# CIRCLES PART - I

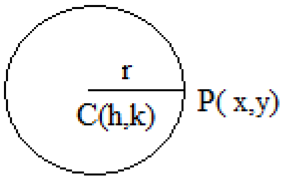
**Equation of A Circle:**

The equation of the circle with centre C (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

**Proof:** Let $P(x_1, y_1)$ be a point on the circle.

$$\text{P lies in the circle} \Leftrightarrow PC = r \Leftrightarrow \sqrt{(x_1-h)^2+(y_1-k)^2} = r$$

$$\Leftrightarrow (x_1-h)^2 + (y_1-k)^2 = r^2.$$




The locus of P is $(x-h)^2 + (y-k)^2 = r^2$.

$\therefore$ The equation of the circle is $(x-h)^2 + (y-k)^2 = r^2$.------(1)

**Note:** The equation of a circle with centre origin and radius r is $(x-0)^2 + (y-0)^2 = r^2$

i.e., $x^2 + y^2 = r^2$ which is the standard equation of the circle.

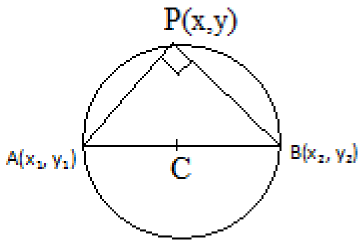
**Note:** On expanding equation (1), the equation of a circle is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$.

**Theorem: If $g^2 + f^2 - c \geq 0$, then the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre $(-g, -f)$ and radius $\sqrt{g^2+f^2-c}$.**

**Note:** If $ax^2 + ay^2 + 2gx + 2fy + c = 0$ represents a circle, then its centre = $\left(-\frac{g}{a}, -\frac{f}{a}\right)$ and its radius $\frac{\sqrt{g^2+f^2-ac}}{|a|}$.

**Theorem:**

The equation of a circle having the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ as diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.



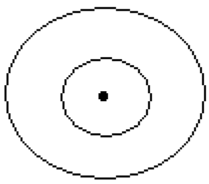

Let P(x,y) be any point on the circle. Given points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Now $\underline{|APB} = \frac{\pi}{2}$. (Angle in a semi circle.)

Slope of AP. Slope of BP =-1

$$\Rightarrow \frac{y-y_1}{x-x_1}\frac{y-y_2}{x-x_2} = -1$$
$$\Rightarrow (y-y_2)(y-y_1) = -(x-x_2)(x-x_1)$$
$$\Rightarrow (x-x_2)(x-x_1)+(y-y_2)(y-y_1) = 0$$

**Definition:** Two circles are said to be concentric if they have same center.



The equation of the circle concentric with the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is of the form $x^2 + y^2 + 2gx + 2fy + k = 0$.

The equation of the concentric circles differs by constant only.

**Parametric Equations of A Circle:**

**Theorem:** If P(x, y) is a point on the circle with centre $C(\alpha,\beta)$ and radius r, then $x = \alpha + r\cos\theta$, $y = \beta + r\sin\theta$ where $0 \le \theta < 2\pi$.

**Note:** The equations $x = \alpha + r\cos\theta$, $y = \beta + r\sin\theta$, $0 \le \theta < 2\pi$ are called parametric equations of the circle with centre $(\alpha, \beta)$ and radius r.

**Note:** A point on the circle $x^2 + y^2 = r^2$ is taken in the form $(r\cos\theta, r\sin\theta)$. The point $(r\cos\theta, r\sin\theta)$ is simply denoted as point$\theta$.

**Theorem:**

(1) If $g^2 - c > 0$ then the intercept made on the x axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $2\sqrt{g^2 - ac}$

2) If $f^2 - c > 0$ then the intercept made on the y axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $2\sqrt{f^2 - bc}$

**Note:** The condition for the x-axis to touch the circle
$x^2 + y^2 + 2gx + 2fy + c = 0$ $(c > 0)$ is $g^2 = c$.

**Note:** The condition of the y-axis to touch the circle
$x^2 + y^2 + 2gx + 2fy + c = 0$ $(c > 0)$ is $f^2 = c$.

**Note:** The condition for the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ to touch the coordinate axes is $g^2 = f^2 = c$.

