CIRCLES PART - II

Theorem: The equation of the tangent to the circle S = 0 at $P(x_1, y_1)$ is $S_1 = 0$.

Theorem: The equation of the normal to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ at $P(x_1, y_1)$ is $(y_1 + f)(x - x_1) - (x_1 + g)(y - y_1) = 0$.

Corollary: The equation of the normal to the circle $x^2+y^2 = a^2$ at $P(x_1, y_1)$ is $y_1x - x_1y = 0$.

Theorem: The condition that the straight line lx + my + n = 0 may touch the circle $x^2 + y^2 = a^2$ is $n^2 = a^2(l^2 + m^2)$ and the point of contact is $\left(\frac{-a^2l}{n}, \frac{-a^2m}{n}\right)$.

Proof:

The given line is lx + my + n = 0 …(1)

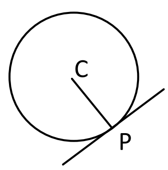
The given circle is $x^2 + y^2 = r^2$…(2)

Centre C = (0,0), radius =r

Line (1) is a tangent to the circle (2)

⇔ The perpendicular distance from the centre C to the line (1) is equal to the radius r.

$$\Leftrightarrow \left|\frac{0-n}{\sqrt{l^2+m^2}}\right| = r$$



$$\Leftrightarrow (n)^2 = r^2(l^2+m^2)$$

Let $P(x_1, y_1)$ be the point of contact.

Equation of the tangent is $S_1=0$, => $x_1x+y_1y - r^2=0$. ---- (3)

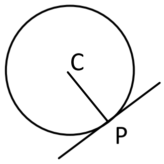
Equations (1) and (3) are representing the same line, therefore, $\frac{x_1}{l} = \frac{y_1}{m} = \frac{-a^2}{n} \Rightarrow x_1 = \frac{-a^2l}{n}, y_1 = \frac{-a^2m}{n}$

Therefore, point of contact is $\left(\frac{-a^2l}{n}, \frac{-a^2m}{n}\right)$

Theorem: The condition for the straight line lx + my + n = 0 may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(g^2 + f^2 – c)(l^2 + m^2) = (lg + mf – n)^2$.

Proof:

The given line is lx + my + n = 0 … (1)



The given circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ … (2)

Centre C = (–g, –f), radius $r = \sqrt{g^2 + f^2 - c}$

Line (1) is a tangent to the circle (2)

⇔ The perpendicular distance from the centre C to the line (1) is equal to the radius r.

$$\Leftrightarrow \left|\frac{-lg - mf + c}{\sqrt{l^2 + m^2}}\right| = \sqrt{g^2 + f^2 - c}$$

$$\Leftrightarrow (lg + mf - n)^2 = (g^2 + f^2 - c)(l^2 + m^2)$$

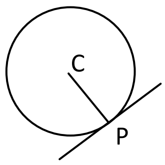
Corollary: The condition for the straight line y = mx + c to touch the circle $x^2 + y^2 = r^2$ is $c^2 = r^2(1 + m^2)$.

The given line is y=mx+c i.e., mx-y+c=0 … (1)

The given circle is $S=x^2 + y^2 = r^2$

Centre C = (0,0), radius =r.

If (1) is a tangent to the circle, then



Radius of the circle = perpendicular distance from centre of the circle to the line.

$$\Rightarrow r = \frac{|c|}{\sqrt{m^2+1}} \Rightarrow r^2 = \frac{c^2}{m^2+1} \Rightarrow r^2\left(m^2+1\right) = c^2$$

Corollary: If the straight line y = mx + c touches the circle $x^2 + y^2 = r^2$, then their point of contact is $\left(-\frac{r^2m}{c}, \frac{r^2}{c}\right)$.

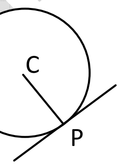
Proof:

The given line is y=mx+c i.e., mx-y+c=0 … (1)

The given circle is $S = x^2 + y^2 = r^2$ …(2)

Centre C = (0,0), radius =r

Let $P(x_1, y_1)$ be the point of contact.



