2 - 2x - 3 = 0$

$\Rightarrow (2x+1)(4x-3) = 0$

$\Rightarrow x = -\frac{1}{2}, \frac{3}{4}$

$\therefore$ The roots of the given equation are

$-\sqrt{3}, -\frac{1}{2}, \frac{3}{4}, \sqrt{3}$

6. Solve $18x^3 + 81x^2 + 21x + 60 = 0$ given that one root is equal to half the sum of the remaining roots.

Sol: Suppose α, β, γ are the roots of

$18x^3 + 81x^2 + 121x + 60 = 0$

Sum $\alpha + \beta + \gamma = \frac{-81}{18} = \frac{-9}{2}$ --------------- (1)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{121}{18}$ ---------- (2)

$\alpha\beta\gamma = \frac{-60}{18} = \frac{-10}{3}$ ------------- (3)

$\because$ One root is equal to half of the sum of the remaining two

Let $\alpha = \frac{1}{2}(\beta + \gamma)$

Substitute in (1)

$\alpha + 2\alpha = -\frac{9}{2} \Rightarrow \alpha = \frac{-3}{2}$

$\because \beta + \gamma = 2\alpha = 2\left(-\frac{3}{2}\right) = -3$

From (3)

$$\left(-\frac{3}{2}\right)(\beta\gamma)=\frac{-10}{3}$$

$$\Rightarrow \beta\gamma=\frac{20}{9}$$

$$\because (\beta-\gamma)^2=(\beta+\gamma)^2-4\beta\gamma$$

$$=(-3)^2-4\left(\frac{20}{9}\right)=\frac{81-80}{9}=\frac{1}{9}$$

$$\therefore \beta-\gamma=\frac{1}{3}$$

$$\underline{\beta+\gamma=-3}$$

Add $2\beta=\frac{1}{3}-3=\frac{-8}{3}\Rightarrow \beta=\frac{-4}{3},\ \gamma=\frac{-5}{3}$

$\therefore$ The roots of the given equation are

$\frac{-3}{2},\frac{-4}{3}$ and $\frac{-5}{3}$

7. Find the condition in order that the equation $ax^4+4bx^3+6cx^2+4dx+c=0$ **many have a pair of equal roots**

Sol: Let $\alpha,\alpha,\beta,\beta$ are the roots of the equation $ax^4+4bx^3+6cx^2+4dx+e=0$

$$\Rightarrow x^4+\frac{4b}{a}x^2+\frac{6c}{a}x^2+\frac{4d}{a}x+\frac{e}{a}=0$$

Sum of the roots, $2(\alpha+\beta)=-\frac{4b}{a}$

$$\Rightarrow \alpha+\beta=-\frac{2b}{a}$$

$$\Rightarrow \alpha\beta=k(say)$$

Equation having roots α,β is

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$\Rightarrow x^2+\frac{2b}{a}x+k=0$$

$$\therefore x^4+\frac{4b}{a}x^3+\frac{6c}{a}x^2+\frac{4d}{a}x+\frac{e}{a}$$

$$=\left(x^2+\frac{2b^2}{a^2}x+k\right)^2$$

$$=x^4+\frac{4b}{a}x^3+\left(\frac{4b^2}{a^2}+2k\right)x^2+\frac{4bk}{a}x+k^2$$

Comparing the coefficient of x

$$\frac{4d}{a}=\frac{4bk}{a}\Rightarrow k=\frac{d}{b}$$

Comparing the coefficient of x^2,

$$\frac{6c}{a}=\frac{4b^2}{a^2}+2k$$

$$\Rightarrow \frac{6c}{a}=\frac{4b^2}{a^2}+\frac{2d}{b}\Rightarrow 6abc=4b^3+2a^2d$$

$$\therefore 3\,abc=2b^3+a^2d$$

Comparing the constant terms

$$k^2=\frac{e}{a}\Rightarrow \frac{d^2}{b^2}=\frac{e}{a}\Rightarrow ad^2=eb^2$$

$\therefore$ The required conditions are

$$3abc=2b^3+a^2d,\ ad^2=eb^2$$

8. (i) Show that $x^5-5x^3+5x^2-1=0$ has three equal roots and find this root.

