RAY OPTICS AND OPTICAL INSTRUMENTS

Important Points:

1. Reflection:

When a light ray traveling from one medium to the other comes back to the same medium at the interface, then it is called Reflection of Light.

2. Laws of Reflection:

i) The incident ray, the reflected ray and the normal drawn at the point of incidence lie in the same plane.

ii) The angle of incidence is equal to angle of reflection.

3. Spherical Mirrors:

A mirror which is a part of a sphere is called Spherical Mirror.

4. Convex Mirror:

The image formed by a convex mirror is always virtual, erect and diminished for any position of the object.

5. Refraction:

When a ray of light passes obliquely from one medium into another, it bends at the interface. It is known as Refraction.

6. Laws of Refraction:

i) The incident ray, the refracted ray and the normal at the point of incidence lie in the same plane.

ii) The ratio of sine of angle of incidence to the sine of angle of refraction is a constant for a given pair of media.

$$ {}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \text{constant} $$

This law is called Snell's law.

7. Absolute Refractive Index (μ):

It is the ratio of velocity of light in vacuum (c_0) to the velocity of light in the medium.

i.e. = $\mu = \frac{C_0}{C_m}$

8. Total Internal Reflection:

a). The light ray must travel from denser to rarer medium.

b). The angle of incidence in the denser medium must be greater than the critical angle.

c). If C is the Critical angle and μ is the refractive index, then $\mu = \frac{1}{\sin C}$

9. Optical Fibre:

i. Principle is total internal reflection.

ii. No loss of light energy.

iii. Optical fibres are used in laparoscopy, endoscopy, and sensors in industry etc

10. Prism:

a) Angle of the prism A = $r_1 + r_2$ and Angle of deviation (d). =I + e – A

I – angle of incidence,

e – angle of emergence,

A – angle of prism,

r_1 – angle of refraction at first retracing face,

r_2 – angle of refraction at second refracting face.

b) $\mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

11. Lenses:

Lens Maker's Formula: For a convex lens in a medium, the focal length of the lens is $\frac{1}{f_{med}} = \left(\frac{\mu_l}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

R_1, R_2 – Radii of curvatures of the curved surfaces.

In air, $\frac{1}{f_{air}} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

This is generally used for paraxial rays.

v. This is not present in mirrors because the light rays do not undergo any path difference.

12. Simple Microscope:

Magnifying power of simple microscope is

i) $m = \left(1 + \frac{D}{f}\right)$ When the image is at near point

ii) $m = \frac{D}{f}$ When the image is at infinity

13. Compound Microscope:

i) Total magnification $m = m_0 \times m_e$ (or) $m = \frac{v_0}{u}\left(1 + \frac{D}{f_e}\right)$ (near point)

ii) When the final image is at infinity $m = \frac{-LD}{f_0 f_e}$

14. Dispersive Power:

Dispersive power of the material of the prism is the ratio of angular dispersion of two extreme colours to their mean deviation.

$$\omega = \frac{\text{Angular dispersion between red and violet colours}}{\text{mean deviation of red } and \text{ violet colours}}$$

Dispersive power $\omega = \dfrac{\delta_v - \delta_R}{\left(\dfrac{\delta_v - \delta_R}{2}\right)}$

Very Short Answer Questions

1. Define focal length and radius of curvature of a concave lens.

A. **Focal Length:** The distance of principle focus from optic center is called focal length of a concave lens.

Radius of Curvature: The radius of the sphere of which the curved surface of the lens forms a part is called the radius of curvature of a concave lens. A concave lens has two radii of curvatures.

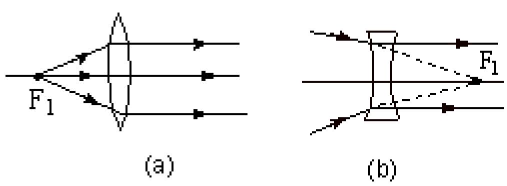
2. What do you understand by the terms 'focus' and 'principal focus' in the context of lenses?

A. Focus:

A point at which rays of light or other radiation converge or from which they appear to diverge, as after refraction in an optical system is the focus of a lens.

Principal Focus:

The point where a beam of light parallel to the principal axis converges or appears to converge is called the principal focus (F).



3. What is Optical Density and how is it different from mass density?

A. Optical Density:

Optical density is a measure of a materials ability to pass light. Optical density is the ratio of speed of light in the two media.

Mass Density:

Mass density is mass per unit volume.

The mass density of an optically denser medium may be lesser than that of an optically rarer medium.

4. What are the laws of reflection through curved mirrors?

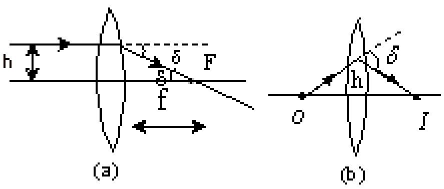
A. i) The angle of reflection is equal to angle of incidence. i.e. $\angle i = \angle r$

ii) The incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in same plane.

5. Define 'power' of a convex lens. What is its unit?

A. Power:

The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre.



$\tan\delta = \frac{h}{f}$ if h = 1, $\tan\delta = \frac{1}{f}$

$\text{Power (P)} = \tan\delta = \frac{1}{f}$ Or $P = \frac{1}{f\,(in\,m)} = \frac{100}{f\,(in\,cm)}D$

Unit: - The S.I. unit of power is dioptre (D).

6. A concave mirror of focal length 10cm is placed at a distance 35cm from a wall. How far from the wall should an object be placed so that its real image is formed on the wall?

A. Distance of image formed from wall = V = – 35 cm

Focal length f = – 10 cm

From mirror formula we have $\frac{1}{u}=\frac{1}{f}-\frac{1}{v}\Rightarrow\frac{1}{u}=-\frac{1}{10}+\frac{1}{35}\Rightarrow u=-14\text{cm}$

The distance of the object from the wall

$x=35-14=21\text{cm}$

7. A concave mirror produces an image of a long vertical pin, placed 40 cm from the mirror, at the position of the object. Find the focal length of the mirror?