Equation of the tangent is $S_1 = 0$, => $x_1x + y_1y - r^2 = 0$.----(3)

Equations (1) and (3) are representing the same line, therefore, $\frac{x_1}{m} = \frac{y_1}{-1} = \frac{-r^2}{c} \Rightarrow x_1 = \frac{-r^2m}{c}, y_1 = \frac{r^2}{c}$

Point of contact is $(x_1, y_1) = \left(-\frac{r^2m}{c}, \frac{r^2}{c}\right)$

Theorem: If P(x, y) is a point on the circle with centre C(α,β) and radius r, then x = α + r cosθ, y = β + r sin θ where 0 ≤θ< 2π.

Note 1: The equations x = α + r cosθ, y = β + r sin θ, 0 ≤θ< 2π are called parametric equations of the circle with centre (α, β) and radius r.

Note 2: A point on the circle $x^2 + y^2 = r^2$ is taken in the form (r cosθ, r sin θ). The point (r cosθ, r sin θ) is simply denoted as point θ.

Theorem: The equation of the chord joining two points θ_1 and θ_2 on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(x+g)\cos\frac{\theta_1+\theta_2}{2} + (y+f)\sin\frac{\theta_1+\theta_2}{2} = r\cos\frac{\theta_1-\theta_2}{2}$ where r is the radius of the circle.

Note1: The equation of the chord joining the points θ_1 and θ_2 on the circle $x^2 + y^2 = r^2$ is $x\cos\frac{\theta_1+\theta_2}{2} + y\sin\frac{\theta_1+\theta_2}{2} = r\cos\frac{\theta_1-\theta_2}{2}$.

Note2: The equation of the tangent at P(θ) on the circle $(x+g)\cos\theta+(y+f)\sin\theta=\sqrt{g^2+f^2-c}$.

Note 3: The equation of the tangent at P(θ) on the circle $x^2 + y^2 = r^2$ is $x \cos\theta + y \sin\theta = r$.

Note 4: The equation of the normal at P(θ) on the circle $x^2 + y^2 = r^2$ is $x \sin\theta - y \cos\theta = r$.

Very Short Answer Questions

1. Find the equation of the tangent at P of the circle S = 0 where P and S are given by $P = (7,-5)$, $S \equiv x^2 + y^2 - 6x + 4y - 12$

Sol. Equation of the circle is

$S \equiv x^2 + y^2 - 6x + 4y - 12 = 0$

Equation of the tangent at P (7,5) is $S_1 = 0$

$\Rightarrow x.7 + y(-5) - 3(x+7) + 2(y-5) - 12 = 0$

$\Rightarrow 7x - 5y - 3x - 21 + 2y - 10 - 12 = 0$

$\Rightarrow 4x - 3y - 43 = 0$

2) Find the equation of the normal at P of the circle S = 0 where P and S are given by

i) $P = (3, -4)$, $S \equiv x^2 + y^2 + x + y - 24$

Sol. Equation of the normal passing through (x_1,y_1) is $(x - x_1)(y_1 + f) - (y - y_1)(x_1 + g) = 0$

$\Rightarrow (x-3)\left(-4+\frac{1}{2}\right) - (y+4)\left(3+\frac{1}{2}\right) = 0$

$\Rightarrow \frac{7}{2}(x-3) - \frac{7}{2}(y+4) = 0$

$\Rightarrow (x-3) + (y+4) = 0$

$\Rightarrow x - 3 + y + 4 = 0$

i.e., $x + y + 1 = 0$

ii) $P = (3,5)$, $S \equiv x^2 + y^2 - 10x - 2y + 6$

Sol . Equation of the normal passing through (x_1,y_1) is $(x - x_1)(y_1 + f) - (y - y_1)(x_1 + g)$

$(x-3)(5-1) - (y-5)(3-5) = 0$

$\Rightarrow 4x - 12 + 2y - 10 = 0$

$\Rightarrow 4x + 2y - 22 = 0$

$\Rightarrow 2x + y - 11 = 0$

Short Answer Questions

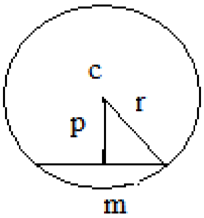
1) Find the length of the chord intercepted by the circle $x^2 + y^2 - x + 3y - 22 = 0$ on the line $y = x - 3$

Sol. Equation of the circle is $S \equiv x^2 + y^2 - x + 3y - 22 = 0$. Center $C\left(\frac{1}{2}, -\frac{3}{2}\right)$ and

