## MOVING CHARGES AND MAGNETISM

## Important Points:

## 1. Fleming's Left Hand Rule:

Stretch fore finger, middle finger and thumb of the left hand in mutually perpendicular directions. If the fore finger represents the direction of magnetic field, middle finger represents electric field, and then the thumb represents the direction of force (or) motion of the conductor.
2. Biot-Savart Law:

Intensity of magnetic induction due to current element $d B=\frac{\mu_{0}}{4 \pi} \frac{\mathrm{idl} \sin \theta}{\mathrm{r}^{2}}$
3 Magnetic field at the centre of a circular coil carrying current $B=\frac{\mu_{0}}{2} \frac{\mathrm{ni}}{\mathrm{r}}$
4 Magnetic induction at a point on the axis of a current carrying circular coil $B=\frac{\mu_{0}}{2} \frac{\mathrm{nir}^{2}}{\left(\mathrm{r}^{2}+\mathrm{x}^{2}\right)^{3 / 2}}$
5. Force on a moving charge in a magnetic field $\mathrm{F}=\mathrm{Bqv} \sin \theta$
6. Force on a current carrying conductor placed in a magnetic field $\mathrm{F}=\mathrm{Bil} \sin \theta$
7. Force per unit length between two parallel current carrying conductors $\frac{F_{2}}{l}=\frac{\mu_{0}}{2 \pi} \frac{i_{1} i_{2}}{r}$

8 Magnetic moment of the current carrying coil is $\mathrm{M}=\mathrm{niA}$
9 Torque on a current carrying coil placed in a magnetic field $\tau=\mathrm{Ni} \mathrm{AB} \operatorname{Sin} \theta$
Where $\theta$ is angle made by the normal to the plane of the coil with the field
10. A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.
11. Galvanometer can be converted in to an ammeter by connecting low resistance parallel to it.
12. Galvanometer is converted into voltmeter by connecting high resistance in series.
13. Shunt resistance $S=\frac{G}{\left(\frac{i}{i_{g}}\right)-1}=\frac{G}{n-1}$
14. Resistance connected in series to the voltmeter $\mathrm{R}=\frac{\mathrm{V}}{\mathrm{i}_{\mathrm{g}}}-\mathrm{G}=\mathrm{G}(\mathrm{n}-1)$

## Very Short Answer Questions

## 1. What is the importance of Oersted's experiment?

A. According to Oersted's experiment the magnetic field is associated with electric current flowing in a conductor. He also noted that the alignment of magnetic needle is tangential to a circle drawn by taking conductor as centre and has its plane perpendicular to the conductor. This observation led to the phenomenon of electromagnetic induction.
2. State Ampere's law and Biot-Savart's Law?
A. Ampere's Law:

The line integral of magnetic induction field $\bar{B}$ around any closed path in vaccum (or) air is equal to $\mu_{0}$ times the total current through the area bounded by the closed path.

$$
\oint \vec{B} \cdot \vec{d} l=\mu_{0} i
$$

## Biot - Savarts Law:

The intensity of magnetic induction $\overline{d B}$ at a distance from a current element of length ' $\mathrm{d} l$ ' carrying a current ' i ' is

i) Directly proportional to the length of the element
ii) Directly proportional to the strength of the current
iii) Directly proportional to the sine of the angle between the direction of current and the line joining the element and the point and
iv) Inversely proportional to the square of the distance between the element and the point

$$
d B \alpha \frac{i d l \sin \theta}{r^{2}}
$$

3. Write the expression for the magnetic induction at any point on the axis of a circular current - carrying coil. Hence, obtain an expression for the magnetic induction at the centre of the circular coil?
A. Magnetic induction field at a point on the axis of a circular coil carrying current at a distance is

$$
B=\frac{\mu_{0} n i R^{2}}{2\left(R^{2}+x^{2}\right)^{3 / 2}}
$$

Here ' $R$ ' = the radius of the coil

$$
\text { ' } n \text { ' = the number of turns }
$$

' i ' = the current in the coil
$\mu_{0}=$ permeability of the medium
At the centre of the coil $(\mathrm{x}=0), \quad B=\frac{\mu_{0} n i}{2 R}$.
4. A circular coil of radius ' $r$ ' having $N$ turns carries a current ' $i$ '. What is its magnetic moment?
A. A circular coil having ' $n$ ' turns carries a current ' $i$ ' and radius ' $r$ ', the magnetic moment is given by $\pi r^{2} n i$.
5. What is the force on a conductor of length ' $l$ ' carrying a current ' $i$ ' placed in a magnetic field of induction ' $B$ '? When does it become maximum?
A. The force acting on a conductor of length ' $l$ ' carrying the current ' $i$ ' placed in a magnetic field of induction $\bar{B}$ is given by
$F=B i l \operatorname{Sin} \theta$

Where ' $\theta$ ' is the angle of inclination of the conductor with the direction of the magnetic field.

When the conductor is perpendicular of the magnetic field i.e. if $\theta=90^{\circ}$, the force becomes maximum

$$
\therefore F_{\max }=B i l
$$

6. What is the force on a charged particle of charge " $q$ " moving with a velocity " $v$ " in a uniform magnetic field of induction $B$ ? When is it become maximum?
A. The force experienced by a particle of charge ' $q$ ' in a uniform magnetic field of induction ' $B$ ' moving with a constant velocity ' $v$ ' making an angle ' $\theta$ ' with the direction of the field is given by

$$
F=q v B \sin \theta=q(\vec{v} \times \vec{B})
$$

The force is perpendicular to the plane containing ' $B$ ' and ' $v$ '.
The force will be maximum when $\theta=90^{\circ}$
i.e. $\quad F_{\max i m u m}=q V B$
7. Distinguish between ammeter and voltmeter.
A.