A. u = 40 cm; v = 40 cm ; f = ?

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}\Rightarrow\frac{1}{40}+\frac{1}{40}=\frac{1}{f}$ f = 20 cm.

8. A small angled prism of 4º deviates a ray through 2.48º. Find the refractive index of the prism?

A. Angle of prism A = 4º

Angle of deviation δ = 2.48º

Refractive index μ = ?

We have $\delta=(\mu-1)A\Rightarrow 2.48=(\mu-1)4$

$0.62=(\mu-1)\Rightarrow\mu=1.62$

9. What 'dispersion'? Which colour gets relatively more dispersed?

A. Dispersion:

The splitting up of a beam of white light in to its constituent colours (VIBGYOR) is called Dispersion.

The colour which gets relatively more dispersed is violet.

10. The focal length of concave lens is 30cm. Where an object should be placed so that its image is $\frac{1}{10}$ of its size?

A. Given focal length of concave lens = 30 cm

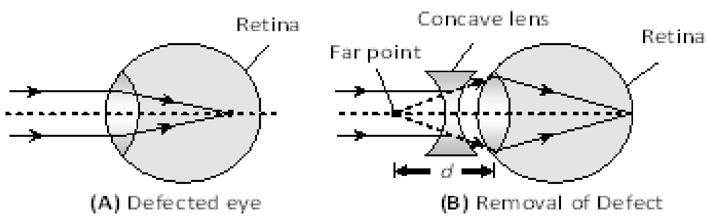
Magnifying power $m = \frac{1}{10}$

We know $m = \frac{-f}{u-f} \Rightarrow \frac{1}{10} = \frac{-30}{u-30}$

$u - 30 = -300 \Rightarrow u = -270\text{cm}$

11. What is myopia? How can it be corrected?

A. Myopia (Short Sightedness):



i) A short-sighted eye can see only nearer objects. Distant objects are not seen clearly.

ii) In this defect image is formed before the retina and Far point comes closer.

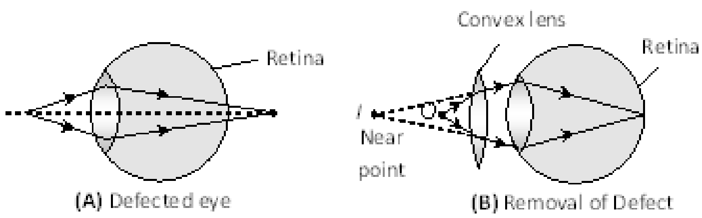
iii) In this defect focal length or radii of curvature of lens reduced or power of lens increases or distance between eye lens and retina increases.

Correction:

This defect can be removed by using a concave lens of suitable focal length.

12. What is Hypermetropia? How can it be corrected?

A. Hypermetropia (long sightness):



i) A long-sighted eye can see distant objects clearly but nearer object are not clearly visible.

ii) Image formed behind the retina and near point moves away.

(iii) In this defect focal length or radii of curvature of lens increases or power of lens decreases or distance between eye lens and retina decreases.

Correction:

This defect can be removed by using a convex lens.

Short Answer Questions

1. A light ray passes through a prism of angle A in a position of minimum deviation. Obtain an expression for

(a) The angle of incidence in terms of the angle of the prism and the angle of minimum deviation

(b) The angle of refraction in terms of the refractive index of the prism.

A. (a) Let ABC is the principal cross section of a prism. BC is prism base and $\angle A$ is refracting angle or angle of prism.

Let PQ is the incident ray falling on the refracting side AB.

Draw normal T at Q. $\angle PQN_1$ is angle of incidence i_1, and r_1 is the angle of refraction at face AB. QR is the path of light ray inside the prism and RS is the emergent ray.

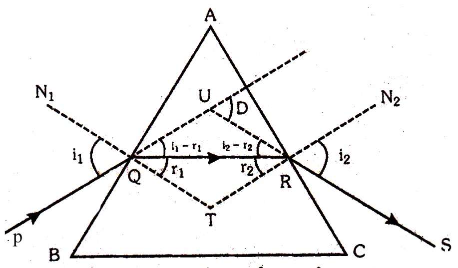
Draw normal N_2T at R.

$\angle N_2RS$ is angle of emergence i_2 and r_2 be the angle of incidence at face AC.

Both normal's will intersect at T.

Extend the lines PQ and RS they will intersect at 'U'.

$\angle D$ is angle of deviation.



Refraction through a prism

Step 1

Consider the quadrilateral AQTR

$\angle A + \angle T = 180^0$ .... (1)

Step 2

Consider the triangle QTR

$r_1 + r_2 + \angle T = 180^0$ .... … (2)

From equation 1 & 2 $r_1 + r_2 = A$ .... (3)

Step 3

Consider the triangle QUR. Angle of deviation D is the external angle of Δle QUR

$\angle D = i_1 - r_1 + i_2 - r_2$

$\angle D = i_1 + i_2 - (r_1 + r_2) \Rightarrow A + D = i_1 + i_2$ (From Equation (3))

$D = i_1 + i_2 - A$ .... (4)

Step 4

When a graph is plotted with angle of incidence i on X- axis and angle of deviation D on Y - axis it is parabola.

As angle of incidence i increases angle of deviation D first decreases and reaches a minimum value, δ then increases.