Sol: Let $f(x)=x^5-5x^3+5x^2-1$

$$f'(x)=5x^4-15x^2+10x$$

$$=5x\left(x^3-3x+2\right)$$

$$f'(1)-5(1)(1-3+2)=0$$

$f(1) = 1 - 5 + 5 - 1 = 0$

$x - 1$ is a factor of $f'(x)$ *and* $f(x)$

$\therefore 1$ is a repeated root of $f(x)$

	1	0	−5	5	0	−1
1	–	1	1	−4	1	1
	1	1	−4	1	1	0
1	–	1	2	−2	−1	
	1	2	−2	−1	0	

$x^3 + 2x^2 - 2x - 1 = 0$

$\Rightarrow 1$ is a root of above equation

($\because$ Sum of the coefficients is zero)

$\therefore 1$ is the required root

(ii) Find the repeated roots of $x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12 = 0$

Sol: Let $f(x) = x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12$

$f'(x) = 5x^4 - 12x^3 - 15x^2 + 54x - 32$

$f'(2) = 5(2)^4 - 12(2)^3 - 15(2)^2 + 54(2) - 32$

$= 80 - 96 - 60 + 108 - 32 = 0$

$f(2) = (2)^5 - 3(2)^4 - 5(2)^3 + 27(2)^2 - 32(2) + 12$

$= 32 - 48 - 40 + 108 - 64 + 12$

$= 152 - 152 = 0$

$\therefore x - 2$ is a common factor of $f'(x)$ and $f(x)$ 2 is a multiple root of $f(x) = 0$

	1	−3	−5	27	−32	12
2	–	2	−2	−14	26	−12
	1	−1	−7	13	−6	0
2	–	2	2	−10	6	
	1	1	−5	3	0	

Let $g(x) = x^3 + x^2 - 5x + 3$

$g'(x) = 3x^2 + 2x - 5 = (3x+5)(x-1)$

$g(1) = 1 + 1 - 5 + 3 = 0$

$\therefore x - 1$ is a common factor of $g'(x)$ *and* $g(x)$

$\therefore 1$ is a multiple root of $g(x) = 0$

	1	1	–5	3
1	–	1	2	–3
	1	2	–3	0
1	–	1	3	
	1	3	0	

$x + 3 = 0 \Rightarrow x = -3$

$\therefore$ The roots are 2, 2, 1, 1, -3

9. Solve the equation $8x^3 - 20x^2 + 6x + 9 = 0$ given that the equation has multiple roots.

Sol: Given equation $8x^3 - 20x^2 + 6x + 9 = 0$ …(1)

Let f(x) = $8x^3 - 20x^2 + 6$

$= 2(12x^2 - 20x + 3)$

$= 2(2x - 3)(6x - 1)$

$f\left(\frac{3}{2}\right) = 8\left(\frac{27}{8}\right) - 20\left(\frac{9}{4}\right) + 6\left(\frac{3}{2}\right) + 9$

$= 27 - 45 + 9 + 9 = 0$

$\therefore f\left(\frac{3}{2}\right) = 0$

$\therefore$ f(x) and f′(x) has a common factor 2x – 3.

$\therefore$ 3/2 is a multiple root of f(x) = 0.

By synthetic division,

$\frac{3}{2}$	8	-20	6	9
	0	12	-12	-9
$\frac{3}{2}$	8	-8	-6	0
	0	12	6	
	8	4	0	

$\therefore 8x^3 - 20x^2 + 6x + 9 = 0$

$\Rightarrow \left(x - \frac{3}{2}\right)^2 (8x + 4) = 0 \Rightarrow x = \frac{3}{2}$ or $x = -\frac{1}{2}$.

$\therefore$ The roots of given equation are $-\frac{1}{2}, \frac{3}{2}, \frac{3}{2}$.

10. Find the polynomial equation whose roots are translates of those of the equation

$x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ *by* -2

Sol: Given equation is

$f(x) = x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$

The required equation is $f(x+2) = 0$

$(x+2)^4 - 5(x+2)^3 + 7(x+2)^2$

$-17(x+2) + 11 = 0$

2	1	-5	7	-17	11
	0	2	-6	2	-30
	1	-3	1	-15	-19
	0	2	-2	-2	A_4
	1	-1	-1	-17	
	0	2	2	A_3	
	1	1	1		
	0	2	A_2		
	1	3			
	A_0	A_1			

Required equation is $x^4+3x^2+x^2-17x-19=0$

11.Find the polynomial equation whose roots are the translates of those of

$x^5-4x^4+3x^2-4x+6=0$ by - 3

Sol: Given equation is

$f(x)=x^5-4x^4+3x^2-4x+6=0$

Required equation is $f(x+3)=0$

$(x+3)^5-4(x+3)^4+3(x+3)^2$

$-4(x+3)+6=0$

3	1	−4	0	3	−4	6
	0	3	−3	−9	−18	−66
	1	−1	−3	−6	−22	−60
	0	3	6	9	9	A_5
	1	2	3	3	−13	
	0	3	15	54	A_4	
	1	5	18	57		
	0	3	24	A_3		
	1	8	42			
	0	3	A_3			
	1	11				
	A_0	A_1				