**Position of Point:**

Let S = 0 be a circle and $P(x_1, y_1)$ be a point I in the plane of the circle. Then

i) P lies inside the circle $S = 0 \Leftrightarrow S_{11} < 0$

ii) P lies in the circle $S = 0 \Leftrightarrow S_{11} = 0$

iii) P lies outside the circle $S = 0 \Leftrightarrow S_{11} = 0$

**Power of a Point:**

Let S = 0 be a circle with centre C and radius r. Let P be a point. Then $CP^2 - r^2$ is called power of P with respect to the circle S = 0.

**Theorem:** The power of a point $P(x_1, y_1)$ with respect to the circle $S = 0$ is $S_{11}$.

**Theorem:** The length of the tangent drawn from an external point $P(x_1, y_1)$ to the circle $S = 0$ is $\sqrt{S_{11}}$.

## Very Short Answer Questions

**1. Find the equation of the circle with centre C and radius r where.**

**i) $C = (1, 7)$, $r = \frac{5}{2}$**

**Sol.** Equation of the circle is

$$(x-h)^2 + (y-k)^2 = (r)^2$$

$$\Rightarrow (x-1)^2 + (y-7)^2 = \left(\frac{5}{2}\right)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 14y + 49 = \frac{25}{4}$$

$$\Rightarrow x^2 + y^2 - 2x - 14y + \frac{175}{4} = 0$$

$$\Rightarrow 4x^2 + 4y^2 - 8x - 56y + 175 = 0$$

**ii) $C = (a, -b)$; $r = a + b$**

Equation of the circle is

$$(x-h)^2 + (y-k)^2 = (r)^2$$

Equation of the circle is

$$(x-a)^2 + (y-(-b))^2 = (a+b)^2$$

$$\Rightarrow x^2 - 2xa + a^2 + y^2 + 2by + b^2 = a^2 + 2ab + b^2$$

$$\Rightarrow x^2 + y^2 - 2xa + 2by - 2ab = 0$$

**2. Find the equation of the circle passing through the origin and having the centre at (-4, -3).**

**Sol.** Centre $(h, k) = (-4, -3)$

Equation of the circle is

$$(x-h)^2 + (y-k)^2 = r^2;$$

$$(x+4)^2 + (y+3)^2 = r^2$$

Circle is passing through origin

$$\therefore (0+4)^2 + (0+3)^2 = r^2$$

$\Rightarrow r^2 = 25$

$\therefore (x+4)^2 + (y+3)^2 = 25$

Hence equation of the circle is

$x^2 - y^2 + 8x + 6y = 0$

**3. Find the equation of the circle passing through (2, -1) having the centre at (2, 3).**

**Sol.** Centre C = (2, 3), point P = (2, -1)

Radius CP = $\sqrt{(2-2)^2 + (3+1)^2} = 4$

Equation of circle be

$(x-2)^2 + (y-3)^2 = 4^2$

**Ans.** $x^2 + y^2 - 4x - 6y - 3 = 0$

**4. Find the equation of the circle passing the through (-2, 3) having the centre at (0, 0).**

**Ans.** $x^2 - y^2 = 13$

**5. Find the value of 'a' if $2x^2 + ay^2 - 3x + 2y - 1 = 0$ represents a circle and also find its radius.**

**Sol.**

The equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a circle, if

$a = b$, $h = 0$ and $g^2 + f^2 - c \geq 0$

If $2x^2 + ay^2 - 3x + 2y - 1 = 0$ represents a circle, then a = 2 and the equation is

$2x^2 + 2y^2 - 3x + 2y - 1 = 0$

$\Rightarrow x^2 + y^2 - \frac{3}{2}x + y - \frac{1}{2} = 0$

$\Rightarrow g = -\frac{3}{4};\ f = \frac{1}{2},\ c = -\frac{1}{2}$

$C = (-g, -f) = \left(\frac{+3}{4}, \frac{-1}{2}\right)$

Radius = $\sqrt{g2 + f2 - c}$

$= \sqrt{\frac{9}{16} + \frac{1}{4} + \frac{1}{2}} = \frac{\sqrt{21}}{4}$

**6. Find the values of a, b if $ax^2 + bxy + 3y^2 - 5x + 2y - 3 = 0$ represent a circle. Also find the radius and centre of the circle.**