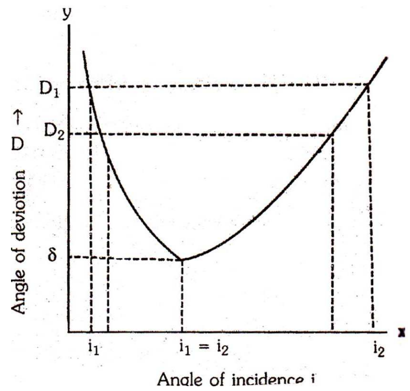
At minimum deviation $D = \delta$, angle of incidence $i_1 = i_2 = i$

And angle of refraction $r_1 = r_2 = r$

$\therefore$ From equation (3) $2r = A \Rightarrow r = \frac{A}{2}$ .... (5)

From eq. (4) $2i = A + \delta$

$\therefore i = \frac{A+\delta}{2}$ .... (6)



b) From Snell's law $\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin r}{\mu}$

$$r = \sin^{-1}\left\{\frac{\sin i}{\mu}\right\}$$

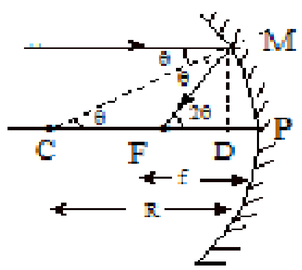
This expression for angle of refraction

2. Define focal length of a concave mirror. Prove that the radius of curvature of concave mirror is double its focal length?

A. Focal Length:

The distance between the pole (P) of the mirror and the principal focus (F) is called **Focal Length** (f).

Relation between f and R:



A light ray incident parallel to the principal axis of a concave mirror gets reflected through the principal focus 'F'. If C is the centre of curvature, CM is the normal to the mirror at M. $\underline{|MCP} = \theta$ and $\underline{|MFP} = 2\theta$

Now $\tan\theta = \frac{MD}{CD}$ and $\tan 2\theta = \frac{MD}{FD}$,

If θ is small, $\tan\theta \approx \theta$ and $\tan 2\theta \approx 2\theta$,

$$\therefore \tan 2\theta \approx 2\tan\theta \text{ (Or) } \frac{MD}{FD} = 2.\frac{MD}{CD}$$

$$\therefore FD = \frac{CD}{2},$$

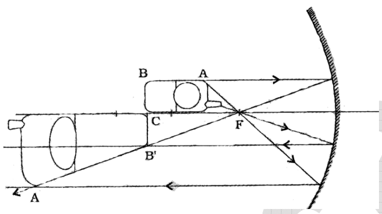
If θ is small, the point 'D' is very close to the point 'P'. Therefore FD = f and CD = R.

$$\therefore f = \frac{R}{2} \text{ or } R = 2f$$

Hence the radius of curvature of a concave mirror is double its focal length.

3. A mobile phone lies along the principle axis of a concave mirror longitudinally. Explain why the magnification is not uniform?

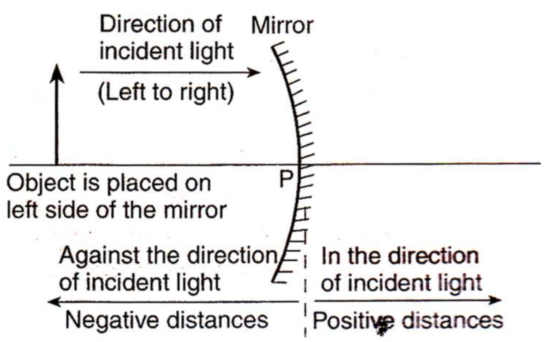
A. The ray diagram for the formation of the image of the phone is shown in figure.



The image of the part which is on the plane perpendicular to principal axis will be on the same plane. The part which is at C will be imaged at C and will be of same size i.e. $B'C = BC$. The other end A of the mobile phone is highly magnified . As distance of image is non linear function of distance of object. The magnification is not uniform.

4. Explain the Cartesian sign convention for mirrors?

A. The New Cartesian Sign Convention for mirrors is shown in figure.



Sign convention for mirrors.

According to the New Cartesian Sign Convention:

The object is always placed on the left side of the mirror so that the direction of incident light is from left to right

i) All the distances are measured from pole of the mirror as origin.

ii) Distances measured in the same direction as that of incident light are taken as positive.

iii) Distances measured against the direction of incident light are taken as negative.

iv) Distances measured upward and perpendicular to the principal axis are taken as positive.

v) Distances measured downward and perpendicular to the principal axis are taken as negative.

vi) The focal length and radius of curvature of concave mirror are taken as negative while those of convex mirror as taken as positive according to sign convention.

5. Define critical angle. Explain total internal reflection using a neat diagram?

A. Critical Angle:

The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90^0 is called the critical angle (i_C)

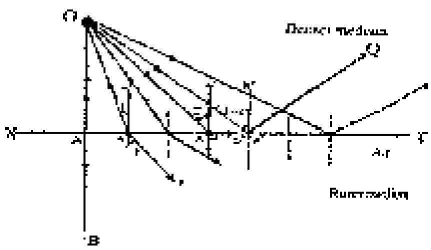
The refractive index of the denser medium (1) with respect to the rarer medium (2) is given by

$$n_{12} = \frac{1}{\sin i_C}$$

Total internal Reflection:

Consider a light ray passing from denser into a rarer medium. The light ray after refraction bends away from the normal. If the angle of incidence is greater than the critical angle, then the light ray is totally reflected into the denser medium. This phenomenon is called total internal reflection.

Conditions for total Internal Reflection:

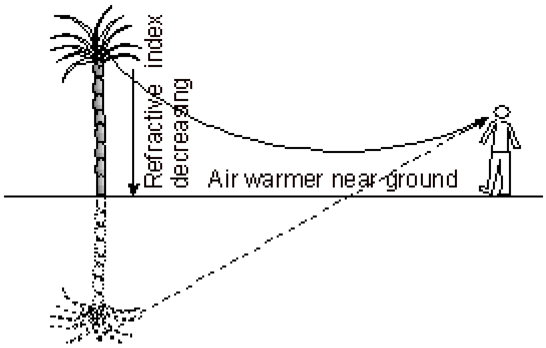


1. The light ray must travel from denser to rarer medium.

2. The angle of incidence in the denser medium must be greater than the critical angle.

6. Explain the formation of mirage?

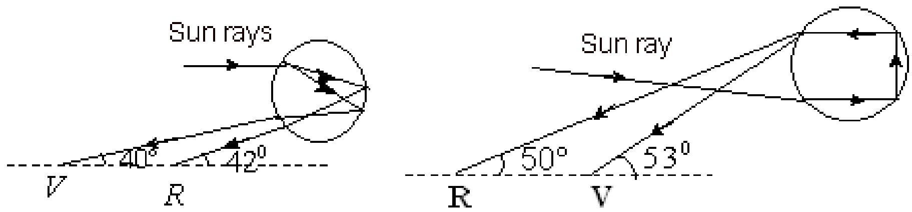
A. Mirage:



In summer, the layers of air near the ground are hotter than the air at higher levels. This hotter air is less dense, and has smaller refractive index than the cold air. In still air, the optical density at different layers of air increases with height. As a result, light from a tall object such as tree, passes the medium whose refractive index decreases towards the ground. Then a ray of light from the object successively bends away from the normal .If the angle of incidence for the air near the ground exceeds the critical angle, total internal reflection takes place.. To a distant observer, the light appears to be coming from somewhere below the ground. Such inverted images of distant tall objects cause an optical illusion to the observer. This phenomenon is called mirage.

7. Explain the formation of a Rainbow?

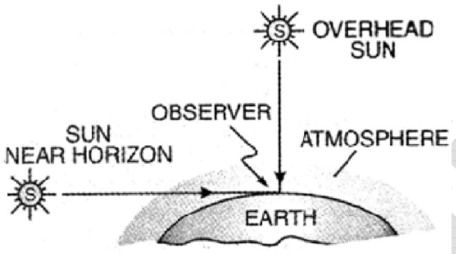
A. Rainbows are due to dispersion of sun light falling on rain drops. Rainbow is observed in a direction facing against the sun. In the common rainbow called primary rainbow, a coloured band with red on the outside and violet on the inner side is formed. It is formed due to two refractions and one reflection of light falling on the raindrops. In the other rainbow called secondary rainbow a colored band, having violet on the outside and red on the inner side is formed. It is formed due to two refractions and two reflections of the sun light falling on the raindrops. The intensity of light is reduced at the second reflection and hence the secondary rainbow is fainter than the primary rainbow. The rainbows are visible only when the altitude of the sun is less than 42^{o}. A complete rainbow can be seen in an aeroplane flying at high altitudes.



Primary Rainbow (a) secondary rainbow (b)

8. Why does the setting Sun appear red?

A. At sunrise or sunset the sun looks almost reddish. The reason is that at the time of sun set or sun rise (when sun is near horizon), the light from the sun has to traverse larger thickness of atmosphere than what it covers, when the sun is overhead as shown in figure.

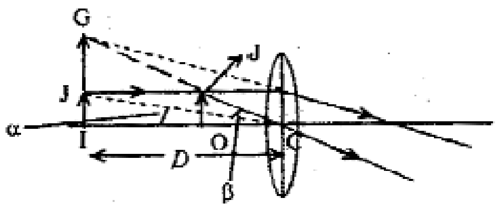


Due to this, more of the blue and shorter wavelength of light is removed by scattering and the least scattered light i.e. red reaches our eye. So the sun looks reddish.

9. With a neat labeled diagram explain the formation of image in a simple microscope?

A. Construction:

A convex lens of short focal length is used as a simple microscope. The lens is arranged in a circular metallic frame provided with a handle. It is known as a magnifying glass.



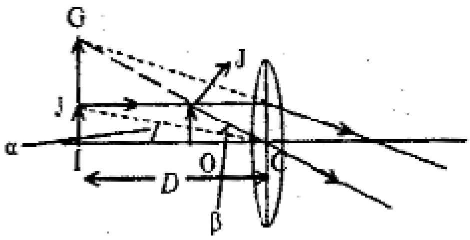
Formation of Image:

Let an object OJ be placed within the principal focus F of the convex lens. The image is virtual, erect and magnified. It is formed on the same side of the object at the least distance of distinct vision 'D'.

If the image is formed at near point, magnifying power $m = 1 + \frac{D}{f}$

If the image is formed at infinity, magnifying power $m = \frac{D}{f}$

10. What is the position of the object for a simple microscope? What is the maximum magnification of a simple microscope for a realistic focal length?



Formation of Image:

An object OJ is placed within the principal focus F of the convex lens. The image is virtual, erect and magnified. It is formed on the same side of the object at the least distance of distinct vision 'D'.

If the image is formed at near point the magnification is maximum.

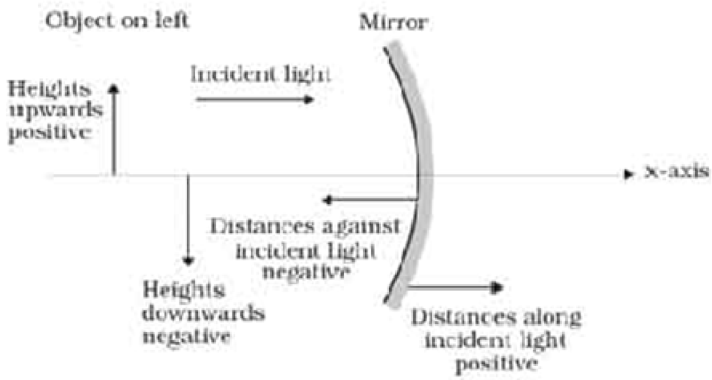
Magnifying power $m = 1 + \frac{D}{f}$

Long Answer Questions

1. **a) What is the Cartesian sign convention? Applying this convention and using a neat diagram, derive an expression for finding the image distance using the mirror equation.**

   b) An object of 5cm height is placed at a distance of 15 cm from a concave mirror of radius of curvature 20cm. Find the size of the image?

A. a) **Cartesian Sign Convention:**



i). All the distances are measured from pole of the mirror as origin.