Required equation is

$x^5+11x^4+42x^3+57x^2-13x-60=0$

12. Find the polynomial equation whose roots are the translates of the roots of the equation

$x^4 - x^3 - 10x^2 + 4x + 24 = 0$ **by 2**

Sol: Given $f(x) = x^4 - x^3 - 10x^2 + 4x + 24 = 0$

Required equation is $f(x-2) = 0$

$(x-2)^4 + (x-2)^3 - 10(x-2)^2$

−2	1	−1	−10	4	24	
	0	−2	6	8	−24	
	1	−3	−4	12	0	A_4
	0	−2	10	−12		
	1	−5	6	0	A_3	
	0	−2	14			
	1	−7	20			
	0	−2	A_2			
	1	−9				
	A_0	A_1				

Required equation is

$x^4 - 9x^3 + 20x^2 = 0$

13. Find the polynomial equation whose roots are the translates of the equation

$3x^5 + 5x^3 + 7 = 0$ **by 4**

Sol: Given $f(x) = 3x^5 - 5x^3 + 7 = 0$

Required equation is $f(x-4) = 0$

$3(x-4)^5 - 5(x-4)^3 + 7 = 0$

−4	3	0	−5	0	0	7
	0	−12	48	−172	688	−2752
	3	−12	43	−172	688	−2745
	0	−12	96	−556	2912	A_5
	3	−24	139	−728	3600	
	0	−12	144	−1132	A_4	
	3	−36	283	−1860		
	0	−12	192	A_3		
	3	−48	475			
	0	−12	A_2			
	3	60				
	A_0	A_1				

Required equation is $x^5 - 60x^4 + 475x^3$

$-1860x^2 + 3600x - 2745 = 0$

14. Transform each of the following equations into ones in which of the coefficients of the second highest power of x is zero and also find their transformed equations.

(i) $x^3 - 6x^2 + 10x - 3 = 0$

Sol: Given equation is $x^3 - 6x^2 + 10x - 3 = 0$

To remove the second term diminish the roots

By $-\frac{a_1}{na_0} = \frac{6}{3} = 2$

2	1	−6	10	−3
	0	2	−8	+4
	1	−4	2	+1
	0	2	−4	A_3
	1	−2	−2	
	0	+2	A_2	
	1	0		
	A_0	A_1		

Required equation is $x^3 - 2x + 1 = 0$

(ii) $x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$

Sol: Given equation is $x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$

Diminishing the roots by $-\frac{a_1}{na_0} = \frac{-4}{4} = -1$

-1	1	4	2	-4	-2
	0	-1	-3	1	3
	1	3	-1	-3	1
	0	-1	-2	3	A_4
	1	2	-3	0	
	0	-1	-1	A_3	
	1	1	-4		
	0	-1	A_2		
	1	0			
	A_0	A_1			

Required equation is $x^4 - 4x^2 + 1 = 0$

(iii) $x^3 - 6x^2 + 4x - 7 = 0$

Sol: Given equation is $x^3 - 6x^2 + 4x - 7 = 0$

Diminishing the roots by $-\frac{a_1}{na_0} = \frac{6}{3} = 2$

2	1	-6	4	-7
	0	2	-8	-8
	1	-4	-4	-15
	0	2	-4	A_3
	1	-2	-8	
	0	2	A_2	
	1	0		
	A_0	A_1		

Required equation is $x^3 - 8x - 15 = 0$

(iv) $x^3 + 6x^2 + 4x + 4 = 0$

Sol: Given equation is $x^3 + 6x^2 + 4x + 4 = 0$

To remove the second term diminish the roots by

−2	1	6	4	4
	0	−2	−8	8
	1	4	−4	12
	0	−2	−4	A_3
	1	2	−8	
	0	−2	A_2	
	1	0		
	A_0	A_1		

Required equation is $x^3 - 8x + 12 = 0$

15.Transform each of the following equations into ones in which the coefficients of the third highest power of x is zero

(i) $x^4 + 2x^3 - 12x^2 + 2x - 1 = 0$

Hint: To remove the rth term in an equation $f(x) = 0$ of degree n diminish the roots by 'h' such that $f^{(n-r+1)}(h) = 0$

Sol: Let $f(x) = x^4 + 2x^3 - 12x^2 + 2x - 1$

To remove the 3rd term, diminish the roots by h such that $f'(h) = 0$

$f(x) = 4x^3 + 6x^2 - 24x + 2$

$f'(x) = 12x^2 + 12x - 24$

$f''(h) = 0 \Rightarrow 12h^2 + 12h - 24 = 0$

$$\Rightarrow h^2 + h - 2 = 0 \Rightarrow (h+2)(h-1) = 0$$

$$\Rightarrow h = -2 \ or 1$$

Case (i):

h = −2	1	2	−12	2	−1
	–	−2	0	24	−52
	1	0	−12	26	−53
	–	−2	4	16	
	1	−2	−8	42	
	–	−2	8		
	1	−4	0		
	–	−2			
	1	−6			