## Ammeter

## Voltmeter

1) A small resistance connected in parallel to 1) A high resistance connected in series to a a galvanometer constitutes ammeter.
2) It measures current in amperes.
3) It measures potential deference in volts.
4) Ammeter is always connected in series in electric circuits.
5) Ammeter has low resistance.
6) Voltmeter is always connected in parallel in electric circuits.
7) Voltmeter has high resistance.

## 8. What is the principle of moving coil galvanometer?

A. When a current carrying coil is placed in a magnetic field, it experiences a torque.
9. What is the smallest value of current that can be measured with a moving coil galvanometer?
A. The smallest current that can be measured with a moving coil galvanometer is $10^{-9} \mathrm{amp}$.
10. How do you convert a moving coil galvanometer into an ammeter?
A. A galvanometer can be converted into an ammeter, by connecting a low resistance called shunt $(S)$ in parallel to the galvanometer.

Shunt resistance $r_{S}=\frac{R_{G}}{n-1}$. Where n is the range of the ammeter.
11. How do you convert a moving coil galvanometer into a voltmeter?
A. A galvanometer can be converted into a voltmeter by connecting a high resistance in series with the galvanometer. Series resistance $r=R_{G}(n-1)$. Where n is the range of the voltmeter.
12. What is the relation between the permittivity of free space $\epsilon_{o}$, permeability of free space $\mu_{o}$ and the speed of light in vacuum?
A. The velocity of light in vacuum $C=\frac{1}{\sqrt{\mu_{0} \epsilon_{0}}}$

Here $\mu_{0}=$ permeability of free space $=4 \pi \times 10^{-7} \mathrm{Hm}^{-1}$

$$
\epsilon_{o}=\text { permittivity of free space }=8.854 \times 10^{-12} \quad \mathrm{C}^{2} / \mathrm{Nm}^{2}
$$

13. A current carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns about the vertical axis?
A. No. A uniform magnetic field cannot be set up in such a manner that the loop turns about the vertical axis. The torque acting on the loop is given by,

$$
\vec{\tau}=\vec{M} \times \vec{B}=I(\vec{A} \times \vec{B})
$$

Since the area vector is along the vertical, the torque on the loop becomes zero.
14. A current carrying circular loop is placed in a uniform external magnetic field. If the loop is free to turn, what is its orientation when it achieves stable equilibrium?
A. In the stable equilibrium state, the current carrying circular loop will orient itself such that $\bar{B}$ is perpendicular to the plane of the loop. In this position the torque on the loop becomes zero.
15. A wire loop of irregular shape carrying current is placed in an external magnetic field. If the wire is flexible, what shape will the loop change to? Why?
A. The loop will take the circular shape with its plane normal to the field in order to minimize the magnetic flux through it. More over for a given perimeter a circle has minimum area.

## Short Answer Questions

## 1. State and explain Biot - Savart law.

## A. Biot-Savart Law:

Biot - Savart Law gives the magnetic field induction at any point around the current carrying conductor of any shape.

## Explanation:

Consider a conductor ' QR ' through which a current ' i ' is passing. The magnetic induction $(\mathrm{dB})$ at any point due to a small element is

1) Directly proportional to the current (i) passing through the conductor.
2) Length of the small element (d $l$ ).
3) Sine of the angle between the element and the line joining small element and the point $(\sin \theta)$ and
4) Inversely proportional to the square of the distance $\left(r^{2}\right)$ between the small element and the point.
$d B \propto \frac{i d l \sin \theta}{r^{2}} \Rightarrow d \theta=\frac{\mu_{0}}{4 \pi} \frac{i d l \sin \theta}{r^{2}}$


Vectorially, $\overrightarrow{d B}=\frac{\mu_{0}}{4 \pi} \frac{i(\overrightarrow{d l} \times \vec{r})}{r^{3}}$

This is known as Biot -Savart law.

## 2. State and explain Ampere's law?

## A. Statement:

The line integral of $\vec{B} . \overrightarrow{d l}$ taken over the entire closed path of induction in a given perpendicular plane is equal to $\mu_{0}$ times the total current enclosed in the closed path.

$$
\oint \vec{B} \cdot \overrightarrow{d l}=\mu_{0} i
$$

## Explanation:

Imagine a long straight current carrying conductor emerging out of and perpendicular to the plane of the paper. The magnetic lines are in the form of concentric circles centered on the wire.

