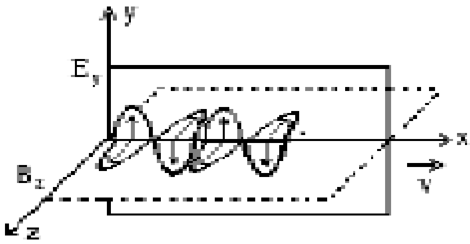
ELECTROMAGNETIC WAVES

Important Points:

1. An accelerated charge produces a sinusoidally varying magnetic field which in turn produces sinusoidal time varying electric field. These two fields are mutually perpendicular to each other.



The electric and magnetic fields shown in the above figure are mathematically represented by

i) $\overline{E}$ = Ey = E0 sin [kx - wt] $= E_0 \sin 2\pi\left(\frac{x}{\lambda} - vt\right)$

Ex = Ez = 0

ii) $\overline{B}$ = Bz = B0 sin [kx - wt] = B0 sin $2\pi\left(\frac{x}{\lambda} - vt\right)$

Bx = By = 0

2. The mutually perpendicular electric & magnetic field constitutes electromagnetic waves which can propagate through empty space. The velocity of the electromagnetic wave in vacuum is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

3. In any isotropic medium the velocity is given by $v = \frac{1}{\sqrt{\mu\varepsilon}}$

4. **Displacement Current:**

The current following due to the varying electric field but not due to the actual flow of charges is called displacement current.

i) $i_d = \varepsilon_0 \frac{dq_e}{dt}$

ii) $i_d = \in_0 A\frac{dE}{dt}$ Where $\frac{dE}{dt}$ is variable electrical field

5. Maxwell's Displacement Current:

i) The rate of change of electrical flux produces a current called displacement current "i d".

ii) The displacement current is also called "induced magnetic field".

iii) Maxwell made the laws of electricity and magnetism symmetrical with the help of displacement current.

iv) Unlike conduction current displacement current exists where there is rate of change of electrical flux.

v) The displacement current is found between the plates of a condenser during its charging or discharging.

vi) It is also found between the plates of a condenser when AC is applied.

vii) It is called current because it produces a magnetic field.

6. Displacement Current:

i) When a charging current "i" which is constant is given, "i d" the displacement current = charging current "i".

ii) When a variable electrical field is applied to the gap $i_d = A\in_0 \frac{dE}{dt}$.

iii) When a variable potential difference is applied to the plates of a condenser of capacity C $i_d = C\frac{dv}{dt}$

vi) Ampere - Maxwell's law or Amperes Law is modified by Maxwell.

v) $\int \vec{B}.\vec{dl} = \mu_0 (i_c + i_d)$ And $\int \vec{B}.\vec{dl} = \mu_0 \left(i_c + \in_0 \frac{d\phi_E}{dt}\right)$

ic = The conduction current found in a conductor carrying current

id = Displacement current which is found between the plates of a condenser which is discontinuous.

7. Maxwell's Equations:

i) $\int \vec{E}.d\vec{A} = q_{net} / \varepsilon_0$ [Gauss law in magnetism]

ii) $\oint B.ds = 0$ [Gauss law in magnetism]

iii) $\int \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$ [Faraday's law]

vi) $\int \vec{B}.d\vec{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 (i_c + i_d)$ [Ampere-Maxwell law]

8. Poynting Vector:

The rate of flow of energy in an electromagnetic wave is described by a vector called pointing vector and it is given by the expression .Its unit is $\vec{S} = \frac{1}{\mu_0}[\vec{\in} \times \vec{B}]$ Wm^{-2}

9. Energy density in electric field: $E_d = \frac{1}{2} \in_0 \in^2$

10. Energy density in magnetic field: $E_d = \frac{B^2}{2\mu_0}$

11. Total energy density: $(E_d)_T = \frac{1}{2}\left[\in_0 \in^2 + \frac{B^2}{\mu_0}\right]$

12. Radiation Pressure:

i) When electromagnetic waves incident on any surface the pressure exerted on the surface is called Radiation Pressure.

ii) If a portion of electromagnetic wave is propagating with speed c, then the linear momentum of electromagnetic wave is $P = \frac{U}{c}$

Where U is the total energy transferred to the surface in a time t.