Transformed equation is

$$x^4 - 6x^3 + 42x - 53 = 0$$

Case (ii):

h = 1	1	2	−12	2	−1
	–	1	3	−9	−7
	1	3	9	−7	−8
	–	1	4	−5	
	1	4	−5	−12	
	–	1	5		
	1	5	0		
	–	1			
	1	6			

Transformed equation is

$$x^4 - 6x^3 - 12x - 8 = 0$$

$\therefore$ The required equation is

$$x^4 - 6x^3 + 42x - 53 = 0$$

Or $x^4 + 6x^3 - 12x - 8 = 0$

(ii) $x^3 + 2x^2 + x + 1 = 0$

Sol: Let $f(x) = x^3 + 2x^2 + x + 1$

To remove the 3[rd] terms, diminish the roots by h such that

$f'(h) = 0,\ f'(x) = 3x^2 + 4x + 1\ f(h) = 0 \Rightarrow 3h^2 + 4h + 1 = 0$

$\Rightarrow (3h+1)(h+1) \Rightarrow h = 1, -\frac{1}{3}$

Case (i):

h = –1	1	2	–1	1
	–	–1	–1	0
	1	1	0	1
	–	–1	0	
	1	0	0	
	–	–1		
	1	–1		

Transformed equation is $x^3 - x^2 + 1 = 0$

Case (ii)

$h = -\frac{1}{3}$	1	2	1	1
	–	$-\frac{1}{3}$	$-\frac{5}{9}$	$-\frac{4}{27}$
	1	$\frac{5}{3}$	$\frac{4}{9}$	$\frac{23}{27}$
	–	$-\frac{1}{3}$	$-\frac{4}{9}$	
	1	$\frac{4}{3}$	0	
	–	$-\frac{1}{3}$		
	1	1		

Transformed equation is $x^3 + x^2 + \frac{23}{27} = 0$

$\Rightarrow 27x^3 + 27x^2 + 23 = 0$

$\therefore$ The required equation is $x^3 - x^2 + 1 = 0$ or $27x^3 + 27x^2 + 23 = 0$

16. Solve the following equations

(i) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

Sol: This is standard reciprocal equation

Dividing with x^2

$$x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0 ....(1)$$

Put $a = x + \frac{1}{x}$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = a^2 - 2$$

Substituting in (1)

$$a^2 - 2 - 10a + 26 = 0$$

$$\Rightarrow a^2 = 10a + 24 = 0$$

$$\Rightarrow (a-4)(a-6) = 0$$

$$a = 4 \text{ or } 6$$

Case (i) a = 4

$$x + \frac{1}{x} = 4$$

$$\Rightarrow x^2 + 1 = 4x$$

$$\Rightarrow x^2 - 4x + 1 = 0$$

$$\Rightarrow x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

Case (ii) a = 6

$$x + \frac{1}{x} = 6$$

$$x^2 + 1 = 6x$$

$$x^2 - 6x + 1 = 0$$

$$x = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm \sqrt{2}}{2}$$

$$x = \frac{2\left(3 \pm 2\sqrt{2}\right)}{2} = 3 \pm 2\sqrt{2}$$

$\therefore$ The roots are $3 \pm 2\sqrt{2}\,, 2 \pm \sqrt{3}$

(ii) $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$

Sol: Given $f(x)$

$= 2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$

This is an odd degree reciprocal equation of first type.

$\therefore -1$ is a root

Dividing $f(x)$ with $x+1$

x = −1						
	2	1	−12	−12	1	2
	0	−2	1	11	1	−2
	2	−1	−11	−1	2	0

Dividing $f(x)\, by\, (x+1)$ we get

$$24x^4 - x^3 - 11x^2 - x + 2 = 0$$

Dividing by x^2

$$2x^2 - x - 11 - \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) - 11 = 0 \text{ --------------- (1)}$$

Put $a = x + \frac{1}{x}$ so that $x^2 + \frac{1}{x^2} = a^2 - 2$

Substituting in (1), request equation is

$$2\left(a^2 - 2\right) - a - 11 = 0$$

$$2a^2 - 4 - a - 11 = 0$$

$$2a^2 - a - 15 = 0$$

$$(a - 3)(2a + 5) = 0$$

$$a = 3 \, or \, -\frac{5}{2}$$

Case (i) a = 3

$$\Rightarrow x + \frac{1}{x} = 3$$

$$\Rightarrow x^2 + 1 = 3x$$

$$\Rightarrow x^2 - 3x + 1 = 0$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

