Least Common Multiple (L.C.M.)

L.C.M.: It is defined as the least number which is exactly divisible each one of the given numbers is called the L.C.M. of that number.

Method of Finding L.C.M.: There are two methods to find L.C.M.

1. Factorisation method
2. Short Cut method

1. L.C.M. by Factorisation method: Resolve the given numbers into prime factors then L.C.M. is the product of highest powers of all the factors.

Ex. Find the L.C.M. of 72, 108 & 2100 ?

Ans:

2|72	2|108	3|2100
2|36	2|54	7|700
2|18	3|27	2|100
3|9	3|9	2|50
|3	|3	5|25
		|5

$72 = 2^3 \times 3^2 = 8 \times 9$

$108 = 2^3 \times 3^3 = 4 \times 27$

$2100 = 2^2 \times 3 \times 5^2 \times 7 = 4 \times 3 \times 25 \times 7 \Rightarrow$ L.C.M. $= 2^3 \times 3^3 \times 5^2 \times 7$

i) Find the L.C.M. of $23 \times 32 \times 5 \times 11$; $24 \times 34 \times 52 \times 7$ and $25 \times 33 \times 53 \times 72 \times 11$ is

a) $2^5 \times 3^4 \times 5^3$ b) $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

c) $2^3 \times 3^2 \times 5 \times 7 \times 11$ d) $2^3 \times 3^2 \times 5$

Ans: L.C.M. $= 2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

2. L.C.M. by Short Cut method:

$2|72-108-2100$
$2|36-54-1050$
$2|18-27-525$
$3|9-27-525$
$3|3-9-175$
$5|1-3-175$
$5|1-3-35$
$|1-3-7$

L.C.M. $= 2\times2\times2\times3\times3\times5\times5\times3\times7$ L.C.M.$= 2^3\times3^3\times5^2\times7 = 37800$

Examples:

i. Find the L.C.M. of 26, 56, 104 and 182.

a) 546 b) 1274 c) 728 d) 784

Ans:

$2|26-56-104-182$
$2|13-28-52-91$
$2|13-14-26-91$
$13|13-7-13-91$
$7|1-7-1-7$
$|1-1-1-1$

L.C.M $= 2\times2\times2\times7\times13 = 56\times13 = 728$

ii. Find the L.C.M. of 148 and 185.

a) 680 b) 740 c) 3700 d) 2960

Ans:

$2|148-185$
$2|74-185$
$5|37-185$
$37|37-37$
$|1-1$

L.C.M $= 2\times2\times5\times3\times37 = 20\times37 = 740.$

iii. Find the least number exactly divisible by 12, 15, 20, and 27.

a) 500 b) 540 c) 550 d) 600

Ans:

$2|\underline{12-15-20-27}$
$2|\underline{6-15-10-27}$
$3|\underline{3-15-5-27}$
$3|\underline{1-5-5-9}$
$5|\underline{1-5-5-3}$
$\underline{1-1-1-3}$

L.C.M = $2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$

Different Therems on L.C.M.

Type-1:

Find the least number which is exactly divisible by x, y and z.

$$\left.\begin{matrix}\text{Required}\\ \text{Number}\end{matrix}\right\} = \text{L.C.M.of}\,(x, y \,\&\, z)$$

Type-2:

Find the least number which when divided by x, y and z leaves the remainder ' r ' in each case.

$$\left.\begin{matrix}\text{Required}\\ \text{Number}\end{matrix}\right\} = \text{L.C.M.of}\,(x, y \,\&\, z) + r$$

Type-3:

Find the least number which when divided by x, y and z leave the remainders p, q and r

$$\left.\begin{matrix}\text{Required}\\ \text{Number}\end{matrix}\right\} = \text{L.C.M.of}\,(x, y \,\&\, z) - K$$

$(x - p) = (y - q) = (z - r) = k$

Examples:

i. Find the least number which when divided by 6, 7, 8, 9 and 12 leaves the semi remainder 1 in each case.

a) 500 b) 505 c) 506 d) 507

Ans: Required Number = (LCM of 6, 7, 8, 9, 12) + 1

$2|\underline{6-7-8-9-12}$

$2|\underline{3-7-4-9-6}$

$2|\underline{3-7-2-9-3}$

$3|\underline{3-7-1-9-3}$

$\underline{|1-7-1-3}-1$

L.C.M. $= 2^3 \times 3^2 \times 7 = 504$

Required Number = (504 + 1) = 505

ii. Find the least number which when divided by 48, 60, 72, 108 and 120 leaves the remainders 38, 50, 62, 98 and 110 respectively.

a) 1950 b) 2050 c) 2120 d) 2150

Ans: Difference between the numbers and the remainder is the same.

(48 – 38) = 10; (60 – 50) = 10; (72 – 62)= 10...