Consider some closed paths around the conductor as shown. Path 1 is circular and path 2 and 3 are of general shape. $d$ l is an elementary path on the path 1 of radius ' $r$ '. Let I be the current.
$\therefore \bar{B} \cdot \overline{d l}=B d l \cos \theta=\frac{\mu_{o} I}{2 \pi r} r d \theta$
Since $\bar{B}$ and $\overline{d l}$ are parallel,
$\bar{B} \cdot \overline{d l}=\frac{\mu_{o} I}{2 \pi} d \theta$

$\therefore \oint \bar{B} \cdot \overline{d l}=\oint \frac{\mu_{o} I}{2 \pi} d \theta=\frac{\mu_{o} I}{2 \pi} \oint d \theta=\mu_{o} I$
$\therefore \oint \bar{B} \cdot \overline{d l}=\mu_{o} I$.
Similarly for other paths,
$\bar{B} \cdot \overline{d l}=\frac{\mu_{\rho} I}{2 \pi} \theta_{A B}$
$\bar{B} \cdot \overline{d l}=\frac{\mu_{o} I}{2 \pi} \theta_{C D}$ and so on

$$
\therefore \oint \cdot \bar{B} \cdot \overline{d l}=\frac{\mu_{o} I}{2 \pi}\left(\theta_{A B}+\theta_{C D}+\ldots . .\right)=\frac{\mu O I}{2 \pi}(2 \pi)
$$

$$
\therefore \oint \bar{B} \cdot \overline{d l}=\mu_{o} I
$$

This is known as Ampere's circuital law.

## 3. Find the magnetic induction due to a long current carrying conductor?

A. Consider a long and straight current carrying conductor. Let I be the current in the conductor. Let P be a point at a distance ' S ' from the conductor. The position vector of P from the element $d l^{1}$ of the conductor is $\bar{r}$.

The field due to $d l^{1}$ is given by
$d B=\frac{\mu_{o} I\left|\overline{d l}{ }^{1} \times \bar{r}\right|}{4 \pi r^{3}}=\frac{\mu_{o} I d l^{1} \sin \alpha}{4 \pi r^{2}}=\frac{\mu_{o} I}{4 \pi r^{2}} d l^{1} \cos \theta$
But $s=r \cos \theta$ and $l^{1}=s \operatorname{Tan} \theta$
$\therefore d l^{1}=s \sec ^{2} \theta d \theta=\frac{s d \theta}{\cos ^{2} \theta}$

$\therefore d B=\frac{\mu_{o}}{4 \pi} \frac{I \cos \theta}{s} d \theta$
Integrating between the limits, $-\pi / 2$ to $\pi / 2, \quad B=\frac{\mu_{o} I}{2 \pi s}$.
4. Derive an expression for the magnetic induction at the centre of a current carrying circular coil using Biot-Savart law?
A. Consider a circular loop with centre ' O ' and radius ' $r$ '. Let ' i ' be the current through the loop .The magnetic field induction at the centre of the loop due to the small element $\mathrm{d} l$ is given by $d B=\frac{\mu_{0}}{4 \pi} \frac{i d l \sin \theta}{r^{2}}$

Here $\theta=90^{\circ} \Rightarrow \sin 90^{\circ}=1$

$$
d B=\frac{\mu_{0}}{4 \pi} \frac{i d l}{r^{2}}
$$

$\therefore$ The magnetic field induction due to the total circular loop is given by

$$
\begin{array}{r}
\int d B=\int \frac{\mu_{0}}{4 \pi} \cdot \frac{i d l}{r^{2}} \Rightarrow B=\frac{\mu_{0} i}{4 \pi r^{2}} \int d l \\
\Rightarrow B=\frac{\mu_{0} i}{4 \pi r^{2}}(2 \pi r) \Rightarrow B=\frac{\mu_{0} i}{2 r}
\end{array}
$$



If the coil has ' n ' turns, then $B=\frac{\mu_{0} n i}{2 r}$
The magnetic induction is perpendicular to the plane of the coil along its axis.
5. Derive an expression for the magnetic induction at a point on the axis of current carrying circular coil using Biot-Savart law?

## A. Derivation:

Let a coil of radius ' $R$ ' having ' $n$ ' turns carries a current ' I ' in the XZ plane. Let P be a point on the axis of coil at a distance ' $x$ ' from the centre $O$ of the coil. Consider a conducting element of $\overline{d l}$ the loop. From Biot - savert law the magnetic induction due to $\overline{d l}$ is given by, $d \bar{B}=\frac{\mu_{0}}{4 \pi} \frac{I|\overline{d l} \times \bar{r}|}{r^{3}}$

Any element of the loop is perpendicular to the position vector from the element to the axial point. Hence $|\overline{d l} \times \bar{r}|=r d l$ and $\mathrm{r}^{2}=\left(x^{2}+\mathrm{R}^{2}\right)$.


As for every current element there is a symmetrically situated opposite element, the component of the field perpendicular to the X -axis cancel each other while along the axis add up.Net induction along X -axis.

$$
d \bar{B}_{x}=d B \cos \theta
$$

But, $\cos \theta=\frac{R}{\left(x^{2}+R^{2}\right)^{1 / 2}}$
$\therefore \mathrm{dB}_{x}=\frac{\mu_{0} \mathrm{Id} l}{4 \pi} \frac{R}{\left(\mathrm{x}^{2}+\mathrm{R}^{2}\right)^{3 / 2}}$

The total magnetic induction due to the coil ( $\mathrm{d} l=2 \pi \mathrm{Rn}$ ),

$$
\mathrm{B}=\frac{\mu_{0} \mathrm{nIR}}{2\left(\mathrm{x}^{2}+\mathrm{R}^{2}\right)^{3 / 2}}
$$