**Sol.** The equation of second degree $ax^2 + 2hy + by^2 + 2gx + 2hy + c = 0$ represents a circle if

$a = b, h = 0, g^2 + f^2 - c \geq 0$

$\therefore ax^2 + bxy + 3y^2 - 5x + 2y - 3 = 0$ represents a circle if $b = 0, a = 3$

Equation of circle is $3x^2 + 3y^2 - 5x + 2y - 3 = 0$

$x^2 + y^2 - \frac{5}{3}x + \frac{2}{6}y - 1 = 0$

$g = -\frac{5}{6}; f = \frac{2}{6}; c = -1$

$C = (-g, -f) = \left(\frac{5}{6}, \frac{1}{3}\right)$

Radius $= \sqrt{g2 + f2 - c}$

$= \sqrt{\frac{25}{36} + \frac{1}{9} + 1} = \frac{\sqrt{65}}{6}$

**7. If $x^2 + y^2 + 2gx + 2fy - 12 = 0$ represents a circle with centre (2, 3) find g, f and its radius.**

**Sol.** Centre C = (-g, -f)= (2,3)

$\therefore g = -2, f = -3, c = -12$

Radius $= \sqrt{g2 + f2 - c}$

$= \sqrt{4 + 9 + 12}$

= 5 units

**8. If $x^2 + y^2 + 2gx + 2fy = 0$ represents a circle with centre (-4, -3) then find g, f and the radius of the circle.**

**Sol.** C = (-g, -f)

C = (-4, -3)

$\therefore g = 4, f = 3$

Radius $= \sqrt{g2 + f2 - c}$

$= \sqrt{16 - 9}$ $\Rightarrow$ 5 units

**9. If $x^2 + y^2 - 4x + 6y + c = 0$ represents a circle with radius 6 then find the value of c.**

**Sol.**

Centre = (-g, -f) = (2,-3)

r = $\sqrt{g^2 + f^2 - c}$ ; g = - 2, f = 3

$\Rightarrow 6 = \sqrt{4 + 9 - c}$

$36 = 13 - c \Rightarrow c = -23$

**10. Find the centre and radius of the circle of each whose equation is given below.**

**i) $x^2 + y^2 - 4x - 8y - 41 = 0$**

**Sol**. Given circle is

$x^2 + y^2 - 4x - 8y - 41 = 0$

g = - 2, f = - 4, c = - 41

Centre = (-g, -f) = (2, 4)

Radius = $\sqrt{g2 + f2 - c}$

= $\sqrt{4 + 16 + 41}$

= $\sqrt{61}$ Units

**ii) $3x^2 + 3y^2 - 5x - 6y + 4 = 0$**

**Sol.** Equation of the circle is

$3x^2 + 3y^2 - 5x - 6y + 4 = 0$

$x^2 + y^2 - \frac{5}{3} \times -\frac{6}{3}y + \frac{4}{3} = 0$ then g = $-\frac{5}{6}$ ; f = - 1 ;and c = $\frac{4}{3}$

Centre = (-g, -f) = $\left(\frac{5}{6}, 1\right)$

r = $\sqrt{g2 + f2 - c}$ = $\sqrt{\frac{25}{36} + 1 - \frac{4}{3}} = \frac{13}{6}$ units

**11. Find the equations of the circles for which the points given below are the end points of a diameter.**

**i) (1, 2), (4, 6)**

**Sol .** Equation of the circle with $(x_1, y_1)$, $(x_2, y_2)$ as ends of a diameter is

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

$\Rightarrow (x-1)(x-4) + (y-2)(y-6) = 0$

$\Rightarrow x^2 - 5x + 4 + y^2 - 8y + 12 = 0$

$\Rightarrow x^2 + y^2 - 5x - 8y + 16 = 0$

**ii) (-4, 3); (3, -4)**

**Sol.** Equations of circle with $(x_1, y_1)$ and

$(x_2, y_2)$ are end points of diameter is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$