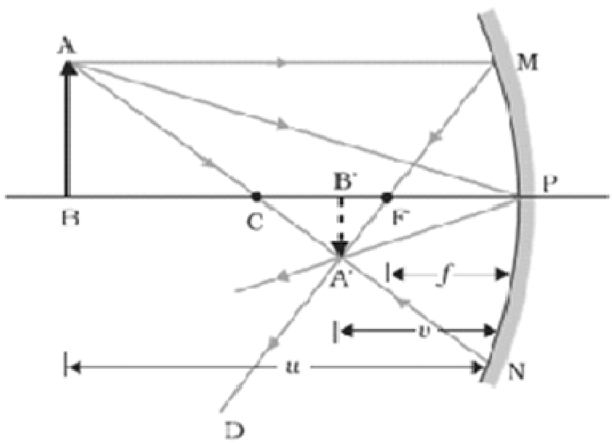
ii). Distances measured in the same direction as that of incident light are taken as positive.

iii). Distances measured against the direction of incident light are taken as negative.

iv). Distances measured upward and perpendicular to the principal axis are taken as positive.

v). Distances measured downward and perpendicular to the principal axis are taken as negative.

Expression for the Mirror Equation:



Above figure shows the image A'B'(in this case, real) of an object A B formed by a concave mirror.

We now derive the mirror equation or the relation between the object distance (u), image distance (v) and the focal length (f).

From Fig, the two right-angled triangles A'B'F and MPF are similar. (For paraxial rays, MP can be considered to be a straight line perpendicular to CP.)

Therefore,

$$\frac{B'A'}{PM}=\frac{B'F}{FP}$$

Or $\dfrac{B'A'}{BA}=\dfrac{B'F}{FP}$ .... (1) $(\because PM = AB)$

Since $\angle APB=\angle A'PB'$, the right angled triangles $A'B'P$ and ABP are also similar. Therefore,

$$\frac{B'A'}{BA}=\frac{B'P}{BP} \qquad \text{............ (2)}$$

Comparing Esq.(1) and (2), we get

$$\frac{B'F}{FP}=\frac{B'P-FP}{FP}=\frac{B'P}{BP} \qquad \text{......... (3)}$$

Applying the sign convention

i.e. $B'P=-v, FP=-f, BP=-u$

Using equation (3) we get

$$\frac{-v+f}{-f}=\frac{-v}{-u}$$

Or $\dfrac{v-f}{f}=\dfrac{v}{u}$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

This relation is known as the mirror equation.

Expression for Image Distance:

From mirror equation

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u} \Rightarrow \frac{1}{v}=\frac{u-f}{uf}$$

Image distance $v=\frac{uf}{u-f}$

b) Height of the object h = 5 cm

Distance of the object u = – 15 cm

Radius of curvature R = – 20 cm

Focal length $f=\frac{R}{2}=-\frac{20cm}{2}=-10cm$

Distance of the object $v=\frac{uf}{u-f}=\frac{-15\times(-10)}{-15+10}=\frac{15}{-5}=-30cm$

$m = \frac{\textbf{size of the image}}{\textbf{size of the object}}=\frac{\mathbf{I}}{\mathbf{O}}=-\frac{\mathbf{v}}{\mathbf{u}}$

Now size of the image $I=-\frac{v}{u}\times 0=-\left(\frac{-30}{-15}\times 5\right)$

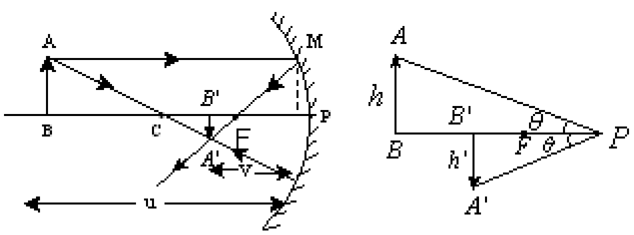
I = – 10 cm

2. **(a) Using a neat labeled diagram derive the mirror equation. Define linear magnification.**

(b) An object is placed at 5cm from a convex lens of focal length 15cm. What is the position and nature of the image?

A. **(a) Mirror Equation:**

Figure shows the ray diagram for the formation of a real image A'B' of an object AB formed by a concave mirror.



$\Delta^{les}\, ABC \text{ and } A'B'C$ are similar and hence

$$\frac{AB}{A'B'}=\frac{CB}{CB'} \qquad \text{...... (1)}$$

From the Δ^{les} ABP and A'B'P are similar

$$\therefore \frac{AB}{A'B'} = \frac{PB}{PB'} \quad ......(2)$$

From equations (1) & (2) we get $\frac{CB}{CB'} = \frac{PB}{PB'}$ (or) $\frac{PB - PC}{PC - PB'} = \frac{PB}{PB'}$

$\therefore \frac{u - R}{R - v} = \frac{u}{v}$ (Or) $uR + vR = 2uv$

Dividing throughout by uvR , we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

Using the sign convention u = –ve, R = –ve and v = –ve into the above equation,

We get $\frac{1}{-u} + \frac{1}{-v} = \frac{2}{-R}$

(Or) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \left(\because f = \frac{R}{2} \right)$

This is known as mirror equation.

Linear Magnification (m):

The ratio of the height of image (h') to the height of the object (h) or the ratio of image distance to the object distance is called linear magnification.

$$m = \frac{h'}{h} = -\frac{v}{u}$$

Here negative magnification implies that image is inverted with respect to object, while positive magnification means that image is erect with respect to object.

(b) u = 5 cm ; f = 15 cm ; v = ?

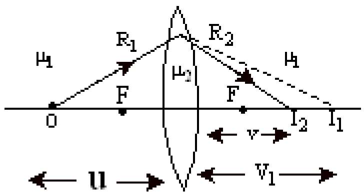
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{5} + \frac{1}{v} = \frac{1}{15}$$

v = - 7.5 cm, Virtual

3. (a) Derive an expression for a thin double convex lens. Can you apply the same to a double concave lens too?

(b) An object is placed at a distance of 20cm from a thin double convex lens of focal length 15cm. Find the position and magnification of the image?

A. (a) Double Convex Lens:



Consider an object O placed at a distance u from a convex lens as shown in figure. Let I_1 be its first image due to refraction through first surface. So from the formula for refraction at curved surface.