Very Short Answer Questions

1. What is the average wavelength of X-rays?

A. X-rays have wavelengths in the range from approximately 10^{-8} m to 10^{-12} m, so the average wavelength of X-rays is 10^{-10} m.

2. Give any one use of infrared rays?

A. Infrared rays from the sun keep the Earth warm and hence help to sustain life on Earth. Infrared lamps are used in physical therapy.

3. If the wavelength of electromagnetic radiation is doubled. What happens to the energy of photon?

A. Energy of electromagnetic radiation, $E = \frac{hc}{\lambda}$

$$E\alpha\frac{1}{\lambda} \Rightarrow \frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{E}{E_2} = \frac{2\lambda}{\lambda} \Rightarrow E_2 = \frac{E}{2}$$

As the wavelength increases, energy of photon decreases. When wavelength is doubled then energy is halved.

4. What is the principle of production of electromagnetic waves?

A. Accelerated charges radiate energy in the form of electromagnetic waves. So it is the source of an electromagnetic wave.

5. What is the ratio of speed of infrared rays and ultraviolet rays in vacuum?

A. Both infrared rays and ultraviolet rays travel with a speed of light in vacuum, hence their ratio is 1:1.

6. What is the relation between the amplitudes of the electric and magnetic fields in free space for an electromagnetic wave?

A. The components of the electric and magnetic fields of plane electromagnetic waves are given by-

$$\overline{E} = E_0 \cos(kx - \omega t)$$
$$\overline{B} = B_0 \cos(kx - \omega t)$$

$\frac{E_0}{B_0} = C =$ Speed of Light

7. What are applications of microwaves?

A. Microwaves are used in -

a) In radar and telecommunications.

b) To analyze the fine details of the molecular structure.

c) Basing on the microwaves, speed guns are designed which are used to time fast balls, and in Tennis serves and automobiles.

d) Microwave Owen is a domestic appliance to cook the food items.

8. Microwaves are used in radars. Why?

A. As the wavelength of microwaves is short, these are used in radar systems which are used in aircraft navigation.

9. Give two uses of infrared rays?

A. i) Infrared rays from the sun keep the Earth warm and hence help to sustain life on Earth.

ii) Infrared rays are used in solar water heaters and cookers.

10. The charging current for a capacitor is 0.6 A. What is the displacement current across its plates?

A. 0.6 A

Short Answer Questions

1. What does an electromagnetic wave consists of? On what factors does its velocity in vacuum, depend?

A. Electromagnetic wave consists of sinusoidally oscillating electric and magnetic fields in space and time. The oscillating electric and magnetic fields $\bar{E}$ and $\bar{B}$ are perpendicular to each other and to the direction of propagation of the electromagnetic wave.

Velocity of electromagnetic wave in vacuum depends on the amplitudes of the electric and magnetic fields E_0 and B_0.

$$C = \frac{E_0}{B_0}$$

2. What is Greenhouse effect and its contribution towards the surface temperature of Earth?

A. **Greenhouse Effect:**

The Earth surface is a source of thermal radiation as it absorbs energy received from sun. The wave length of this radiation lies in the infrared region. But a large portion of this radiation is absorbed by greenhouse gases like Co_2, CH_4, N_2O, O_3. This heats up the atmosphere which in turn gives more energy to Earth. As a result the surface of Earth becomes warmer. This increases the intensity of radiation from the surface. This process is repeated until no radiation is available for absorption. The net result is heating up to Earth's surface and atmosphere this is known as greenhouse effect. Without the green house effect the temperature of the Earth is $-18^0 C$.

Concentration of greenhouse gases has enhanced due to human activities. As a result the average temperature of Earth has increased by the middle of the next century the temperature may be increased by .This global warming may cause problems for human life, plants & animals

Infrared radiation plays an important role in maintaining the average temperature of the Earth through the greenhouse effect. Incoming visible light is absorbed by the Earth's surface and re-radiated as infrared radiations. This radiation is trapped by the greenhouse gasses like carbon dioxide and water vapour. The carbon dioxide strongly absorbs infrared and does not allow as much of it to escape into space.