At the centre of the coil i.e. at $x=0, B=\frac{\mu_{0} n I}{2 R}$

## 6. Obtain an expression for the magnetic dipole moment of a current loop?

A. The magnetic induction on the axial line of a circular current carrying loop at a distance ' $x$ ' from its centre is,

$$
B=\frac{\mu_{0} I R^{2}}{2\left(R^{2}+x^{2}\right)^{3 / 2}}
$$

If $x \ggg>$ the intensity of magnetic induction field on axis of coil is

$$
B=\frac{\mu_{0} I R^{2}}{2 x^{3}}, \text { multiplying both numerator } \& \text { denominator by }
$$

$$
B=\frac{\mu_{0} I 2 \pi R^{2}}{4 \pi x^{3}}=\left(\frac{\mu_{0}}{4 \pi}\right) \frac{2 I\left(\pi R^{2}\right)}{x^{3}}=\frac{\mu_{0}}{4 \pi}\left(\frac{2 I A}{x^{3}}\right)\left(\because A=\pi R^{2}\right)
$$

Comparing the above equation with intensity of magnetic induction field on axis of a short bar magnet, $B=\frac{\mu_{0}}{4 \pi}\left(\frac{2 M}{x^{3}}\right)$ the magnetic moment of current carrying coil is $\mathrm{M}=\mathrm{iA}$.

## 7. Derive an expression for the magnetic dipole moment of a revolving electron?

A. Consider an electron revolving in a circular orbit of radius $r$ with a speed ' $v$ ' and frequency $v$. Consider a point P on the circle. The electron crosses the point once in every revolution. In one revolution, the electron travels distance $2 \pi r$. The number of revolutions made by electron in one second is,

$$
v=\left(\frac{v}{2 \pi r}\right)
$$

Electric current $I=e v=\left(\frac{e v}{2 \pi r}\right)$
This produces magnetic field and taking the orbit of the electron as circular loop, $\left[A=\pi r^{2}\right]$, its magnetic dipole moment is,
$\mu_{l}=I A=\frac{e v}{2 \pi r}\left(\pi r^{2}\right)=\left(\frac{e r v}{2}\right)$
Multiplying and dividing with mass of the electron $m_{e}, \quad \mu_{l}=\frac{e}{2 m_{e}}\left(m_{e} v r\right)=\frac{e}{2 m_{e}} l$
Where ' $l$ ' is the angular momentum of the electron
Vectorially $\mu_{l}=-\frac{e}{2 m_{e}} \bar{l}$
The negative sign indicates that the angular momentum of the electron is is opposite to the direction of magnetic moment.

From Bohr's postulate, angular momentum of the electron, $l=\frac{n h}{2 \pi}$, Where n is an integer.

$$
\mu_{l}=\frac{n h e}{4 \pi m_{e}} \quad \text { or }\left(\mu_{l}\right)_{\mathrm{Min}}=\frac{h e}{4 \pi m_{e}}=\frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}}=9.27 \times 10^{-24} \mathrm{Am}^{2}
$$

This value is called Bohr magneton.

## 8. Explain how crossed E and B fields serve as a velocity selector.

A. Consider a charged particle ' q ' moving with velocity ' $\bar{v}$ ' in presence of both electric and magnetic fields. It experiences a force is given by

$\bar{F}=q(\bar{E}+\bar{v} \times \bar{B})=\bar{F}_{E}+\bar{F}_{B}$
The electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of the particle as shown in the figure.

Hence $\bar{E}=E \hat{j} ; \bar{B}=B \hat{k} \quad$ and $\quad \bar{v}=v \hat{i}$
$\bar{F}_{E}=q \bar{E}=q E \hat{j}$
$\bar{F}_{B}=q(\bar{v} \times \bar{B})=q(v \hat{i} \times B \hat{k})=-q v B \hat{j}$
$\therefore \bar{F}=q(E-v B) \hat{j}$
Thus, electric and magnetic forces are in opposite directions as shown in figure.
The values of E and B are adjusted such that magnitudes of the two forces are equal. Then, total force on the charge is zero and the charge will move in the fields un-deflected.

This happens when,

$$
q E=q v B \text { Or } v=\frac{E}{B}
$$

Hence the crossed E and B fields serve as a velocity selector.
9. What are the basic components of a cyclotron? Mention its uses?

## A. Cyclotron:

Cyclotron is a device used to accelerates positively charged particles (like $a$-particles, deuterons etc.) to acquire enough energy to carry out nuclear disintegration etc.

$$
E_{\max }=\left(\frac{q^{2} B^{2}}{2 m}\right) r^{2}
$$

## Principle:

It is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency.

## Description:

It consists of two hollow $D$-shaped metallic chambers $D_{1}$ and $D_{2}$ called Dees. The two Dees are placed horizontally with a small gap separating them. The Dees are connected to the source of high frequency electric field. The Dees are enclosed in a metal box containing a gas at a low pressure of the order of $10^{-3} \mathrm{~mm}$ mercury. The whole apparatus is placed between the two poles of a strong electromagnet $N S$. The magnetic field acts perpendicular to the plane of the Dees.


1) Cyclotron Frequency: Time taken by ion to describe a semicircular path is given by

$$
t=\frac{\pi r}{v}=\frac{\pi m}{q B}
$$

If $T=$ time period of oscillating electric field then, the cyclotron frequency
2) Maximum Energy of Particle: Maximum energy gained by the charged particle $E_{\max }=\left(\frac{q^{2} B^{2}}{2 m}\right) r^{2}$ where $r_{0}=$ maximum radius of the circular path followed by the positive ion.

## Uses:

Cyclotron is used

1) To bombard nuclei with energetic particles and study the resulting nuclear reactions.
2) To implant ions into solids and modify their properties or even synthesise new materials.
3) In hospitals to produce radioactive substances which can be used in diagnosis and treatment.