For first surface $\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$ ...... (1)

The image I_1 acts as object to second surface, and form final image I_2.Hence For second surface

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \quad ..... (2)$$

From equations (1) and (2) , $\mu_1\left[\frac{1}{v} - \frac{1}{u}\right] = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(Or) $\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{v} - \frac{1}{u} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

With $\mu_r = \frac{\mu_2}{\mu_1} (or) \frac{\mu_L}{\mu_M}$

Now if object is at infinity, image will be formed at the focus i.e. for $u = -\infty$, v = f, so that above equation becomes $\frac{1}{f} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

This is known as Lens-maker's formula and for a lens $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ which is known as the 'lens-formula'. This formula is valid for both convex as well as concave lens.

(b) u = - 20 cm ; f = 15 cm. ; v = ?

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{-20}=\frac{1}{15}$$

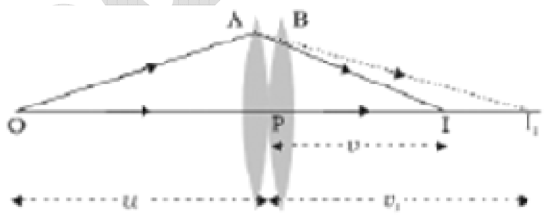
v = 60 cm.

Magnification $m=\frac{v}{u}=\frac{60}{20}=3$

4. Obtain an expression for the combined focal length for two thin convex lenses kept in contact and hence obtain an expression for the combined power of the combination of the lenses?

A. Consider that two thin lenses A and B of focal lengths f_1 and f_2 are placed in contact with each other. Suppose that a point object O lies on the principal axis of the two lenses. Figure shows the formation of the final image I of the object O in two steps.

In the first step, the lens A produces I_1 as the real image of the object O. If $PO = u$ and $PI_1 = v_1$ then for lens A, we have



$$-\frac{1}{u}+\frac{1}{v_1}=\frac{1}{f_1} \qquad \text{........... (1)}$$

In the second step, the lens B produces the final image I of the image I_1(produced by lens A , which acts as virtual object for it.)

$$-\frac{1}{v_1}+\frac{1}{v}=\frac{1}{f_2} \qquad \text{............. (2)}$$

Adding the equation (1) and (2), we have

$$-\frac{1}{u}+\frac{1}{v_1}-\frac{1}{v_1}+\frac{1}{v}=\frac{1}{f_1}+\frac{1}{f_2}$$

Or $$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f_1}+\frac{1}{f_2} \qquad \text{............ (3)}$$

The equivalent focal length of two lens system if it f, then we have

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad \text{........... (4)}$$

From the equations (3) and (4), we have

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \qquad \text{............. (5)}$$

Here, f is called focal length of the equivalent lens.

Thus, reciprocal of the focal length of an equivalent lens is equal to sum of the reciprocals of the focal length of the two lenses placed in contact.

If the focal lengths f_1 and f_2 are in metre, then the equation (5) becomes

$$P = P_1 + P_2$$

Here, P_1 and P_2 are powers of the two lenses and P is the power of the equivalent lens.

5. (a) Define Snell's Law. Using a neat labeled diagram derive an expression for the refractive index of the material of an equilateral prism?

(b) A ray of light, after passing through a medium, meets the surface separating the medium from air at an angle of 45^0 and is just not refracted. What is the refractive index of the medium?

A. (a) Snell's Law:- The ratio of sine of angle of incidence in air or vacuum and the sine of angle of refraction in a medium is called absolute refractive index of the medium with respect to air or vacuum.

$n_{21}=\frac{\sin i}{\sin r}$ Where n_{21} is the refractive index of the medium (2) with respect to the first medium (1).

Refractive index of the prism:

ABC is the cross section of a glass prism and A is the angle of the prism.

PQ is the incident light ray. QR is the refracted light ray and RS is the emergent light ray.

Let i and e are the angles of incidence and emergence. *Let* r_1 and r_2 are the angles of refraction.

Angle of the prism:- In the quadrilateral AQNR, $\angle A + \angle N = 180°$

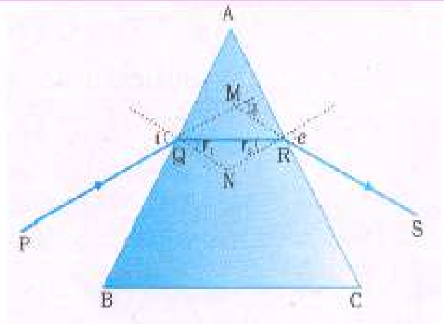
In the ΔQNR, $r_1 + r_2 + \angle A = 180°$

$\therefore r_1 + r_2 + \angle N = \angle A + \angle N$

$r_1 + r_2 = \angle A$ ....(1)

Angle of Deviation:

In the ΔMQR, $\angle M$ QR = $i - r_1$ and $\angle M$ RQ = $e - r_2$



A ray of light passing through a triangular glass prism.

Also, Angle of deviation $\angle\delta = \angle M$ QR + $\angle M$ RQ = $i - r_1 + e - r_2$

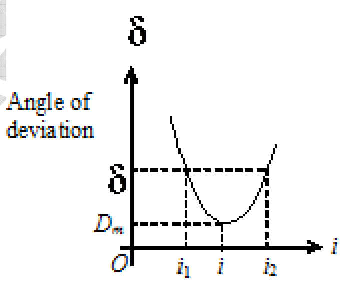
$\therefore \angle\delta = (i + e) - (r_1 + r_2)$

From equation (1)

$i + e = \text{A} + \delta$ ....(2)

i - δ Curve:

A graph drawn between the angle of incidence (i) on X-axis and the angle of deviation (δ) on Y-axis is a parabola as shown.



As the angle of incidence increases the angle of deviation first decreases, becomes minimum and again increases.

Angle of Minimum Deviation: - The angle of incidence for which the deviation produced by the prism is minimum is called angle of Minimum Deviation.