Long Answer Questions

1. Give the brief history of discovery of knowledge of electromagnetic waves?

A. History of Electromagnetic Waves:

i) In the 1865, Maxwell predicted the electromagnetic waves theoretically. According to him, an accelerated charge sets up a magnetic field in its neighborhood.

ii) In 1887, Hertz produced and detected electromagnetic waves experimentally at wavelength of about 6m.

iii) Latter J.C. Bose became successfully produced electromagnetic waves of wavelength in the range 5mm to 25mm.

iv) In 1896, Marconi discovered that if one of the spark gap terminals is connected to an antenna and the other terminal is earthed, the electromagnetic waves radiated could go up to several kilometers.

v) The antenna and the Earth wires form the two plates of a capacitor which radiates radio frequency waves. These waves could be received at a large distance by making use of an antenna Earth system as detector.

vi) Using these arrangements in 1899 Marconi first established wireless communication across the English Channel i.e. across a distance of about 50 km.

2. State six characteristics of electromagnetic waves. What is Green house effect?

A. Characteristics of Electromagnetic Waves:

i) Electromagnetic wave consists of sinusoidally oscillating electric and magnetic fields in space and time. The oscillating electric and magnetic fields and are perpendicular to each other and to the direction of propagation of the electromagnetic wave.

ii) Velocity of electromagnetic wave in vacuum depends on the amplitudes of the electric and magnetic fields E_0 and B_0, $C = \frac{E_0}{B_0}$

iii) Electromagnetic waves travel through vacuum with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

Where μ_0 = permeability of free space

ε_0 = permittivity of free space

iv) Electromagnetic waves have linear momentum as well as energy.

v) Electromagnetic waves obey the principle of superposition.

vi) Electromagnetic radiation is classified by wavelength into radio wave, microwave, infrared, visible light, ultraviolet, X-rays and gamma rays.

Green House Effect:

The Earth surface is a source of thermal radiation as it absorbs energy received from sun. The wave length of this radiation lies in the infrared region. But a large portion of this radiation is absorbed by greenhouse gases like Co_2, CH_4, N_2O, O_3. This heats up the atmosphere which in turn gives more energy to Earth. As a result the surface of Earth becomes warmer. This increases the intensity of radiation from the surface. This process is repeated until no radiation is available for absorption. The net result is heating up to Earth's surface and atmosphere this is known as greenhouse effect. Without the green house effect the temperature of the Earth is -18^0C.

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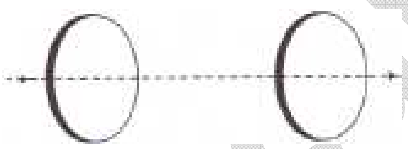
PROBLEMS

1. Figure shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 cm. The capacitor is being charged by external sources (not shown in the figure). The charging current is constant and equal to 0.15 A.

(a) Calculate the capacitance and the rate of charge of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain?



Use the concept of parallel plate capacitor and capacitance of the capacitor $C = \frac{K\varepsilon_0 A}{d}$

A. Radius of plates R = 12 cm = 12×10^{-2} m

Separation of two circular plates d = 5 cm = 5×10^{-2} m

Current I = 0.15 A

(a) Capacity of parallel plate capacitor $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \pi r^2}{d}$

Where, A is the area of plates.

$$C = \frac{8.854 \times 10^{-12} \times 3.14 (12 \times 10^{-2})^2}{5 \times 10^{-2}}$$

$$C = \frac{8.854 \times 3.14 \times 144 \times 10^{-12} \times 10^{-4}}{5 \times 10^{-2}} = 8.01 \times 10^{-12} F = 8.01 \text{ pF}$$

Charge on the plates of the capacitor q = CV

$$\frac{dq}{dt} = C.\frac{dV}{dt}$$

$$I = C.\frac{dV}{dt} \qquad \left[\because \frac{dq}{dt} = I\right]$$

$$\frac{dV}{dt} = \frac{I}{C} = \frac{0.15}{8.01\times10^{-12}} = 18.7 \text{ x } 10^9 \text{ V/s.}$$

Thus, the rate of change of potential is 18.7 x 10^9 V/s.

(b) The displacement current is equal to the conduction current I_d = 0.15 A.

(c) Yes, Kirchhoff's first rule is valid because we take the current to be the sum of conduction currents and the displacement currents.

2. A parallel plate capacitor (shown in figure) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V AC supply with a (angular) frequency of 300 rad/s?