## Long Answer Questions

1. Deduce an expression for the force on a current carrying conductor placed in a magnetic field. Derive an expression for the force per unit length between two parallel current carrying conductors.

## A. Force on a current carrying conductor:

Consider a conductor carrying a current ' i' in a magnetic field $\bar{B}$. Consider a small element dl of the wire. The free electron drift with a speed $\mathrm{v}_{\mathrm{d}}$ opposite to the direction of the current. The relation for current is given by


$$
\mathrm{i}=\mathrm{j} \mathrm{~A}=\operatorname{nev}_{\mathrm{d}} \mathrm{~A}
$$

Where $A$ is the area of cross-section of the wire and $n$ is the number of free electron is per unit volume. Each electron experiences an average magnetic force

$$
\overrightarrow{\mathrm{f}}=-\mathrm{e} \overrightarrow{\mathrm{v}}_{\mathrm{d}} \times \overrightarrow{\mathrm{B}}
$$

The number of free electrons in the small element considered is nAdl. Thus the magnetic force on the wire of length dl is

$$
\mathrm{d} \overline{\mathrm{~F}}=(\mathrm{nAd} l)\left(-\mathrm{e} \overline{\mathrm{~V}}_{\mathrm{d}} \times \overline{\mathrm{B}}\right) \text { Or } \quad \mathrm{d} \overrightarrow{\mathrm{~F}}=\mathrm{nAev}_{\mathrm{d}} \mathrm{~d} \vec{l} \times \overrightarrow{\mathrm{B}}
$$

$\therefore \mathrm{d} \overrightarrow{\mathrm{F}}=\mathrm{i} . \mathrm{d} \overrightarrow{\mathrm{l}} \times \overrightarrow{\mathrm{B}}$
For a straight wire of length $l$ carrying a current ' i ' in a magnetic field $\bar{B}$ the force is given by

$$
\overrightarrow{\mathrm{F}}=\mathrm{i} \bar{l} \times \overline{\mathrm{B}}=\mathrm{i}(\bar{l} \times \overline{\mathrm{B}}) \text { Or } \quad \overline{\mathrm{F}}=\mathrm{i} l \mathrm{~B} \sin \theta
$$

## Force between two parallel current carrying conductors:

Consider two very long parallel conductors $X$ and $Y$ carrying currents $i_{1}$ and $i_{2}$ in same direction as shown in the figure. Let 'r' be the separation between the conductors in the plane of the paper.

The magnitude of magnetic Induction field at any point P on the conductor Y due to current $\mathrm{i}_{1}$ in conductor X is $B_{X}=\frac{\mu_{0}}{2 \pi} \frac{i_{1}}{r}$.

From right hand grip rule the direction of $\mathrm{B}_{\mathrm{X}}$ is perpendicular to the plane of the paper and is directed inwards


Now imagine the conductor $Y$ carrying current $i_{2}$ in this magnetic induction field $\mathrm{B}_{\mathrm{X}}$.

Force for length $l$ of conductor $Y$ is given by $F_{y}=B_{x} i_{2} 1 \sin \theta=B_{x} i_{2} 1 \quad\left(\because \theta=90^{\circ}\right)$

Now $\mathrm{F}_{\mathrm{y}}=\frac{\mu_{0}}{2 \pi} \frac{\mathrm{i}_{1} \mathrm{i}_{2}}{\mathrm{r}} l$

Force per unit length $=\frac{\mu_{0}}{2 \pi} \frac{i_{1} i_{2}}{r}$

According to Fleming's left hand rule, the force $F_{y}$ will be in the plane of the paper and it is directed towards the conductor X. Similarly the force on conductor X for length $l$ is given by

$$
\mathrm{F}_{\mathrm{x}}=\frac{\mu_{0}}{2 \pi} \frac{\mathrm{i}_{1} \mathrm{i}_{2}}{\mathrm{r}} l
$$

This force will be in the plane of the paper and is directed towards conductor Y. Hence the two conductors attract each other and the force of attraction per unit length would be

$$
\mathrm{F}=\frac{\mu_{0}}{2 \pi} \frac{\mathrm{i}_{1} \mathrm{i}_{2}}{\mathrm{r}}
$$

Thus if current through two parallel conductors are in the same direction they attract each other, if the currents are in opposite directions the conductors repel each other.
2. Obtain an expression for the torque on a current loop placed in a uniform magnetic field. Describe the construction and working of a moving coil galvanometer?

## A Torque on a current loop in a magnetic field:

Consider a rectangular current carrying coil suspended in uniform magnetic field of induction B. Let ' $l$ ' be the length and ' $b$ ' is the breadth of the coil. ' $A$ ' is the area of cross - section and ' N ' be the number of turns of the coil. Let 'I' be the current passing through the coil.

According to Fleming's left hand rule, the arms QR and PS of the coil experience a force
F = B I $l \mathrm{~N}$ each. These two equal, opposite, parallel and non - collinear forces forms a couple.
Hence the torque on the coil is given by
Torque $\tau=\mathrm{F} \times$ perpendicular distance. $=(\mathrm{BI} / \mathrm{N})(\mathrm{b})=\mathrm{B}$ IA n
If ' $\theta$ ' is the angle made by the normal to the plane of the coil with magnetic field, then
Torque on the coil $\tau=B I A N \sin \theta$
In vectorial form, $\vec{\tau}=N I(\vec{A} \times \vec{B})$


## Moving Coil Galvanometer:

## Principle:

When a rectangular current carrying coil is suspended in uniform magnetic field, it experiences a torque.