If $\delta = D_m$ then $i = e = i$ and $r_1 = r_2 = r$

From equations (1) and (2), $i = \frac{A + D_m}{2}$ and $r = \frac{A}{2}$

Using Snell's law, $n_{21} = \frac{\sin i}{\sin r}$

$$\therefore n_{21} = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Where n_{21} is the refractive index of the material of the prism with respect to air.

(b) $i_C = 45^0 ; n_{12} = ?$

$$n_{12} = \frac{1}{\sin i_C} = \frac{1}{\sin 45} = 1.414$$

6. Draw a neat labeled diagram of a compound microscope, explain its working. Derive an expression for its magnification?

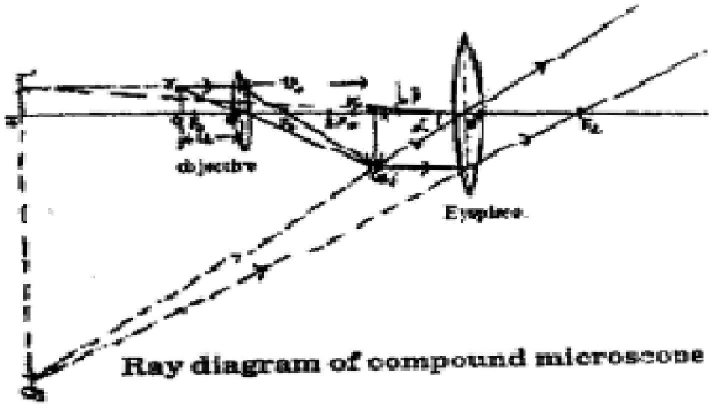
A. Construction:

Compound microscope consists of two coaxially arranged convex lenses of different focal lengths. The distance between the lenses can be adjusted by rack - pinion arrangement. The lens of less focal length nearer to the object is called objective. The lens of large focal length nearer to the eye is called eye piece.

Working:

The object OJ is placed in front of the objective in between F_0 and $2F_0$ of the objective. The image I_1G_1 is formed on the other side of the objective beyond $2F_0$. It is a real, magnified and inverted image. This image acts like an object for the eye piece. This image is made to form below the focus F_e of the eye piece.

Hence the final virtual, magnified and inverted image is formed at the near point.



Magnifying Power:

It is defined as the ratio between the angle subtended by the image formed at near point at the eyepiece and the angle subtended by the object at the eyepiece when the object is imagined at near point.

$$m = \frac{\alpha}{\beta} \simeq \frac{\mathrm{Tan}\,\alpha}{\mathrm{Tan}\,\beta}$$

$$\mathrm{Tan}\,\alpha = \frac{IJ}{IO''} \quad and \quad \mathrm{Tan}\,\beta = \frac{IJ'}{IO''}$$

$$\therefore m = \frac{IG}{IJ'} = \frac{IG}{OJ}$$

Also, $m = m_0 m_e$

When the image formed at near point, $m = \frac{V_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

If the image is at infinity then, $m = \frac{V_0}{u_0} \times \frac{D}{f_e}$

If the object OJ is adjusted such that the image formed by it is very close to the eyepiece, then V_0 is taken as the distance between the lenses or the length of the microscope L

Also, $m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \cong \frac{LD}{f_0 f_e}$ Where L is the length of the microscope.

PROBLEMS

1. A light wave of frequency 4×10^{14}Hz and a wavelength 5×10^{-7}m passes through a medium. Estimate the refractive index of the medium?

Sol: Given

$$v = 4\times10^{14}\ Hz$$

$$\lambda = 5\times10^{-7}\,m$$

$$v_m = v\lambda = 4\times10^{14}\times5\times10^{-7}$$

$$= 2\times10^{8}\,ms^{-1}$$

$$\frac{\mu_m}{\mu_a} = \frac{v_a}{v_m} = \frac{3\times10^8}{2\times10^8} = 1.5$$

$$\frac{\mu_m}{1} = 1.5 \quad \text{(Or)}\,\mu_m = 1.5$$

2. A ray of light is incident at an angle of 60^0 on the face of a prism of angle 30^0. The emergent ray makes an angle of 30^0 with the incident ray. Calculate the refractive index of the material of the prism?

Sol: $i = 60^0,\ A = 30^0,\ \delta = 30^0$

From

$$\delta = i + e - A$$

$$\Rightarrow 30 = 60 + e - 30 \Rightarrow e = 0$$

$$\Rightarrow r_2 = 0$$

From $r_1 + r_2 = A \Rightarrow r_1 = A = 30^0$

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60^0}{\sin 30^0} = \frac{\sqrt{\frac{3}{2}}}{\frac{1}{2}}$$

$$= \sqrt{3} = 1.732$$

3. Two lenses of power –1.75D and +2.25D respectively, are placed in contact. Calculate the focal length of the combination?

Sol: Given

$P_1 = -1.75\ D$

$P_2 = 2.25\ D$

$$\Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = P_1 + P_2 = -1.75 + 2.25$$

(Or) $\frac{1}{F} = 0.5 \Rightarrow F = \frac{1}{0.5} = 2m$ = 200 cm

4. Some rays falling on a converging lens are focused 20cm from the lens. When a diverging lens is placed in contact with the converging lens, the rays are focused 30 cm from the combination. What is the focal length of the diverging lens?

A. focal length of converging lens f_1 = 20 cm

Focal length of diverging lens f_2 = ?

Focal length of the combination f = 30 cm

We have $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{30} = \frac{1}{20} + \frac{1}{f_2} \Rightarrow \frac{1}{f_2} = \frac{1}{30} - \frac{1}{20}$

f_2 = – 60 cm

5. A double convex lens of focal length 15cm is used as a magnifying glass is order to produce an erect image which is 3 times magnified. What is the distance between the object and the lens?

A. focal length ‘f’ = +15 cm

Magnifying power m = +3

$$m = \frac{f}{u+f} \Rightarrow 3 = \frac{15}{u+15} \Rightarrow u + 15 = +5 \Rightarrow u = -10cm$$

6. A compound microscope consists of an object lens of focal length 2cm and an eyepiece of focal length 5cm. When an object is placed at 2.2cm from the object lens, the final image is at 25cm from the eye lens. What is the distance between the lenses? What is the total linear magnification?