Case (ii) $a=-\frac{5}{2}$

$$x+\frac{1}{x}=-\frac{5}{2}$$

$$\Rightarrow \frac{x^2+1}{x}=-\frac{5}{2}$$

$$\Rightarrow 2x^2+2=-5x$$

$$\Rightarrow 2x^2+5x+2=0$$

$$\Rightarrow (2x+1)(x+2)=0$$

$$\Rightarrow x=-\frac{1}{2},-2$$

$\therefore$ The roots are $-1,-\frac{1}{2},-2,\frac{3\pm\sqrt{5}}{2}$

17. If α,β,γ are the roots of $x^3+px^2+qx+r=0$, then form the cubic equation whose roots are $\alpha(\beta+\gamma),\beta(\gamma+\alpha),\gamma(\alpha+\beta)$

Sol: Let α,β,γ be the roots of the given equation,

We have, $\alpha+\beta+\gamma=-p, \alpha\beta+\beta\gamma+\gamma\alpha=q$

$\alpha\beta\gamma=-r$

Let $y=\alpha(\beta+\gamma)$

$=\alpha\beta+\alpha\gamma+\gamma\beta-\beta\gamma$

$$y=q-\frac{\alpha\beta\gamma}{\alpha}$$

$$y=q+\frac{r}{a}\Rightarrow y\alpha=q\alpha+r$$

$$\alpha(y-q)=r \Rightarrow \alpha=\frac{r}{y-q}$$

Since 'α' be the roots of $x^3+px^2+qx+r=0$

$$\frac{r^3}{(y-q)^3}+p\left(\frac{r^2}{y-q}\right)^2+q\left(\frac{r}{y-q}\right)+r\ =0$$

$$\Rightarrow r^3+pr^2(y-p)+qr(y-q)^2+(y-q)^3\ r=0$$

$$r^3+pr^2y-pr^2q+qr\left(y^2+q^2-2qy\right)+r\left(y^3-q^3-3y^2q+3yq^2\right)=0$$

$$r^3+pr^2y-pr^2q+qry^2+q^3r-2q^2ry+ry^3-rq^3-3y^2\ qr+3yq^2\ r=0$$

$$r\left(y^3+r^2+p^2rq+qy^2+q^3-2q^2y\right)-q^3-3y^2q+3yq^2)\ =0$$

$$y^3-2qy^2+y\left(pr-q^2\right)-p^2rq+r^2\ \ =0$$

$$\therefore x^3-2qx^2+\left(q^2+pr\right)x-r\left(pq-r\right)=0$$

18. Given that the sum of two roots of $x^4-2x^3+4x^2+6x-21=0$ is zero, find the roots of the equation

Sol: Let $\alpha, \beta, \gamma, \delta$ are the roots of given equation, since sum of two is zero

$$\alpha+\beta=0$$

Now $\alpha+\beta+\gamma+\delta=2 \Rightarrow \gamma+\delta=2$

Let $\alpha\beta=p, \gamma\delta=q$

The equation having the roots α, β is

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$\therefore x^2+p=0$$

The equation having the roots γ, δ is

$$x^2 - (\gamma + \delta)x + \gamma\delta = 0$$

$$\therefore x^2 - 2x + q = 0$$

$$\therefore x^4 - 2x^3 + 4x^2 + 6x - 21$$

$$= (x^2 + p)(x^2 - 2x + q)$$

$$= x^4 - 2x^3 + x^2(p+q) - 2px + pq$$

Comparing the like terms

$$p + q = 4, -2p = 6$$

$$-3 + q = 4 \;\; p = -3$$

$$q = 7$$

$$\therefore x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3} \; and \; x^2 - 2x + 7 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4-28}}{2}$$

$$= \frac{2 \pm 2\sqrt{6}i}{2}$$

$$= 1 \pm \sqrt{6}i$$

$\therefore$ Roots are $-\sqrt{3}, \sqrt{3}, 1 - i\sqrt{6} \; and \; 1 + i\sqrt{6}$

19. Solve $x^4 - 5x^3 + 5x^2 + 5x - 6 = 0$ given that the product of two of its roots is 3

Sol: Let $\alpha, \beta, \gamma, \delta$ be the root of the given equation

Product of the roots $\alpha\beta\gamma\delta 33 = -6$

Given $\alpha\beta = 3$ ($\because$ Products of two roots is 3)

$$\therefore \alpha\beta\gamma\delta = -6$$

$\gamma\delta = -2$

Let $\alpha + \beta = p, \gamma + \delta = q$

The equation having the roots α, β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - px + 3 = 0$