## Construction:

It consists of a permanent horse-shoe magnet (NS) with concave shaped magnetic poles. These poles produce radial magnetic field. A rectangular coil of ' N ' turns of thin insulated copper wire is suspended between the poles of the magnet. A cylindrical soft iron core is placed inside the coil without touching it. The coil is suspended by means of a fine phosphor bronze wire ( P ) from a torsion head $\left(\mathrm{T}_{1}\right)$. Phosphor bronze wire has low rigidity modulus and high Young's modulus. Hence it can easily be twisted and cannot be elongated. Lower end of the coil is connected to a binding screw $\left(\mathrm{T}_{2}\right)$ through a phosphor bronze spring $(\mathrm{G})$.

A small circular plane mirror $(\mathrm{M})$ is attached to the suspension wire. It is used to find the deflection in the coil with the help of lamp and scale arrangement.


## Working:

Let 'I' be the current through the coil.
Deflecting torque on the coil=BIAN.
Let k be the restoring torque per unit twist.
Restoring torque $=\mathrm{k} \phi$
In equilibrium position, deflecting couple= restoring couple
Or B IA n $=\mathrm{k} \phi \quad$ Or $\quad I=\left(\frac{k}{B A n}\right) \phi$
Hence the current through the galvanometer is proportional to the deflection in the coil.
3. How can a galvanometer be converted to an ammeter? Why is the parallel resistance smaller that the galvanometer resistance?

## A. Conversion of Galvanometer into an Ammeter:

A galvanometer can be converted into an ammeter by connecting a suitable small resistance in parallel called shunt resistance. Ammeter is used for measuring the current in an electric circuit.

Let ' $R_{G}$ ' be the resistance of the galvanometer and ' $r_{s}$ ' be the shunt resistance. Let ' $R$ ' be the effective resistance. Then,

$$
R=\frac{R_{G} r_{S}}{R_{G}+r_{S}} \text { And } \quad V=i_{g} R_{G}=i_{s} r_{S}=i R
$$



Conversion of galvanometer into ammeter

$$
\text { Expression for } \mathbf{i}_{\mathbf{g}}: \quad i_{g}=\frac{i r_{S}}{R_{G}+r_{S}}
$$

Expression for $\mathbf{i}_{\mathbf{S}}$ : $\quad i_{s}=\frac{i R_{G}}{R_{G}+r_{S}}$
Expression for shunt: $\quad r_{S}=\frac{i_{g} R_{G}}{i-i_{g}}$

Or $r_{s}=\frac{R_{G}}{n-1}$ Where $n=\frac{i}{i_{g}}$ is called the range of the ammeter.

## Necessity of Small Parallel Resistance:

The current or total resistance of circuit should not change due to the presence of ammeter. Therefore the ammeter should have very low resistance. If the parallel resistance is smaller than the galvanometer, more current flows through the shunt. Hence the galvanometer can be protected from high currents.
4. How can a galvanometer be converted to a voltmeter? Why the series resistance is is greater than the galvanometer resistance?

## A. Conversion of Galvanometer into Voltmeter:

Voltmeter is used to measure the potential difference between any two points in a circuit. A galvanometer can be converted into voltmeter by connecting a suitable high resistance in series with it.

Let ' $R_{G}$ ' be the resistance of the galvanometer and ' $r$ ' be the parallel resistance. Let ' $R$ ' be the effective resistance. $\quad R=r+R_{G}$


Then from the figure, the potential difference between A and B is given by,
$V=i_{g}\left[r+R_{G}\right]$ Or $r=R_{G}(n-1)$
Where $n=\frac{V}{i_{g} R_{G}}$ is called the range of the voltmeter.

## Necessity of High Series Resistance:

Voltmeter has to draw a negligible amount of current from the main current. Therefore it should have very high resistance. Thus, series resistance is greater than galvanometer resistance.
5. Derive an expression for the force acting between two very long parallel current carrying conductors and hence the define ampere?
A. Derivation:

Consider two infinitely long parallel conductors $X$ and $Y$ carrying currents $i_{1}$ and $i_{2}$ in same direction as shown in the figure. Let 'r' be the separation between the conductors in the plane of the paper.

The magnitude of magnetic Induction field at any point P on the conductor Y due to current $\mathrm{i}_{1}$ in conductor X is $B_{X}=\frac{\mu_{0}}{2 \pi} \frac{i_{1}}{r}$.

The direction of $\mathrm{B}_{\mathrm{X}}$ is perpendicular to the plane of the paper and is directed inwards


Now imagine the conductor Y carrying current $\mathrm{i}_{2}$ in this magnetic induction field $\mathrm{B}_{\mathrm{X}}$.
Force for length $l$ of conductor $Y$ is given by $F_{y}=B_{x} i_{2} 1 \sin \theta=B_{x} i_{2} 1 \quad\left(\because \theta=90^{\circ}\right)$
Now $\mathrm{F}_{\mathrm{y}}=\frac{\mu_{0}}{2 \pi} \frac{\mathrm{i}_{1} \mathrm{i}_{2}}{\mathrm{r}} l$
Force per unit length $=\frac{\mu_{0}}{2 \pi} \frac{i_{1} i_{2}}{r}$
According to Fleming's left hand rule, the force $\mathrm{F}_{\mathrm{y}}$ will be in the plane of the paper and it is directed towards the conductor $X$. Similarly the force on conductor $X$ for length $l$ is given by

$$
\mathrm{F}_{\mathrm{x}}=\frac{\mu_{0}}{2 \pi} \frac{\mathrm{i}_{1} \mathrm{i}_{2}}{\mathrm{r}} l
$$

This force will be in the plane of the paper and is directed towards conductor Y. Hence the two conductors attract each other and the force of attraction per unit length would be

$$
\mathrm{F}=\frac{\mu_{0}}{2 \pi} \frac{i_{1} i_{2}}{\mathrm{r}}
$$

Thus if current through two parallel conductors are in the same direction they attract each other; if the currents are in opposite directions the conductors repel each other.