Radius $r = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{1}{4} + \frac{9}{4} + 22} = \sqrt{\frac{1 + 9 + 88}{4}} = \sqrt{\frac{98}{44}}$

Equation of the line is $y = x - 3 \Rightarrow x - y - 3 = 0$

P = distance from the centre to the line



$= \frac{\left|\frac{1}{2} + \frac{3}{2} - 3\right|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

Length of the chord $= 2\sqrt{r^2 - p^2} = 2\sqrt{\frac{98}{4} - \frac{1}{2}} = 2\sqrt{\frac{98-2}{2}} = \sqrt{96} = 4\sqrt{6}$ units.

2) Find the length of the chord intercepted by the circle $x^2 + y^2 - 8x - 2y - 8 = 0$ the line $x + y + 1 = 0$

Sol. Equation of the circle is $x^2 + y^2 - 8x - 2y - 8 = 0$

Centre is C(4,1) and radius $r = \sqrt{16 + 1 + 8} = 5$

Equation of the line is $x + y + 1 = 0$

P = distance from the centre to the line

$= \left|\frac{4 + 1 + 1}{\sqrt{1+1}}\right| = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

Length of the chord $= 2\sqrt{r^2 - p^2}$

$= 2\sqrt{25 - 18} = 2\sqrt{7}$ units.

3) Find the length of the chord formed by $x^2 + y^2 = a^2$ on the line $x \cos\alpha + y \sin\alpha = p$.

Sol. Equation of the circle is $x^2 + y^2 = a^2$

Centre C (0, 0), radius = a

Equation of the line is $X \cos\alpha + y \sin\alpha - p = 0$

P = distance from the centre to the line $= \frac{|0 + 0 - p|}{\sqrt{\cos^2\alpha + \sin^2\alpha}} = p$

Length of the chord $= 2\sqrt{a^2 - p^2}$

4) Find the equation of circle with centre (2, 3) and touching the line 3x - 4y + 1= 0.

Sol.

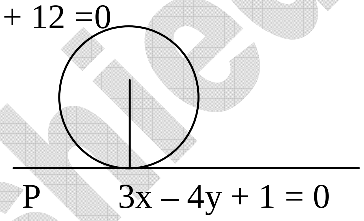
Centre C=(2,3).

Radius r =Perpendicular distance from C to 3x-4y +1=0= $\left|\frac{3(2) - 4(3) + 1}{\sqrt{3^2 + 4^2}}\right|$

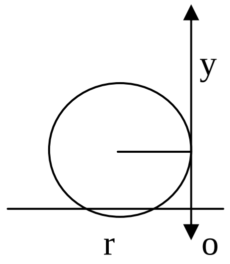
Equation of circle $(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-3)^2 = 1$

$x^2 + y^2 - 4x - 6y + 12 = 0$



5) Find the equation of the circle with Centre (-3,4) and touching y-axis.



(-3, 4)

Sol. Centre of the circle is C(-3,4).

Since the circle touches y – axis',

Radius r = distance of c from y-axis = $|3|$ = 3

Equation of the circle is $(x+3)^2 + (y-4)^2 = 9$

$x^2 + 6x + 9 + y^2 - 8y + 16 – 9 = 0$

i.e., $x^2 + y^2 + 6x – 8y + 16 = 0$

6) Find the equation of tangents of the Circle $x^2 + y^2 + 8x – 2y + 12 = 0$ at the points whose ordinates are 1.

Sol. Equation of the circle is

$S \equiv x^2 + y^2 – 8x – 2y + 12 = 0$

Let the point P be $(a, 1)$

Since P is a point on the circle ,

$a^2 + 1 - 8a - 2 + 12 = 0$

$a^2 - 8a + 11 = 0$

$a = \frac{8 \pm \sqrt{64 - 44}}{2} = \frac{8 \pm 2\sqrt{5}}{2} = 4 \pm \sqrt{5}$