Required equation of circle be

$(x+4)(x-3) + (y-3)(y+4) = 0$

$X^2 + y^2 + x + y - 24 = 0$

**12. Obtain the parametric equation of each of the following circles.**

**i) $x^2 + y^2 = 4$**

Sol. C (0, 0), r = 2

Parametric equations are

$x = r\cos\theta = 2\cos\theta$

$y = r\sin\theta = \sin\theta,\ 0 \le \theta < 2\pi$

**ii) $4(x^2 + y^2) = 9$**

**Sol.** $x^2 + y^2 = \frac{9}{4}$

Centre C (0, 0), r = $\frac{3}{2}$

Parametric equations are

$x = \frac{3}{2}\cos\theta,\ y = \frac{3}{2}\sin\theta,\ 0 \le 2\pi$

**iii) $(x-3)^2 + (y-4)^2 = 8^2$**

**Sol.** Parametric equations are

$x = h + r\cos\theta$, $y = k + r\sin\theta$, $0 \le \theta < 2\pi$

Here (h,k) is the centre of the circle.

$x = 3 + 8\cos\theta$, $y = 4 + 8\sin\theta$, $0 \le \theta < 2\pi$

$x = 2 + 5\cos\theta$, $y = 3 + 5\sin\theta$, $0 \le \theta < 2\pi$

**13. Locate the position of the point P with respect to the circle S=0 when**

**i) P(3,4) and $S \equiv x^2 + y^2 - 4x - 6y - 12 = 0$**

**Sol.** $S \equiv x^2 + y^2 - 4x - 6y - 12$

Given point P(3,4)

$S_{11} = 3^2 + 4^2 - 4.3 - 6.4 - 12$

$= 9 + 16 - 12 - 24 - 12$

$= -23 < 0$

P (3, 4) lies inside the circle

**ii) P(1,5) and $S \equiv x^2 + y^2 - 2x - 4y + 3 = 0$**

**Sol.** $S_{11} = (1)^2 + (5)^2 - 2(-1) - 4(5) + 3 = 7$

$S_{11} > 0$ ∴P is outside the circle

**14. Find the power of the point P with Respect to the circle S = 0 When**

**i) P = (5,-6), and $S \equiv x^2 + y^2 + 8x + 12y + 15$**

**Sol.** Power of the point $= S_{11}$

$= 25 + 36 + 40 - 72 + 15 = 116 - 72 = 44$

**ii)** P = (2,4) and $S \equiv x^2 + y^2 - 4x - 6y - 12$

Power of the point = 4 + 16 - 8 - 24 - 12

= - 24.

**15. Find the length of tangent from P to the circle S = 0 when i) P =(-2,5) and S ≡ $x^2$+ $y^2$-25.**

**Sol.** Length of tangent = $\sqrt{S_{11}}$

$=\sqrt{(-2)^2+(5)^2-25}$= 2 units

i)P =(-2,5) and S ≡ $x^2$+$y^2$ -5x +4y -5

**Sol.** Length of the tangent $\sqrt{S_{11}}$

$=\sqrt{4+25-10+20-5}$

$=\sqrt{34}$ units

**16. If the length of the tangent from (5, 4) to the circle $X^2$ + $y^2$ $^+$2ky = 0 is 1 then find k.**

**Sol.** Length of tangent =$\sqrt{S_{11}}$=$\sqrt{(5)^2+(4)^2}-8k$

But length of tangent =1

∴ 1 = $\sqrt{25+16+8k}$

Squaring both sides we get 1 =41 + 8K

K = -5 units.

**17) If the length of the tangent from (2, 5) to the circle $x^2$+$y^2$ – 5x +4y + k =0 is $\sqrt{37}$ then fine k.**

**Sol.** Length of tangent = $\sqrt{S_{11}}$

$=\sqrt{(2)^2+(5)^2-5\text{ x }2+4\text{ x}5+k}$

= 37 = 39 +k

K = - 2 units.