## Definition of Ampere:

If $\mathrm{i}_{1}=\mathrm{i}_{2}=1 \mathrm{~A}$
$\mathrm{r}=1 \mathrm{~m}, \mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Then $\mathrm{F}=2 \times 10^{-7} \mathrm{~N} / \mathrm{m}$,

Ampere is that current which when flowing through each of two parallel conductors of infinite length and placed in free space (or vacuum) at a distance of one metre from each other produces between them a force of $2 \times 10^{-7}$ Newton per meter length of each conductor.

## PROBLEMS

1. A current of 10 A passes through two very long wires held parallel to each other and separated by a distance of 1 m . What is the force per unit length between them?
A. $\mathrm{i}_{1}=\mathrm{i}_{2}=10 \mathrm{~A} ; \mathrm{d}=1 \mathrm{~m} ; \mathrm{F}=? \frac{\mathrm{~F}}{l}=\frac{\mu_{0}}{2 \pi} \frac{\mathrm{i}_{1} \mathrm{i}_{2}}{\mathrm{r}}=\frac{2 \times 10^{-7} \times 10 \times 10}{1}=2 \times 10^{-5} \mathrm{~N} / \mathrm{m}$
2. A moving coil galvanometer can measure a current of $10^{-6} \mathrm{~A}$. What is resistance of the shunt required to measure 1 A ?
A. $i=1 \mathrm{~A} ; \mathrm{i}_{\mathrm{g}}=10^{-6} \mathrm{~A} ; \mathrm{G}=1 \mathrm{ohm}$.
$S=\frac{G \times i_{g}}{\left(i-i_{g}\right)}=\frac{1 \times 10^{-6}}{\left(1-10^{-6}\right)} \approx 10^{-6} \mathrm{ohm}$
3. A circular wire loop of radius 30 cm carries a current of 3.5 A . Find the magnetic field at a point on its axis 40 cm away from the centre?
A. $n=1 ; i=3.5 \mathrm{~A} ; \mu_{0}=4 \pi \times 10^{-7} \mathrm{Hm}^{-1}$;

$$
\mathrm{x}=40 \mathrm{~cm}=40 \times 10^{-2} \mathrm{~m}
$$

$$
\mathrm{B}=\frac{\mu_{0} \mathrm{nir}^{2}}{2\left(\mathrm{r}^{2}+\mathrm{x}^{2}\right)^{3 / 2}}
$$

$\therefore B=\frac{4 \pi \times 10^{-7} \times 3.5 \times\left(30 \times 10^{-2}\right)^{2}}{2 \times\left[\left(30 \times 10^{-2}\right)^{2}+\left(40 \times 10^{-2}\right)^{2}\right]^{3 / 2}}=15.84 \times 10^{-7} \mathrm{~T}$
4. A circular coil of wire consisting of 100 turns each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field $B$ at the centre of the coil?
A. $N=100, r=8 \mathrm{~cm}=0.8 \mathrm{~m}, I=0.40 \mathrm{~A}$
$\therefore B=\frac{\mu_{0} N I}{2 r}=\frac{4 \pi \times 10^{-7} \times 100 \times 0.40}{2 \times 0.08}=3.1 \times 10^{-4} T$.
5. A long straight wire carries a current of 35 A . What is the magnitude of the field $B$ at a point 20 cm from the wire?
A. $I=35 A, r=20 \mathrm{~cm}=0.20 \mathrm{~m}$
$\mu=4 \pi \times 10^{-7} T m A^{-1}$
$B=\frac{\mu_{0} I}{2 \pi r}=\frac{4 \pi \times 10^{-7} \times 35}{2 \pi \times 0.20}=3.5 \times 10^{-5} \mathrm{~T}$.
6. A long straight wire in the horizontal plane carries a current of 50 A in the north to south direction. Give the magnitude and direction of $B$ at a point 2.5 m east of the wire?
A. $\quad I=50 A, r=2.5 \mathrm{~m}$
$B=\frac{\mu_{0} I}{2 \pi r}=\frac{4 \pi \times 10^{-7} \times 50}{2 \pi \times 2.5}=4 \times 10^{-6} T . \quad$ (Vertically upwards)
7. A horizontal over head power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?
A. $\quad I=90 \mathrm{~A}, r=1.5 \mathrm{~m}, \mu=4 \pi \times 10^{-7} \mathrm{TmA}^{-1}$
$B=\frac{\mu_{0} I}{2 \pi r}=\frac{4 \pi \times 10^{-7} \times 90}{2 \pi \times 1.5} T=1.2 \times 10^{-5} \mathrm{~T}$. (Towards south)
8. What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of $30^{\circ}$ with the direction of a uniform magnetic field of 0.15 T ?
A. $\quad I=8 A, \theta=30^{\circ}, B=0.15 T$