$a_1 = 4 + \sqrt{5}$, $a_2 = 4 - \sqrt{5}$

Co – ordinates of P are $(4 + \sqrt{5}, 1)$ and Q $(4 - \sqrt{5}, 1)$

Equation of the tangent at P$(4 + \sqrt{5}, 1)$ is $S_1 = 0$

$x(4 + \sqrt{5}) + y.1 – 4(x + 4 + \sqrt{5}) – (y + 1) + 12 = 0$

$\Rightarrow 4x + \sqrt{5}x + y – 4x – 16 - 4\sqrt{5} - y - 1 + 12 = 0$

$\Rightarrow \sqrt{5}x – 5 - 4\sqrt{5} = 0 \Rightarrow \sqrt{5}(x - \sqrt{5} - 4) = 0$

$\Rightarrow x - \sqrt{5} - 4 = 0 \Rightarrow x = 4 + \sqrt{5}$

Equation of the tangent at Q(4 -$\sqrt{5}$, 1) is $S_2 = 0$

$\Rightarrow x(4 - \sqrt{5}) + y.1 - 4(x + 4 - \sqrt{5}) - (y+1)+12=0$

$\Rightarrow 4x - \sqrt{5}x + y - 4x - 16 + 4\sqrt{5} - y - 1 + 12 = 0$

$\Rightarrow -\sqrt{5}x + 4\sqrt{5} - 5 = 0 \Rightarrow -\sqrt{5}(x - 4 + \sqrt{5}) = 0$

$\Rightarrow x - 4 + \sqrt{5} = 0 \Rightarrow \quad x = 4 - \sqrt{5}$

7) Find the equation of tangents the circle $x^2 + y^2 - 10 = 0$ at the points whose abscissa are 1.

Sol. Equation of the circle is $S = x^2 + y^2 = 10$

Let the point be (1, y)

$1 + y^2 = 10 \Rightarrow y^2 = 9$

$Y = \pm 3$.

Co – ordinates of P are (1,3) and (1, -3)

Equation of the tangent at P (1, 3) is $S_1=0$.

$\Rightarrow x.1 + y.3 = 10$

$\Rightarrow x + 3y - 10 = 0$

Equation of the tangent of P(1, -3) is $S_2=0$

$\Rightarrow x.1 + y(-3) = 10 \Rightarrow x - 3y - 10 = 0$

Long Answer Questions

1) If $x^2 + y^2 = c^2$ and $\frac{x}{a} + \frac{y}{b} = 1$ intersect at A and B, then find AB. Hence deduce the condition, that the line touches the circle.

Sol. Equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

Equation of the circle $x^2 + y^2 = c^2$.

Centre C = (0,0) and radius r = c

Perpendicular from c to the line is

$$d = \left|\frac{0.\frac{1}{a} + 0.\frac{1}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

Length of chord AB $= 2\sqrt{r^2 - d^2}$

$$= 2\sqrt{c^2 - \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}}} = 2\sqrt{c^2 - \left(\frac{a^2b^2}{a^2 + b^2}\right)}$$

Line touches circle when $r^2 = d^2$ or r = d

Length of the chord = 0

$$\rightarrow c = \sqrt{\frac{a^2b^2}{a^2 + b^2}}$$

2) If y = mx + c and $x^2 + y^2 = a^2$ i) intersect at A and B. ii) AB = 2λ, then show that $C^2 = (1 + m^2)(a^2 - \lambda^2)$

Equation of circle $x^2 + y^2 = a^2$

Sol . C = (0, 0) r = a

Length of chord $= 2\sqrt{r^2 - d^2}$

$\Rightarrow 2\sqrt{r^2 + d^2} = 2\lambda$

$\Rightarrow a^2 - d^2 = \lambda^2$----------------- **(1)**

Equation of the line y = mx + c $\Rightarrow$ mx – y + c = 0

Perpendicular distance from c to line is $d = \left|\frac{0 - 0 + c}{\sqrt{m^2 + 1}}\right|$

$\Rightarrow d^2 = \frac{c^2}{m^2+1}$

Therefore, from (1)

$\Rightarrow a^2 - \frac{c^2}{m^2+1} = \lambda^2$

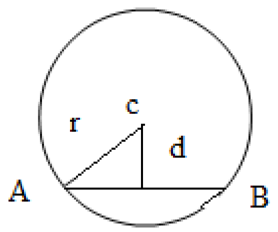
$\therefore$ $c^2 = (a^2 - \lambda^2)(1 + m^2)$ which is the required condition

3) Find the equation of the circle with centre (-2, 3) cutting a chord length 2 units on $3x + 4y + 4 = 0$

Sol. Equation of the line is $3x + 4y + 4 = 0$

d = Length of the perpendicular form (-2, 3) to 3x+4y +4 = 0

$= \frac{|3(-2) + 4.3 + 4|}{\sqrt{9+16}} = \frac{10}{5} = 2$



If r is the radius of the circle then Length of the chord = $2\sqrt{r^2 - d^2} = 2$ (given)

$r^2 - d^2 = 1$ then $r^2 - 2^2 = 1 \Rightarrow r^2 = 5$.