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

A. Radius of plates R = 6 cm = 6 x 10^{-2} m

Capacitance of capacitor C = 100 pF = 100 x 10^{-12} F = 10^{-10} F

Voltage of capacitor V = 230 V

Frequency of capacitance = 300 rad/s.

(a) The rms value of current $I_{rms} = \frac{V_{rms}}{X_C}$

$$X_C = \frac{1}{\omega C} = \frac{1}{300\times10^{-10}} = \frac{10^{10}}{300}\Omega$$

$$I_{rms} = \frac{230\times300}{10^{10}}$$

$$= 69 \text{ x } 10^{-7} = 6.9 \text{ x } 10^{-6} \text{ A}$$

$$I_{rms} = 6.9\,\mu\text{A.}$$

(b) Yes, the conduction current is equal to displacement current

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} \text{ (By the definition of displacement current)}$$

$$I_d = \varepsilon_0 \frac{d}{dt}(EA) \quad (\phi_E = EA)$$

$$I_d = \varepsilon_0 A \frac{dE}{dt} \quad \left(E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}\right)$$

$$I_d = \varepsilon_0 A \frac{d}{dt}\left(\frac{Q}{\varepsilon_0 A}\right)$$

$$I_d = \varepsilon_0 A . \frac{1}{\varepsilon_0 A} . \frac{dQ}{dt} = \frac{dQ}{dt} = I \Rightarrow I_d = I.$$

(c) The distance of point from the axis between the plates

$$r = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$$

Radius of plates R = 6 cm = 6×10^{-2} m

The magnetic field at a point between the plates

$$B = \frac{\mu_0}{2\pi R^2} . r . I_d$$

$$B = \frac{\mu_0 r}{2\pi R^2} I \quad (I_d = I)$$

If $I = I_0$, maximum value of current then $I = \sqrt{2}\, I_{rms}$

$$B = \frac{\mu_0 r}{2\pi R^2} \sqrt{2}\, I_{rms}$$

$$B = \frac{4\pi \times 10^{-7} \times 0.03 \times \sqrt{2} \times 6.9 \times 10^{-6}}{2\pi \times 0.06 \times 0.06}$$

$$B = 1.63 \times 10^{-11} \text{ T.}$$

3. What physical quantity is the same for X-rays of wavelength $10^{-10}m$ red light of wavelength 6800 A⁰ and radio waves of wavelength 500 m?

A. The wave speed in vacuum is the same for all radiations $(c = 310^{8} ms^{-1})$.

4. A plane electromagnetic wave travels in vacuum along Z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

A. Vectors $\vec{E}\, and\, \vec{B}$ lie in x-y plane in mutually perpendicular directions.

Frequency, $v = 30\ MHz = 30 \times 10^{6}\ Hz$

$$\therefore Wavelength, \lambda = \frac{c}{v} = \frac{3\times10^{8}}{30\times10^{6}} = 10m.$$

5. A radio can tune to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

A. $v_1 = 7.5MHz = 7.5\times10^{6}\, Hz$

$v_2 = 12MHz = 12\times10^{6}\, Hz$

$$\therefore\ \lambda_1 = \frac{c}{v_1} = \frac{3\times10^{8}}{7.5\times10^{6}} = 40m$$

$$\lambda_2 = \frac{c}{v^2} = \frac{3\times10^{8}}{12\times10^{6}} = 25m$$

Thus the wavelength band is 40m-25m.

6. A charged particle oscillates about its mean equilibrium position with a frequency of $10^{9}\, Hz.$ What is the frequency of the electromagnetic waves produced by the oscillator?

A. According to Maxwell, a charged particle oscillating with 19 frequency of $10^{9}\, Hz.$, produces electromagnetic waves of the same frequency $10^{9}\, Hz.$

7. **The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is** $B_0 = 510nT$. **What is the amplitude of the electric field part of the wave?**

A. $B_0 = 510\ nT = 510 \times 10^{-9} T$

Amplitude of the electric field,

$E_0 = cB_0 = 3\times10^8 \times 510\times10^{-9} = 153NC^{-1}$

8. **Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/C and that its frequency is f = 50.0 MHz. (a) Determine, B_0, ω, k and λ. (b) Find expression for E and B?**