Sol: $f_0 = 2cm, \quad f_e = 5cm$

$u_0 = 2.2\ cm, \qquad v_e = D = 25\ cm$

For Eyepiece

From $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

$$\Rightarrow \frac{1}{5} = \frac{1}{(-25)} - \frac{1}{(-u_e)}$$

$$\Rightarrow \frac{1}{u_e} = \frac{1}{5} + \frac{1}{25} = \frac{6}{25} \Rightarrow u_e = \frac{25}{6} cm$$

For Objective:

From

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{2} = \frac{1}{u_o} - \frac{1}{(-2.2)}$$

$$\Rightarrow \frac{1}{v_0} = \frac{1}{2} - \frac{1}{2.2} = \frac{0.2}{4.4}$$

$v_o = 22\ cm$

$$\therefore L = v_o + u_e = 22 + \frac{25}{6} = \frac{157}{6}$$

L = 26.16 cm

From

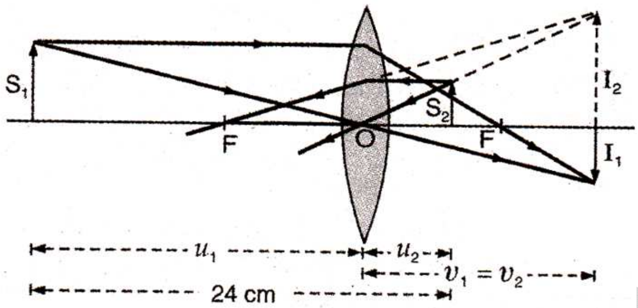
$$m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

$$\Rightarrow m = \frac{22}{2.2}\left(1 + \frac{25}{5}\right) = 10 \times 6 = 60$$

7. The distance between two point sources of light is 24 cm where should you place a converging lens of focal length 9cm, so that the images of both sources are formed at the same point?

A. focal length of converging lens (f) = 9 cm

Distance between two points source = 24 cm



For source S_1,

$$\frac{1}{f}=-\frac{1}{u}+\frac{1}{v_1}$$

$u_1=-x, f=9cm$

$$\frac{1}{v_1}=\frac{1}{f}+\frac{1}{u_1}=\frac{1}{9}+\frac{1}{-x}=\frac{1}{9}-\frac{1}{x} \text{ .... (1)}$$

For point source S_2

$$\frac{1}{f}=\frac{1}{u_2}+\frac{1}{v_2} \Rightarrow u_2=-(24-x), f=9cm$$

Also given $v_2 = -v_1$

$$\therefore \frac{1}{9}=\frac{-1}{-(24-x)}-\frac{1}{v_1} \Rightarrow \frac{1}{v_1}=-\frac{1}{9}+\frac{1}{(24-x)} \text{ .... (2)}$$

From (1) & (2)

$$\frac{1}{9}+\frac{1}{(24-x)}=\frac{1}{9}-\frac{1}{x} \Rightarrow \frac{1}{(24-x)}+\frac{1}{x}=\frac{2}{9} \Rightarrow x^2-24x+108=0$$

$$x=-\frac{(-24)\pm\sqrt{(-24)^2-4\times1\times108}}{2}=\frac{24\pm12}{2}=18cm \text{ or } 6cm$$

8. Find two position of an object, placed in front of a concave mirror of focal length 15cm, so that the image formed is 3 times the size of the object?

Sol: $m = \frac{-v}{u} = 3 \Rightarrow v = 3 \quad \Rightarrow v = -3u$

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\Rightarrow \frac{1}{15} = \frac{1}{4} + \frac{1}{-3u} = \frac{1}{u}\left(1 - \frac{1}{3}\right)$

$\frac{1}{15} = \frac{2}{3u} \Rightarrow \ u = 10\ cm$

9. When using a concave mirror, the magnification is found to be 4 times when the object is 25 cm from the mirror as it with the object at 40 cm from the mirror. The image being real in each case. What is the focal length?

A. **Case** (i) object distance (u) = 25 cm

Magnifying power = 4 m

From formula $\mathrm{m} = \frac{\mathrm{f}}{\mathrm{u}+\mathrm{f}} \Rightarrow 4\mathrm{m} = \frac{\mathrm{f}}{25+\mathrm{f}}$ .... (1)

Case (ii) object distance u = 40 cm

Magnifying power = m

$$\therefore \mathrm{m} = \frac{\mathrm{f}}{40+\mathrm{f}} \qquad \text{............... (2)}$$

From (1) and (2) $\frac{4\mathrm{m}}{\mathrm{m}} = \frac{\frac{\mathrm{f}}{25+\mathrm{f}}}{\frac{\mathrm{f}}{40+\mathrm{f}}} \Rightarrow 4 = \frac{40+\mathrm{f}}{25+\mathrm{f}} \Rightarrow 100 + 4\mathrm{f} = 40 + \mathrm{f} \Rightarrow \mathrm{f} = -20$ cm

10. The focal length of the objective and eyepiece of a compound microscope are 4cm and 6cm respectively. If an object is placed at distance of 6cm from the objective, what is the magnification produced by the microscope?

Sol: $f_o = 4\ cm, f_e = 6\ cm$

$u_o = 6cm,$ D = 15 cm

<u>**For objective**</u>

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\Rightarrow \frac{1}{4} = \frac{1}{v_o} - \frac{1}{(-6)}$$

$$\frac{1}{v_o} = \frac{1}{4} - \frac{1}{6} = \frac{2}{24} \Rightarrow v_o = 12\ cm$$

$$m = \left(\frac{v_o}{u_o}\right)\left(1 + \frac{D}{f_e}\right)$$

$$= \left(\frac{12}{6}\right)\left(1 + \frac{25}{6}\right) = 2\left(\frac{31}{6}\right) = 10.33.$$