$\therefore$ Equation of the circle is $(x+2)^2 + (y-3)^2 = r^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 5$

i.e., $x^2 + y^2 + 4x - 6y + 8 = 0$

4) Find the equation of tangent and normal at (3,2) of the circle $x^2 + y^2 - x - 3y - 4 = 0$

Sol.

Equation of the circle is $S \equiv x^2 + y^2 - x - 3y - 4 = 0$

Equation of the tangent at P(3,2) is S1 =0

$\Rightarrow x.3 + y.2 - \frac{1}{2}(x + 3) - \frac{3}{2}(y + 2) - 4 = 0$

$6x + 4y - x - 3 - 3y - 6 - 8 = 0$

$\Rightarrow 5x + y - 17 = 0$

Let the normal be $x-5y+c=0$ ($\because$ normal is perpendicular to the tangent)

Since normal passes through P (3, 2)

$\Rightarrow 3 - 10 + c = 0 \Rightarrow c = 7$

Hence Equation of the normal is $x-5y+7 = 0$

5) Find the equation of the tangent and normal at (1,1) to the circle $2x^2+ 2y^2- 2x - 5y + 3 =0$

Sol. Equation of the circle is

$2x^2 + 2y^2 - 2x - 5y + 3 = 0$

$S \equiv x^2 + y^2 - x - \frac{5}{2} y - \frac{3}{2} = 0$

Equation of the tangent at P(1,1) is $S_1= 0$

$x.1 + y.1 - \frac{1}{2} (x + 1) - \frac{5}{4} (y+1) \frac{3}{2} = 0$

$4x + 4y - 2 (x + 1) - 5 (y+1) + 6 = 0$

$4x + 4y - 2x - 2 - 5y - 5 + 6 = 0$

$2x - y - 1 = 0$

Equation of the normal can be taken as

$x + 2y + k = 0$

The normal passes through Center P(1,1)

$\Rightarrow 1+2 + k = 0 \Rightarrow k = -3$

Equation of the normal is $x + 2y - 3 = 0$

6) Prove that the tangent at (3, -2) of the circle $x^2 + y^2 = 13$ touches the circle $x^2 + y^2+ 2x - 10y - 26 = 0$ and find its point of contact.

Sol.

Equation of the circle is $S \equiv x^2 + y^2=13$

Equation of the tangent at P(3, -2) is $S_1=0$

$x.3 + y(-2) = 13$

$\Rightarrow 3x - 2y - 13 = 0$ ----------- (1)

Equation of the second circle is

$x^2 + y^2+2x - 10y - 26 = 0$

Centre is c(-1, 5) and

Radius r = $\sqrt{1+25+26} = \sqrt{52} = 2\sqrt{13}$

d = length of the perpendicular from c(-1, 5) to (1)

$d = \frac{|-3-10-13|}{\sqrt{9+4}} = \frac{26}{\sqrt{13}} = 2\sqrt{13}$ = radius

∴ The tangent to the first circle also touches the second circle.

Equation of the circle

$x^2 + y^2 + 2x - 10y - 26 = 0$

Center c = (-1,5)

Equation of normal is $2x+3y+k = 0$. This normal is passing through (-1,5)

$\Rightarrow 2(-1) + 3(5) + k = 0 \Rightarrow K = -13$

∴ Normal is $2x + 3y - 13 = 0$ -------------- (2)

Solving (1) & (2)

Point of intersection is (5,1)

∴ Point of contact is (5,1)

7) Show that the tangent at (-1,2) the Circle $x^2 + y^2-4x-8y+7=0$ touches the Circle $x^2 + y^2+4x+6y=0$ and also Find its point of contact.