## Short Answer Questions

**1. If the abscissa of points a, B are the roots of the equation, $x^2 + 2ax - b^2 = 0$ and ordinates of A, B are roots of $y^2 + 2py - q^2 = 0$, then find the equation of a circle for which AB is a diameter.**

**Sol.**

Let $A(x_1,y_1)$, $B(x_2,y_2)$ be the points.

Given $x_1, x_2$ are the roots of $x^2 + 2ax - b^2 = 0$, therefore, $x_1+x_2 = -2a$ and $x_1x_2 = -b^2$

Given $y_1, y_2$ are the roots of $y^2 + 2py - q^2 = 0$ therefore, $y_1+y_2 = -2p$ and $y_1y_2 = -q^2$

Equation of the circle with $(x_1, y_1)$, $(x_2, y_2)$ as ends of a diameter is

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

i.e. $x^2 + y^2 - (x_1+x_2)x - (y_1+y_2)y + x_1x_2 + y_1y_2 = 0$

$x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0.$

**2. Show that A (3,-1) lies on $x^2 + y^2 - 2x + 4y = 0$ and find the other end of diameter through A.**

**Sol.**

Equation of the circle is $x^2 + y^2 - 2x + 4y = 0$---------- (1)

substituting A (3,-1) in eq. (1)

$3^2 + (-1)^2 - 2(3) + 4(-1) = 9 + 1 - 6 - 4 = 0$

Therefore A (3,-1) is a point on the given circle.

Centre of the circle is C =(1,-2)

Let B(h, k) be the other end of the diameter.

Then centre C = midpoint of diameter AB

$$(1,-2) = \left(\frac{h+3}{2}, \frac{k-1}{2}\right)$$

(h,k) = (-1,-3)

**3. Find the equation of a circle which passes through (2,-3) and (-4, 5) and having the centre on $4x + 3y + 1 = 0$**

**Sol.**

Let S(a,b) be the centre of the circle.

S(a,b) is a point on the line $4x + 3y + 1 = 0$

$\Rightarrow 4a + 3b + 1 = 0$ ----------(1)

A(2,-3)and B(-4,5) are two points on the circle.

Therefore, SA=SB $\Rightarrow SA^2 = SB^2$

$\Rightarrow (a - 2)^2 + (b + 3)^2 = (a + 4)^2 + (b - 5)^2$

$\Rightarrow 3a - 4b + 7 = 0$ ----(2)

Solving (1) and (2), we get

(a,b) = (-1,1) = centre.

Raidus= $SA = \sqrt{(2+1)^2 + (-3-1)^2}$

$= 5$

Equation of the circle is $(x + 1)^2 + (y - 1)^2 = 5^2$

$= x^2 + y^2 + 2X - 2y - 23 = 0$

**4. Find the equation of a circle which passes through (4, 1) (6,5) and having the centre on $4x + 3y - 24 = 0$**

**Ans.** $x^2 + y^2 - 6x - 8y + 15 = 0$

**5. Find the question of a circle which is concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ and passing through (-2, 14).**

**Sol.** Equation of the circle concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ is $x^2 + y^2 - 6x - 4y + k = 0$

It is passing through (-2,14)

$\therefore (-2)^2 - (14)^2 - 6(-2) - 4(14) + k = 0$

$156 + k = 0$

$k = -156$

If the circle is

$x^2 + y^2 - 6x - 4y - 156 = 0$

**6. Find the question of the circle whose centre lies on the X – axis and passing through (-2,3) and (4,5).**

**Sol**. Let the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ________(i)

Centre is (-g,-f)

But centre is on x-axis, f = 0

( i) is passing through

(-2, 3) and (4,5)

4+9-4g +6f +c = 0

⇨ -4g + c = -13 ____(ii)

And

16+25+8g+10f + c =0

⇨ 8g+c = -41 ____(iii)