$\left.\begin{array}{|l|}\hline \text{Required} \\ \text{Number} \\ \hline \end{array}\right\} = (\text{L.C.M. of } 48, 60, 72, 108 \text{ and } 120) = 10$

= 2160 – 10 = 2150

iii. What is the least number which when divided by 8, 12, 18 and 24 leaves the remainders 4, 8, 14 and 20 respectively?

a) 78 b) 68 c) 58 d) None

Ans: Difference between the numbers and the remainder is the same.

8 – 4 = 4; 12 – 8 = 4; 18 – 14 = 4

$\left.\begin{array}{|l|}\hline \text{Required} \\ \text{Number} \\ \hline \end{array}\right\} = (\text{L.C.M. of } 8, 12, 18 \text{ and } 24) - 4$

= 72 – 4 = 68.

iv. Five bells begins to toll together and toll at interval of 36, 45, 72, 81, and 108 seconds. After what interval of time they will keep on tolling together?

a) 3240 seconds b) 3080 seconds

c) 3140 seconds d) 3200 seconds

The interval time = L.C.M. of 36, 45, 72, 81, and 108 = 3240 seconds

v. How often will five bells toll together in one hour, if they start together and toll at interval of interval of 5, 6, 8, 12, 20 seconds respectively?

a) 29 b) 30 c) 31 d) 120

Ans: The time after which the bells ring together } = L.C.M. of 5, 6, 8, 12 and 20

= 120 seconds.

The number of times they will toll together in one hour = $\frac{360\cancel{0}}{12\cancel{0}} = 30$

One is added because the bells begin tolling together = (30 + 1) = 31 times.

vi. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 and 12 seconds respectively. In 30 minutes How many times do they toll together?

a) 4 b) 10 c) 15 d) 16

Ans: L.C.M. of 2, 4, 6, 8, 10, 12 is 120 seconds.

The bell will toll together after every 120 seconds = i.e. 2 min

In 30 minutes they will toll together

$= \left(\frac{30}{2} + 1\right) = 16 \text{times}$

vii. An electronic device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 10 a.m. The time when they will next make a beep together at the earliest is?

a) 10.30 a.m b) 10.31 a.m. c) 10.59 a.m. d) 11 a.m.

Ans: L.C.M. of 60 & 62 seconds is 1860 seconds = 31 minutes

∴ They will beep together at 10.31 a.m.

viii. The traffic lights at three different road crossings change after every 48, 72 and 108 seconds respectively. If they all change simultaneously at 8.20.00 hours when they will agian change simultaneously?

a) 8: 27: 12 hrs b) 3: 27: 24 hrs

c) 8 : 27: 36 hrs d) 8 : 27 : 48 hrs

Ans: Interval of change = L.C.M. of (48,72 &108) seconds = 432 seconds

= 7 minutes 12 seconds

∴ Next simultaneous change will take place at

= 8 : 20 : 00

7 : 20

8: 27 : 20 hrs

ix. Find the smallest number which when increased by 11 is divided by 18, 24, 60 and 150.

a) 1700 b) 1780 c) 1789 d) 1790

Ans: L.C.M. of 18, 24, 60 and 150 is

$2|18-24-60-150$

$2|9-12-30-75$

$2|9-6-15-75$

$3|9-3-15-75$

$5|3-1-5-25$

$|3-1-1-5$

L.C.M = 18, 24, 60 and 150 is $2^3 \times 3^2 \times 5^2 = 1800$

Required Number is 1800 – 11 = 1789

x. The least number of five digits when divided by 6, 8, 10, 12 leaves no remainder is?

a) 10080 b) 10070 c) 10065 d) None

Ans:

$$\begin{array}{l|l} 2 & 6-8-10-12 \\ 2 & 3-4-5-6 \\ 2 & 3-2-5-3 \\ 3 & 3-1-5-3 \\ & 1-1-5-1 \end{array}$$

L.C.M. of 6, 8, 10, 12 is $2^3 \times 3 \times 5 = 120$

Least number of five digits = 10000; now divide 10000 by 120

$$\begin{array}{r} 120\overline{)10000}(83 \\ 960 \\ \hline 400 \\ 360 \\ \hline 40 \end{array}$$

$\therefore$ Least number of five digits = 1000 + (120 – 40) = 10080

xi. Find the greatest number of four digits which is exactly divisible by 6, 8, 10 and 12.

a) 9990 b) 9960 c) 9950 c) 9860

Ans: L.C.M. 6, 8, 10 and 12 is 120

Greatest number of four digits = 9999

$$\begin{array}{r} 120\overline{)9999}(83 \\ 960 \\ \hline 399 \\ 360 \\ \hline 39 \end{array}$$

Required number = 9999 – 39 = 9960