Sol. S≡ $x^2 + y^2 - 4x - 8y + 7 = 0$ equation of the tangent at (-1, 2) to S =0 is $S_1=0$

$\Rightarrow x(-1) + y(2) - 2(x-1) - 4(y+2) + 7 = 0$

$\Rightarrow -3x - 2y + 1 = 0 \Rightarrow 3x + 2y - 1 = 0.$

Equation of the second circle is $x^2 + y^2 + 4x + 6y = 0$

Centre C =(-2,-3) radius r $= \sqrt{g^2+f^2-c} = \sqrt{4+9} = \sqrt{13}$

Perpendicular distance from C to the line is d = $\left|\frac{3(-2)+2(-3)-1}{\sqrt{3^2+2^2}}\right| = \left|\frac{-13}{\sqrt{13}}\right| = \sqrt{13}$

d = r

Hence $3x + 2y - 1 = 0$ is also tangent to

$x^2 + y^2+4x+6y = 0$

Point of contact (foot of perpendicular)

Let (h, k) be foot of perpendicular from (-2,-3) to the line 3x+2y-1 =0

$\frac{h+2}{3} = \frac{k+3}{2} = \frac{|3(-2)+2(-3)-1|}{9+4} \Rightarrow \frac{h+2}{3} = 1$ and $\frac{k+3}{2} = 1$

h = 1, k = -1 therefore (1, -1) is point of contact.

8) Find the equation of the tangents to the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ which are parallel to $x + y - 8 = 0$

Sol. Equation of the circle is

$S = x^2 + y^2 - 4x + 6y - 12 = 0$

Centre is C(2, -3); r = radius $= \sqrt{4 + 9 + 12} = 5$

Equation of the given line is x +y-8=0

Equation of the line parallel to above line is x+y+k=0

If x+y+k=0 is a tangent to the circle then

Radius = perpendicular distance from the centre.

$$5 = \frac{|2 - 3 + k|}{\sqrt{1+1}}$$

$\Rightarrow |k - 1| = 5\sqrt{2} \Rightarrow k-1 = \pm\ 5\sqrt{2} \Rightarrow k = 1 \pm\ 5\sqrt{2}$

Equation of the tangent is

$x+y+1 \pm\ 5\sqrt{2} = 0$

9) Find the equations of the tangents to the circle $x^2+y^2 + 2x - 2y - 3 = 0$ which are perpendicular to $3x - y + 4 = 0$

Sol. S = $x^2+y^2 +2x-2y-3=0$, centreC(-1, 1)

And radius $r = \sqrt{1+1+3} = \sqrt{5}$

Equation of the line perpendicular to 3x-y+4 =0 is

x+ 3y +k = 0

$$\sqrt{5} = \left|\frac{-1+3+k}{\sqrt{1+9}}\right| \Rightarrow 5 = \frac{(k+2)^2}{10}$$

$\Rightarrow 50 = k^2+4k+4 \quad \Rightarrow k^2+4k-46 = 0$

$$\Rightarrow k = \frac{-4 \pm \sqrt{16+184}}{2}$$

$$k = \frac{-4 \pm 10\sqrt{2}}{2} = -2 \pm 5\sqrt{2}$$

Equation of the required tangent is

$x + 3y - 2 \pm\ 5\sqrt{2} = 0$

10) Find the equation of the tangents to the circle $x^2+y^2 - 4x - 6y + 3 = 0$ which makes an angle 45^0 with x – axis.

Sol. Equation of the circle is $S = x^2+y^2-4x-6y+3=0$

Centre C (3, 3), radius $r=\sqrt{4+9-3} = \sqrt{10}$

Slope of the tangent $m = \tan 45^0 = 1$

Equation of the tangent can be taken as $y=x+c$ i.e., $x-y+c = 0$

Length of the perpendicular from centre c to tangent $= \dfrac{|2-3+c|}{\sqrt{2}}$

$\therefore \sqrt{10} = \dfrac{|c-1|}{\sqrt{2}} \Rightarrow (c-1)^2 = 20$

$c-1 = \pm\sqrt{20} = \pm\ 2\sqrt{5}$

$c = 1 \pm\ 2\sqrt{5}$

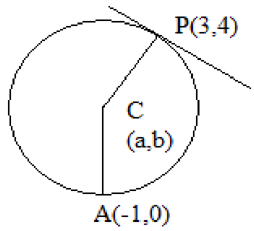
Equation of the tangents $x - y + 1 \pm\ 2\sqrt{5} = 0$

11)Find the equation of the circle passing through (-1, 0) and touching x + y – 7=0 at (3,4)

Sol.