(iii) – (ii) we get

12g = -28

3g = -7 => g = $-\frac{7}{3}$

From (ii) c = $-\frac{67}{3}$,

From (i) required equation will be

$3(x^2 + y^2) - 14x - 67 = 0.$

# Long Answer Questions

**1. Find the equation of circle passing through each of the following three points.**

**i) (3, 4); (3, 2); (1, 4)**

Let the equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

It is passing through (3, 4); (3, 2); (1, 4)

∴Given points satisfy above equation then

$9 + 16 + 6g + 8f + c = 0$

$25+6g + 8f + c = 0$ _____(i)

$9 + 4 + 6g + 4f + c = 0$

$13 + 6g + 4f + c = 0$ _____(ii)

$1+16+2g +8f +c = 0$

$17+2g +8f +c = 0$________(iii)

(ii) – (i) we get

$-12 -4f = 0$ (or) $f = - 3$

(ii) – (iii) we get $-4 +4g -4f = 0$

$g - f = 1 \Rightarrow g = - 2$

Now substituting g, f in equation (i) we get

$25 + 6 (-2) +8 (-3) + c = 0$

We get $c = 11$

Required equation of circle be

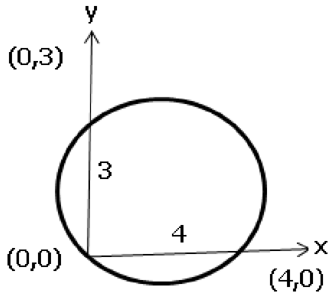
$X^2 + y^2 -4x - 6y + 11 = 0$

Practice problem (5,7); (8,1); (1,3)

**Ans.** $3(x^2+y^2) - 29x - 19y + 56 = 0$

**2. i) Find the equation of the circle passing through (0,0) and making intercepts 4,3 on X – axis and Y –axis respectively.**

**Sol.**




Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

Given circle is making intercepts 4, 3 on x, y –axes respectively.

Therefore, (4,0) and (0,3) are two points on the circle.

Circle is passing through

(0,0), (4,0) and (0,3).

$(0,0) \Rightarrow 0 + 0 + 2g(0) + 2f(0) + c = 0$

$C = 0$

$(4,0) \Rightarrow 16+0+8g+2f.0+c = 0$

$G = 2$ as $c = 0$

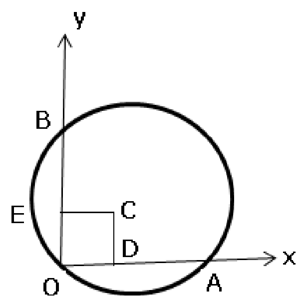
$(0,3) \Rightarrow 0+ 9+2g.0+6f +c = 0$

$f = -\frac{3}{2}$ as $c = 0$

Required equation of circle is $X^2 + y^2 - 4x - 3y = 0$

**ii) Find the equation of the circle passing through (0, 0) and making intercept 6 units on X- axis and intercept 4 units on Y – axis.**

**Sol.**



OA = 6 units

OB = 4 units

Let D,E be the mid points of OA and OB.

Then OD = 3 units OE = 2 units

∴ Co –ordinates of centre c are( 3,2)

Radius OC = $\sqrt{3^2 + 2^2}$

= $\sqrt{13}$

Equation of circle with ( h,k) as centre be radius is $(x-h)^2 + (y-k)^2 = r^2$

∴ Required equation of circle be

$(x-3)^2 + (y-2)^2 = 13$

$x^2 + y^2 - 6x - 4y = 0.$

**3. Show that the following four points in each of the following are concyclic and find the equation of the circle on which they lie.**

**i) (1, 1), (-6, 0), (-2,2), (-2,-8)**

First find the equation of the circle passing through the points (1, 1), (-6, 0), (-2, 2)

The circle passing through (1, 1), (-6, 0), (-2, 2) is $x^2 + y^2 + 4x + 6y - 12 = 0$

Substitute (-2,-8) in above equation, then

$(-2)^2 + (-8)^2 + 4(-2) + 6(-8) - 12 = 0$

4+64-8-48-12 =0

⇨ 0 =0.