The equation having the root γ, δ is

$x^2 - (\gamma + \delta)x + \gamma\delta = 0 \; x^2 - qx - 2 = 0$

$\therefore x^4 - 5x^3 + 5x^2 + 5x - 6$

$= (x^2 - px + 3)(x^2 - qx - 2)$

$= x^4 - (p+q)x^3 + (1 + pq)x^2$

$+ (2p - 3q)x - 6$

Comparing the like terms,

$p + q = 5, 2p - 3q = 5$

$\therefore 2p - 3q = 5$

$3p + 3q = 15$

$5p = 20 \Rightarrow p = 4$

$\therefore p = 1$

Now $x^2 - 4x + 3 = 0 \Rightarrow (x-3)(x-1) = 0$

$\Rightarrow x = 1, 3$

$x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = -1, 2$

$\therefore$ The roots are -1, 2, 1, 3

20. Solve $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$. Given that it has two pairs of equal roots

Sol: Given equation is

$x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$

Let the roots be $\alpha, \alpha, \beta, \beta$

Sum of the roots , $2(\alpha + \beta)$

$= -4 \Rightarrow \alpha + \beta = -2$

Let $\alpha\beta = p$

The equation having root α, β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e., $x^2 + 2x + p = 0$

$\therefore x^4 + 4x^3 - 2x^2 - 12x + 9$

$= \left[x^2 - (\alpha + \beta)x + \alpha\beta\right]^2$

$= x^4 + 4x^3 + (2p + 4)x^2 + 4px + p^2$

Comparing coefficients of x on both sides

$4p = -12 \Rightarrow p = -3$

$x^2 + 2x + p = 0$

$\therefore x^4 + 4x^3 + (2p + 4)x^2 + 4px + p^2$

Comparing coefficients of x on both sides

$4p = -12 \Rightarrow p = -3$

$x^2 + 2x + p = 0 \Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x+3)(x-1)=0$

$\Rightarrow x=-3,1$

$\therefore$ The root s of the given equation are $-3,-3,1,1$

21. **Solve** $6x^4-13x^3-35x^2-x+3=0$ **given that one of its root is** $2+\sqrt{3}$

Sol: $2+\sqrt{3}$ is a root $\Rightarrow 2-\sqrt{3}$ is also a root

The equation having roots

$2\pm\sqrt{3}$ *is* $x^2-4x+1=0$

$\therefore x^2-4x+1$ is a factor of

$6x^4-13x^3-53x^2-x+3=0$

	6	−13	−35	−1	3
4	–	24	44	12	–
−1	–	–	−6	−11	−3
	6	11	3	0	0

$6x^2+11x+3=0$

$\Rightarrow 6x^2+9x+2x+3=0$

$\Rightarrow 3x(2x+3)+1(2x+3)=0$

$\Rightarrow (3x+1)(2x+3)=0$

$\Rightarrow x=-\frac{1}{3},\frac{-3}{2}$

$\therefore$ The roots of the given equation are $-\frac{1}{3},\frac{-3}{2},2\pm\sqrt{3}$

22. Find the polynomial equation of degree 4 whose roots are the negatives root of

$x^4 - 6x^3 + 7x^2 - 2x + 1 = 0$

Sol: Let $f(x) \equiv x^4 - 6x^3 + 7x^2 - 2x + 1$

The required equation is $f(-x) = 0$

i.e., $(-x)^4 - 6(-x)^3 + 7(-x)^2 - 2(-x) + 1 =)$

$\therefore x^4 + 6x^3 + 7x^2 + 2x + 1 = 0$

25. Find the algebraic equation of the degree 4 whose roots are 3 times the roots of the equation

$6x^4 - 7x^3 + 8x^2 - 7x + 2 = 0$

Sol: Let $f(x) \equiv 6x^4 - 7x^3 + 8x^2 - 7x + 2$

The required equation is $f\left(\frac{x}{3}\right) = 0$

i.e., $6\left(\frac{x}{3}\right)^4 - 7\left(\frac{x}{3}\right)^3 + 8\left(\frac{x}{3}\right)^2 - 7\left(\frac{x}{3}\right) + 2 = 0$

i.e., $\frac{6x^4}{81} - \frac{7x^3}{27} + \frac{8x^2}{9} - \frac{7x}{3} + 2 = 0$

$\therefore 6x^4 - 21x^3 + 72x^2 - 189x + 162 = 0$

23. Form the equation whose roots are m times the roots of the equation

$x^3 + \frac{x^2}{4} - \frac{x}{16} + \frac{1}{72} = 0$ **and deduce the case when** $m = 12$