A. Amplitude of an electromagnetic wave, E_0 = 120 N/C

Frequency of wave f = 500 MHz = 50 x 10^6 Hz

(a) Speed of light in vacuum c = E_0/B_0

$$B_0 = E_0/c = \frac{120}{3\times10^8} = 40 \times 10^{-8}$$

Or $B_0 = 400 \times 10^{-9}$ T = 400 nT

Angular frequency of wave

$$\omega = 2\pi f = 2 \times 3.14 \times 50 \times 10^6$$

$$\omega = 3.14 \times 10^8 \text{ rad/s}$$

Wave number of electromagnetic waves

$$K = \omega/c = \frac{3.14\times10^8}{3\times10^8} = 1.05 \text{ rad/m}$$

Wavelength of electromagnetic wave

$$\lambda = c/f = \frac{3\times10^8}{50\times10^6} = 6.00 \text{ m.}$$

(b) Expression of electric field E = E_0 sin (kx - ωt)

$$E = 120 \sin (1.05 \times 3.14 \times 10^8 t)$$

Expression of magnetic field B

$$B = B_0 \sin(kx - \omega t)$$

$B = 4 \times 10^{-7} \sin (1.05 \times 3.14 \times 10^{8} t)$

9. The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula E = hν (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

A. Energy of photon E = hν

$$= \frac{hc}{\lambda} = \frac{12400\, A^0}{\lambda\ in\ A^0} eV$$

(i) For γ rays $\lambda = 10^{-12} m$

$$= 10^{-10} \times 10^{-2} m$$

$$= 10^{-2} A^0$$

$$\therefore E = \frac{12400}{10^{-2}} eV = 12.4 \times 10^5 eV$$

(ii) For x - rays $\lambda = 10^{-10} m$

$$= 1 A^0$$

$$\therefore E = \frac{12400}{1} eV$$

$$= 12400\, eV = 12.4 \times 10^3 eV$$

(iii) For visible rays, $\lambda = 10^{-6} m$

$$= 10^{-10} \times 10^4 m$$

$$= 10^4 A^0$$

$$\therefore E = \frac{12400}{10^4} eV = 124 \times 10^2 \times 10^{-4}$$

$$= 124 \times 10^{-2} = 12.4 \times 10^{-1} eV$$

(iv) For infrared rays, $\lambda = 10^{-5} m$

$$= 10^{-10} \times 10^5 m$$

$$= 10^5 A^0$$

$$E = \frac{12400}{10^5} ev$$

$=12400\times10^{-5}eV$

$=124\times10^{-3}eV=12.4\times10^{-2}eV$

(v) For microwaves, $\lambda=10^{-2}\ m$

$$\lambda=10^{-10}\times10^{8}\ m=10^{8}\ A^{0}$$

$$E=\frac{12400}{10^{8}A^{0}}eV=12.4\times10^{-5}eV$$

10. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0 x 10^{10} Hz and amplitude 48 V/m

(a) What is the wavelength of the wave?.

(b) What is the amplitude of the oscillating magnetic field?

(c) Show that the average energy density of the E field equals the average energy density of the B field. [c = 3 x 10^{8} m/s]

A. Frequency of oscillation = 2 x 10^{10} Hz

c = 3 x 10^{8} m/s

Electric field amplitude E_0 = 48 V/m

(a) Wavelength of waves $\lambda=\frac{c}{f}=\frac{3\times10^{8}}{2\times10^{10}}$ = 1.5 x 10^{-2} m.

(b) Using the formula, $c = E_0/B_0$

The amplitude of the oscillating magnetic field

$$B_0 = E_0/c = \frac{48}{3\times10^{8}} = 1.6\times10^{-7}\ T.$$

(c) The average energy density of electric field

$$U_E=\frac{1}{4}\varepsilon_0E_0^2 \ ......\ (i)$$

We know that $E_0/B_0 = c$

Putting in Eq. (i) $U_E=\frac{1}{4}\varepsilon_0 .c^2B_0^2$ ...... (ii)

Speed of electromagnetic waves $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Putting in Eq. (ii), we get

$$U_E = \frac{1}{4}\varepsilon_0 B_0^2 . \frac{1}{\mu_0 \varepsilon_0}$$

$$U_E = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{B_0^2}{2\mu_0} = \mu_B$$

Thus, the average energy density of the E field equals the average energy density of B field.