Hence the points are concyclic. And the equation of the circle is

$x^2 + y^2 + 4x + 6y - 12 = 0$

**4. If (2, 0), (0,1) (4,5), (0,c) are concyclic and then find c.**

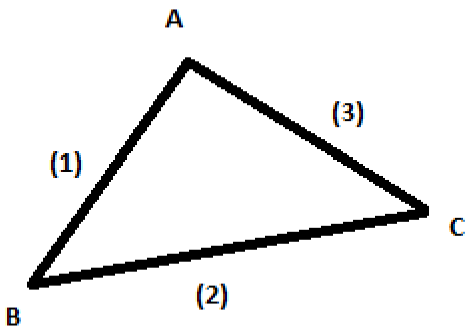
$1^{st}$ find the equation of the circle passing through (2, 0), (0, 1) (4, 5) then substitute (0, c).

**Ans**. 1 or $\frac{14}{3}$

**5. Find the equation of the circum circle of the triangle formed by the straight lines given in each of the following.**

i) $2x + y = 4; x + y = 6; x + 2y = 5$

**Sol.**




Given lines are

$2x + y = 4$---------(1)

$x + y = 6$---------(2)

$x + 2y = 5$---------(3)

On solving (1) and (2), we get

B = (-2,8)

On solving (1) and (3), we get

A = (1,2)

On solving (3) and (2), we get

C = (7,-1)

Let S(h,k) be the circum centre of the triangle ABC

Then SA = SB = SC.

$SA = SB \Rightarrow SA^2 = SB^2$

$\Rightarrow (1 - h)^2 + (2 - k)^2 = (-2 - h)^2 + (8 - k)^2$

$\Rightarrow h^2 + k^2 - 2h - 4k + 5 = h^2 + k^2 + 4h - 16k + 68$

$\Rightarrow 6h-12k+63 = 0$ --------(4)

$SA = SC \Rightarrow SA^2 = SC^2$

$\Rightarrow (1-h)^2 + (2-k)^2 = (7-h)^2 + (-1-k)^2$

$\Rightarrow h^2 + k^2 - 2h - 4k + 5 = h^2 + k^2 - 14h + 2k + 50$

$\Rightarrow 12h - 6k - 45 = 0$--------(5)

Solving (4) and (5), We get $S = (17/2, 19/2)$

Now radius = SA

$$= \sqrt{\left(1 - \frac{17}{2}\right)^2 + \left(2 - \frac{19}{2}\right)^2} = \frac{225}{\sqrt{2}}$$

Equation of the circle is

$$\left(x - \frac{17}{2}\right)^2 + \left(y - \frac{19}{2}\right)^2 = \frac{225}{2}$$

$\Rightarrow x^2 + y^2 - 17x - 19y + 50 = 0.$

ii) $x + 3y - 1 = 0$; $x + y + 1 = 0$; $2x + 3y + 4 = 0$

Ans. $x^2 + y^2 + 12x + 12y + 7 = 0$

**6. Show that the locus of the point of intersection of the lines $x\cos\alpha + y\sin\alpha = a$, $x\sin\alpha - y\cos\alpha = b$ ($\alpha$ is a parameter) is a circle.**

**Sol.** Equations of the given lines are $x\cos\alpha + y\sin\alpha = a$

$y\sin\alpha - y\cos\alpha = b$

Let $p(X_1, Y_1)$ be the point of intersection

$x_1\cos + y_1\sin\alpha = a$ – (1)

$x_1\sin\alpha - y_1\cos\alpha = b$ –(2)

Squaring and adding (1) and (2)

$(x_1\cos\alpha + y_1\sin\alpha)^2 + (x_1\sin\alpha - y_1\cos\alpha)^2$

$= \alpha^2 + b^2$

$x_1^2\cos^2\alpha + Y_1^2\sin^2\alpha + 2x_1y_1$

$\cos\alpha\sin\alpha + X_1^2\sin^2\alpha + Y_1^2\cos^2\alpha$

$-2 x_1 y_1 \cos\alpha \sin\alpha = a^2 + b^2$

$x_1^2 (\cos^2\alpha + \sin^2\alpha) + y_1^2 (\sin^2\alpha + \cos^2\alpha)$

$= a^2 + b^2$

$x_1^2 + y_1^2 == a^2 + b^2.$

Locus of $p(x_1, y_1)$ is which represents a circle

$X^2 + y^2 = a^2 + b^2$

**7. Show that the locus of a point such that the ratio of distance of it from two given Point is constant k($\neq \pm 1$) is a circle.**