Sol: Let $f(x) \equiv x^3 + \frac{x^2}{4} - \frac{x}{16} + \frac{1}{72}$

The required equation is $f\left(\frac{x}{m}\right) = 0$

i.e., $\left(\frac{x}{m}\right)^3 + \frac{1}{4}\left(\frac{x}{m}\right)^2 - \frac{1}{16}\left(\frac{x}{m}\right) + \frac{1}{72} = 0$

i.e., $\frac{x^3}{m^3} + \frac{x^2}{4m^2} - \frac{x}{16m} + \frac{1}{72} = 0$

$\therefore x^3 + \frac{m}{4}x^2 - \frac{m^2}{16}\times + \frac{m^2}{72} = 0$

When m = 12

$x^3 + \frac{12}{4}x^2 - \frac{144}{16}\times + \frac{144\times12}{72} = 0$

$\therefore x^3 + 3x^2 - 9x + 24 = 0$

24 Find the algebraic equation of degree 5 whose roots are the translates of the root of $x^5 + 4x^3 - x^2 + 11 = 0$ **by -3**

Sol: Let $f(x) \equiv x^5 + 4x^3 - x^2 + 11$

The required equation is $f(x+3) = 0$

3	1	0	4	–1	0	11
	–	3	9	39	114	342
3	1	3	13	38	114	353
	–	3	18	93	393	
3	1	6	31	131	507	
	–	3	27	174		
3	1	9	58	305		
	–	3	36			
3	1	12	94			
	–	3				
	1	15				

The required equation is

$x^5 + 15x^4 + 94x^3 + 305x^2 + 507x + 353 = 0$

25. Find the algebraic equation of degree 4 whose roots are $4x^4+32x^3+83x^2+76x+21=0$ **by 2**

Sol: Let $(x)\equiv 4x^4+32x^3+83x^2+76x+21$

The required equation is $f(x-2)=0$

-2	4	32	83	76	21
	–	-8	-48	-70	-21
-2	4	24	35	6	9
	–	-8	-32	-6	
-2	4	16	3	0	
	–	-8	-16		
-2	4	8	-13		
	–	-8			
	4	0			

The required equation is

$4x^3-13x^2+9=0$

26. Find the polynomial equation whose roots are the reciprocals of the roots of the equation

$x^4+3x^3-6x^2+2x-4=0$

Sol: Let $f(x)\equiv x^4+3x^3-6x^2+2x-4$

The required equation is $f\left(\frac{1}{x}\right)=0$

i.e., $\left(\frac{1}{x}\right)^4+3\left(\frac{1}{x}\right)^3-6\left(\frac{1}{x}\right)^2+2\left(\frac{1}{x}\right)-4=0$

i.e., $\frac{1}{x^4}+3\frac{1}{x^3}-6\frac{1}{x^2}+2\frac{1}{x}-4=0$

i.e., $1+3x-6x^2+2x^3-4x^4=0$

27. Find the polynomial equation whose roots are the squares of the roots of

$x^3 - x^2 + 8x - 6 = 0$

Sol: Let $f(x) \equiv x^3 - x^2 + 8x - 6$

The required equation is $f\left(\sqrt{x}\right) = 0$

i.e., $\left(\sqrt{x}\right)^3 - \left(\sqrt{x}\right)^2 + 8\sqrt{x} - 6 = 0$

$\Rightarrow x\sqrt{x} + x + 8\sqrt{x} - 6 = 0$

$\Rightarrow \sqrt{x}(x+8) = x + 6$

Squaring on both sides

$\Rightarrow x\left(x^2 + 16x + 64\right) = x^2 + 12x + 36$

$\Rightarrow x^3 + 16x^2 + 64x - x^2 - 12x - 36 = 0$

$\therefore x^3 + 15x^2 + 52x - 36 = 0$

27. Solve the equation $4x^3 - 13x^2 - 13x + 4 = 0$

Sol: $4x^3 - 13x^2 - 13x + 4 = 0$ is a reciprocal equation of first class and of odd degree. Thus -1 is a root of the given equation

–1	4	–13	–13	4
	–	–4	17	–4
	4	–17	4	0

$4x^2 - 17x + 4 = 0 \Rightarrow 4x^2 - 16x - x + 4 = 0$

$\Rightarrow 4x(x-4) - 1(x-4) = 0$

$\Rightarrow (x-4)(4x-1) = 0$

$\Rightarrow x = 4 \text{ or } \frac{1}{4}$

$\therefore$ The roots are $-1, 4, \frac{1}{4}$

28. **Solve** $x^4 + x^3 - 4x^2 + x + 1 = 0$

Sol: Given equation is $x^4 + x^3 - 4x^2 + x + 1 = 0$

This is a reciprocal equation of even degree and of first class

Given equation can be written as

$x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

i.e., $\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 4 = 0$ -------------- (1)

Put $x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

$\therefore$ (1) Becomes $\left(y^2 - 2\right) + \left(y\right) - 4 = 0$

$\Rightarrow y^2 + y - 6 = 0$

$\Rightarrow (y+3)(y-2) = 0$

$\Rightarrow y = -3, 2$

Case (i) $y = -3 \Rightarrow x + \frac{1}{x} = -3$

$\Rightarrow \frac{x^2 + 1}{x} = -3$

$\Rightarrow x^2 + 1 = -3x$

$\Rightarrow x^2 + 3x + 1 = 0$

$\therefore x = \dfrac{-3 \pm \sqrt{9-4}}{2} = \dfrac{-3 \pm \sqrt{5}}{2}$

Case (ii) $y = 2 \Rightarrow x + \dfrac{1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x$