As $F=I l B \sin \theta$
Force per unit length,
$=\frac{F}{l}=I B \sin \theta=8 \times 0.15 \times \sin 30^{\circ}=0.6 \mathrm{Nm}^{-1}$.
9. A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T . What is the magnetic force on the wire?
A. $\quad l=3.0 \mathrm{~cm}=0.03 \mathrm{~m}, I=10 \mathrm{~A}$,

$$
\theta=90^{\circ}, B=0.27 T
$$



$$
F=I l B \sin \theta=10 \times 0.03 \times 0.27 \times \sin 90^{\circ}=8.1 \times 10^{-2} \mathrm{~N} .
$$

10. Two long and parallel straight wires $A$ and $B$ carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm . Estimate the force on a 10 cm section of wire $A$ ?
A. Force per unit length of each wire is

$$
F=\frac{\mu_{0} I_{1} I_{2}}{2 \pi r}=\frac{4 \pi \times 10^{-7} \times 8 \times 5}{2 \pi \times 4 \times 10^{-2}}=2 \times 10^{-4} \mathrm{Nm}^{-1}
$$



Force on 10 cm section of wire A, $F l=2 \times 10^{-4} \times 10 \times 10^{-2}=2 \times 10^{-5} \mathrm{~N}$.
11. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm . If the current carried is 8.0 A , estimate the magnitude of inside the solenoid near its centre?
A. Number of turns per unit length of the solenoid is

$$
n=\frac{\text { Number of turns per layer } \times \text { Number of layers }}{\text { Length of solenoid }}=\frac{400 \times 5}{0.80}=2500 \mathrm{~m}^{-1}
$$

Magnetic field inside the solenoid is

$$
B=\mu_{0} n I=4 \pi \times 10^{-7} \times 2500 \times 8=8 \pi \times 10^{-3} T=2.5 \times 10^{-2} T
$$

12. A square coil of side 10 cm consists of 20 turns and carries a current of 12 A . The coil is suspended vertically and the normal to the plane of the coil and makes an angle of $30^{\circ}$ with the direction of a uniform horizontal magnetic field of magnitude 0.80 T . What is the magnitude of torque experience by the coil?
A. $A=0.10 m \times 0.10 \mathrm{~m}=0.01 \mathrm{~m}^{2}, N=20$,
$I=12 A, \theta=30^{\circ}, B=0.80 T$

Torque on the coil

```
\tau=NIBA亪 }
=20\times12\times0.80\times0.01\times\operatorname{sin}3\mp@subsup{0}{}{\circ}=0.96 Nm}
```

13. Two moving coil meters have the following particulars:

$$
R_{1}=10 \Omega, N_{1}=30, A_{1}=3.6 \times 10^{-3} m^{2}, B_{1}=0.25 T \quad R_{2}=10 \Omega, N_{2}=42, A_{2}=1.8 \times 10^{-3} m^{2}, B_{2}=0.50 T
$$

The spring constants are identical for the two meters. Determine the ratio of
(i) Current sensitivity and (ii) Voltage Sensitivity of $M_{2}$ and $M_{1}$.

## Current Sensitivity

A. $\frac{\alpha}{I}=\frac{N B A}{k}$
$\therefore \frac{\text { Current sensitivity of } M_{2}}{\text { Current sensitivity of } M_{1}}=\frac{N_{2} B_{2} A_{2} / k}{N_{1} B_{1} A_{1} / k}=\frac{N_{2} B_{2} A_{2}}{N_{1} B_{1} A_{1}}=\frac{42 \times 0.50 \times 1.8 \times 10^{-3}}{30 \times 0.25 \times 3.6 \times 10^{-3}}=1.4$
Voltage sensitivity $\frac{\alpha}{V}=\frac{\alpha}{I R}=\frac{N B A}{k R}$
$\frac{\text { Voltage sensitivity of } M_{2}}{\text { Voltage sensitivity of } M_{1}}=\frac{N_{2} B_{2} A_{2} / k R_{2}}{N_{1} B_{1} A_{1} / k R_{1}}=\frac{N_{2} B_{2} A_{2}}{N_{1} B_{1} A_{1}} \times \frac{R_{1}}{R_{2}}=\frac{7}{5} \times \frac{10}{14}=1$
14. In a chamber, a uniform magnetic field of $6.5 \mathrm{G}\left(1 G=10^{-4} T\right)$ is maintained. An electron is shot into the field with a speed of $4.8 \times 10^{6} \mathrm{~ms}^{-1}$ normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. Given that ( $\left.e=1.6 \times 10^{-19} \mathrm{C}, m_{e}=9.1 \times 10^{-31} \mathrm{~kg}.\right)$
A. The perpendicular magnetic field exerts a force on the electron perpendicular to its path and makes it move along a circular path.
$\therefore$ Magnetic force on the electron $=$ Centripetal force

$$
e v B \sin 90^{\circ}=\frac{m_{e} v^{2}}{r} \quad \text { Or } \quad r=\frac{m_{e} v}{e B}
$$

Now $B=6.5 \mathrm{G} \times 10^{-4} T, v=4.8 \times 10^{6} \mathrm{~ms}^{-1}$
$\therefore r=\frac{4.8 \times 10^{-31} \times 4.8 \times 10^{6}}{1.6 \times 10^{-19} \times 6.5 \times 10^{-4}}=4.2 \times 10^{-2} \mathrm{~m}=4.2 \mathrm{~cm}$