**Sol.** Let $P(x_1, y_1)$ be a point on the locus Let A(a, 0), B(-a, 0)be two given points

Given $\frac{PA}{PB} = k, (\neq \pm 1)$

$\therefore \frac{\sqrt{(x_1 - a)^2 + y_1^2}}{\sqrt{(x_1 + a)^2 + y_1^2}} = K$

By Squaring and cross multiplying, we get

$(x_1 - a)^2 + y_1^2 = k^2 [(x_1 + a)^2 + y_1^2]$

$\Rightarrow (1-k^2)(x_1^2 + y_1^2 + a^2) + (-1-k^2)(2ax_1) = 0$

$\Rightarrow x_1^2 + y_1^2 - 2 \frac{(1+k)^2}{1-k^2} ax + a^2 = 0$

$\therefore$ Locus of $p(x_1, y_1)$ is

$x^2 + y^2 - 2\left(\frac{1+k^2}{1-k^2}\right)ax + a^2 = 0$

Which represents a circle. (Here k $\neq \pm$)

**8. If a point P is moving such that the Lengths of tangents drawn from P to $X^2 + y^2 + 6x + 18y + 26 = 0$ are in the Ratio 2:3, then find the equation of the Locus of P.**

**Sol.** Let p(x, y) be any point on the locus.

Let. $S \equiv X^2 + y^2 + 4x + 6y - 12 = 0$

Lengths of tangents from P to S=0 is

$PT_1 = \sqrt{x^2 + y^2 - 4x - 6y - 12}$

Let. $S^1 = x^2 + y^2 + 6x + 18y + 26 = 0$

Length Tangent from P to $S^1 = 0$ is

$PT_2 = \sqrt{x^2 + y^2 + 6x + 18y + 26}$

Given $\frac{PT_1}{PT_2} = \frac{2}{3}$

$\Rightarrow \frac{PT_1^2}{PT_2^2} = \frac{4}{9}$

$9\ PT_1^2 = 4.PT_2^2$

$9(x^2 + y^2 - 4x - 6y - 12)$

$= 4(x^2 + y^2 + 6x + 18y + 26)$

$9x^2 + 9y^2 - 36x - 54y - 108$

$= 4x^2 + 4y^2 + 24x + 72y + 104$

Locus of P is $5x^2 + 5y^2 - 60x - 126y - 212 = 0$

**9. If a point P is Moving such that the Lengths of the tangents drawn from P to the circles $x^2 + y^2 + 8x + 12y + 15 = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ are equal then find the equation of the locus of**

**Sol.**

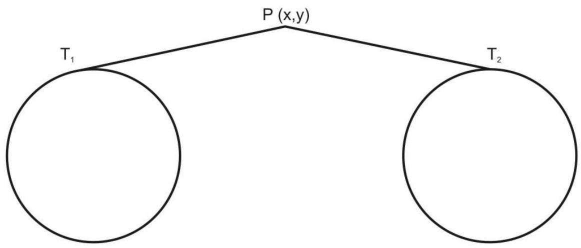
$S = x^2 + y^2 + 8x + 12y + 15 = 0$

$S^1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$

$P(x_1, y_1)$ is any point on the locus and $PT_1$, $PT_2$ are the tangents from P to the two circles.

Given condition is

$PT_1 = PT_2 \Rightarrow PT_1^2 = PT_2^2$




$$x_1^2 + y_1^2 + 8x_1 + 12y_1 + 15$$
$$= x_1^2 + y_1^2 - 4x_1 - 6y_1 - 12$$
$$12x_1 + 18y_1 + 27 = 0$$
$$(or)\, 4x_1 + 6y_1 + 9 = 0$$

Locus of $P(x_1, y_1)$ is $4x + 6y + 9 = 0$