Let C(a,b) be the centre of the circle A(-1,0) and P(3,4)

Equation of the tangent is $x+y-7 = 0$ ----------- (i)



Now CA = CP

$\Rightarrow CA^2 = CP^2$

$\Rightarrow (a+1)^2 + b^2 = (a-3)^2 + (b-4)^2$

$\Rightarrow 8a + 8b - 24 = 0 \Rightarrow a+b-3 = 0$ ---------- (2)

Line CP is perpendicular to tangent (1)

$\therefore$Product of their slopes = -1

$\left(\dfrac{b-4}{a-3}\right)(-1) = -1 \Rightarrow a - b + 1 = 0$ ------------- (3)

Solving (2) and (3) , a=1 and b=2.

Centre C =(1, 2)

radius r = CA = $\sqrt{(1+1)^2+2^2}=\sqrt{8}$

Equation of the circle is $(x-1)^2+(y-2)^2=8$

i.e., $x^2+y^2-2x-4y-3=0$

12) Find the equations of the circles passing through (1, -1)and touching the lines $4x + 3y + 5 = 0$ and $3x - 4y - 10 = 0$

Sol. Suppose equation of the circle is $x^2+y^2+2gx+2fy+c=0$

⊥ From (-g, -f) to the line is equal to the radius of the circle

$$\left|\frac{-4g-3f+5}{5}\right|=\left|\frac{-3g+4f-10}{5}\right|$$

$-7g+f-5=0$ (or) $-g-7f+15=0$

$F = 7g + 5$

Now $\left|\frac{-4g-3f+5}{5}\right|^2$ $(-g-1)^2+(-f+1)^2$

$\Rightarrow \frac{(-4g-21g-15+5)^2}{5}$

$=(-g-1)^2+(-7g-5+1)^2$

$(5g+2)^2$ $g^2+1+2g+16+49$

$=g^2+56g$

Solving, we get

$25g^2+38g+13=0$

$g=-1, \frac{-26}{50}$

If $g = -1$; $f = -2$

Circle is passing through (1, -1)

$\therefore x^2+y^2-2x-4y-4=0$

Required equation of circle is $x^2+y^2-2x-4y-4=0$

13. Show that x +y +1 = 0 touches the circle $x^2+y^2-3x + 7y + 14 = 0$ and find its point of contact.

Sol.

S=$x^2+y^2-3x + 7y + 14 = 0$

Centre (3/2,-7/2)

Radius = $\frac{1}{\sqrt{2}}$

Perpendicular distance from centre to the line **x +y +1 = 0----------(1) is** $\frac{\left|\frac{3}{2}-\frac{7}{2}+1\right|}{\sqrt{1+1}}=\frac{1}{\sqrt{2}}$ =radius of the circle.

$\therefore$Given line touches the given circle.

Let (x_1,y_1) be the point of contact. Then equation of the tangent is S_1=0.

$$\Rightarrow X_1X+Y_1Y-\frac{3}{2}(X+X_1)+\frac{7}{2}(Y+Y_1)+14=0$$

$$\Rightarrow\left(x_1-\frac{3}{2}\right)x+\left(Y_1+\frac{7}{2}\right)y+\left(-\frac{3}{2}X_1+\frac{7}{2}Y_1+14\right)=0 \text{ --------- (2)}$$

Equations (1) and (2) representing same line, therefore

$$\frac{\left(x_1-\frac{3}{2}\right)}{1}=\frac{\left(Y_1+\frac{7}{2}\right)}{1}=\frac{\left(-\frac{3}{2}X_1+\frac{7}{2}Y_1+14\right)}{1}$$

$$\Rightarrow x_1-y_1=5 \quad \text{and} \quad \frac{5}{2}x_1-\frac{7}{2}y_1=\frac{31}{2}\Rightarrow 5x_1-7y_1=31$$

solving these equations $y_1=-3$ and $x_1=2$

Therefore the point of contact is (2,-3)