$\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0$

$\therefore x = 1, 1$

$\therefore$ The roots are $1, 1, \dfrac{-3 \pm \sqrt{5}}{2}$

29. **Solve** $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$

Sol: Given equation is $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$

Is a reciprocal equation of odd degree and of class two

$\therefore 1$ is a root of the given equation

$\Rightarrow (x-1)$ is a factor of

$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$

1	1	−5	9	−9	5	−1
	–	1	−4	5	−4	1
	1	−4	5	−4	1	0

$x^4 + 4x^3 + 5x^2 - 4x + 1 = 0$

$\Rightarrow x^2 - 4x + 5 - \dfrac{4}{x} + \dfrac{1}{x^2} = 0$

$\Rightarrow \left(x^2 + \dfrac{1}{x^2}\right) - 4\left(x + \dfrac{1}{x}\right) + 5 = 0$ -----------(1)

Put $x + \dfrac{1}{x} = y \Rightarrow x^2 + \dfrac{1}{x^2} = y^2 - 2$

$\therefore$ (1) Becomes $\left(y^2-2\right)-4y+5=0$

$\Rightarrow y^2-4y+3=0$

$\Rightarrow (y-1)(y-3)=0$

$\Rightarrow y=1,3$

Case (i) $y=1 \Rightarrow x+\dfrac{1}{x}=1$

$\Rightarrow \dfrac{x^2+1}{x}=1$

$\Rightarrow x^2+1=x$

$\Rightarrow x^2-x+1=0$

$\therefore x=\dfrac{1\pm\sqrt{1-4}}{2}=\dfrac{1\pm\sqrt{3}i}{2}$

Case (ii) $y=3 \Rightarrow x+\dfrac{1}{x}=3$

$\Rightarrow \dfrac{x^2+1}{x}=3 \Rightarrow x^2+1=3x$

$\Rightarrow x^2-3x+1=0$

$\therefore x=\dfrac{3\pm\sqrt{9-4}}{2}=\dfrac{3\pm\sqrt{5}}{2}$

$\therefore$ The roots of the given equation are

$1, \dfrac{1\pm\sqrt{3}i}{2}, \dfrac{3\pm\sqrt{5}}{2}$

30. Solve the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

Sol: Given equation is $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$ is a reciprocal equation of second class and of even degree

$\therefore x^2 - 1$ is a factor of

$6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

1	6	−25	31	0	−31	25	−6
	–	6	−19	12	12	−19	6
−1	6	−19	12	12	−19	6	0
	–	−6	25	−37	25	−6	
	6	−25	37	−25	6	0	

$6x^4 - 25x^3 + 37x^2 - 25x + 6 = 0$

$\Rightarrow 6x^2 - 25x + 37 - \frac{25}{x} + \frac{6}{x^2} = 0$

$\Rightarrow 6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) + 37 = 0$ ----------- (1)

Put $x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

$\therefore$ (1) Becomes $6\left(y^2 - 2\right) - 25(y) + 37 = 0$

$\Rightarrow 6y^2 - 12 - 25y + 37 = 0$

$\Rightarrow 6y^2 - 25 + 25 = 0$

$\Rightarrow 6y^2 - 15y - 10y + 25 = 0$

$\Rightarrow 3y(2-5) - 5(2y-5) = 0$

$\Rightarrow (2y-5)(3y-5) = 0$

$\Rightarrow y = \frac{5}{2}, \frac{5}{3}$

Case (i): $y=\frac{5}{2}\Rightarrow x+\frac{1}{x}=\frac{5}{2}\Rightarrow\frac{x^2+1}{x}=\frac{5}{2}$

$\Rightarrow 2x^2+2=5x$

$\Rightarrow 2x^2-5x+2=0$

$\Rightarrow(2x-1)(x-2)=0$

$\therefore x=\frac{1}{2},2$

Case (ii) $y=\frac{5}{2}\Rightarrow x+\frac{1}{x}=\frac{5}{3}\Rightarrow\frac{x^2+1}{x}=\frac{5}{3}$

$\Rightarrow 3x^2+3=5x$

$\Rightarrow 3x^2-5x+3=0$

$\Rightarrow x=\frac{5\pm\sqrt{25-36}}{6}$

$\therefore x=\frac{5\pm\sqrt{11}i}{6}$

$\therefore$ The roots of the given equation are

$\pm 1,\frac{1}{2},2,\frac{5\pm\sqrt{11}i}{6}$